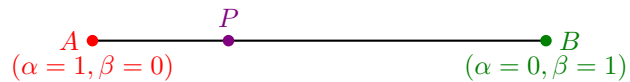


# Barycentric Coordinates

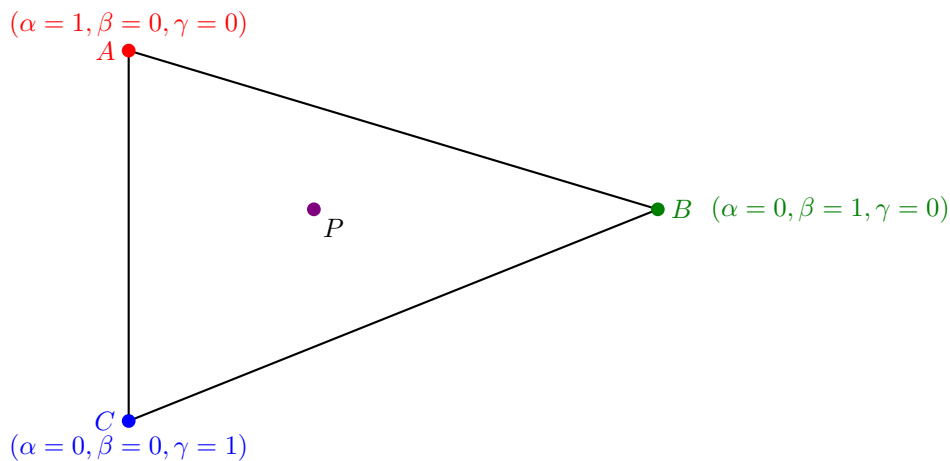
CS 130

1. Want to interpolate vertex data along a segment



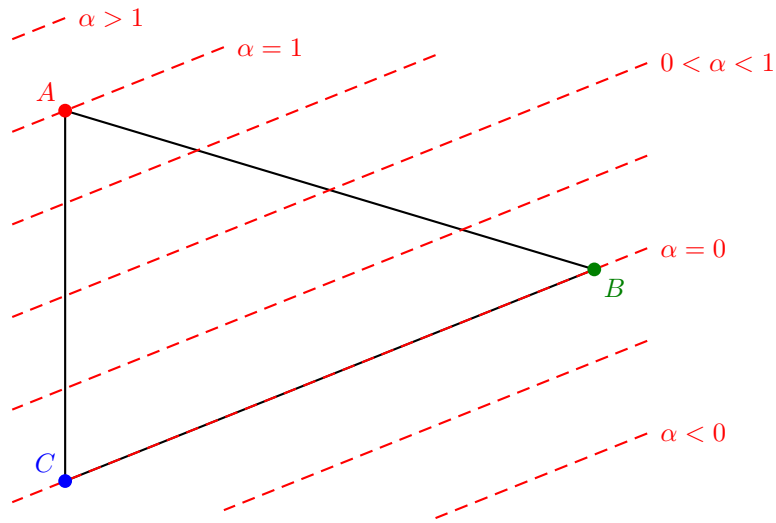
- Define  $f(\mathbf{x})$  for all points  $\mathbf{x}$  on the line
- Value at endpoints:  $f_A, f_B$ .
- Interpolation should get the endpoints right:  $f(A) = f_A, f(B) = f_B$
- $f(P) = \alpha f(A) + (1 - \alpha)f(B)$ .
- $0 \leq \alpha \leq 1$ .
- Symmetry: define  $\beta = 1 - \alpha$ .
- $f(P) = \alpha f(A) + \beta f(B)$ , with  $\alpha + \beta = 1$ .
- $\alpha = \frac{\text{len}(PB)}{\text{len}(AB)}, \beta = \frac{\text{len}(AP)}{\text{len}(AB)}$

2. Extend this to a triangle

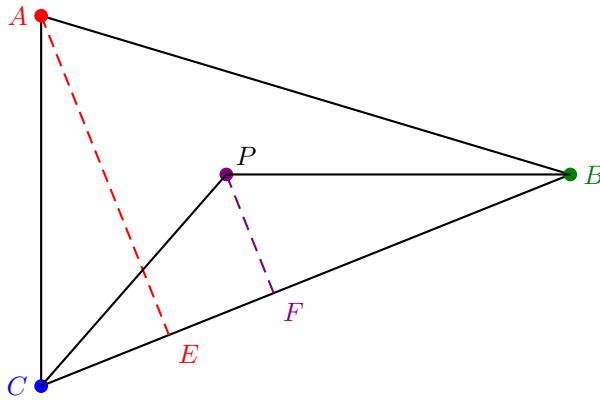


- Define  $f(\mathbf{x})$  for all points  $\mathbf{x}$  on the triangle
- Value at vertices:  $f_A, f_B, f_C$ .
- Interpolation should get the vertices right:  $f(A) = f_A, f(B) = f_B, f(C) = f_C$
- $f(P) = \alpha f(A) + \beta f(B) + \gamma f(C)$ , with  $\alpha + \beta + \gamma = 1$ .

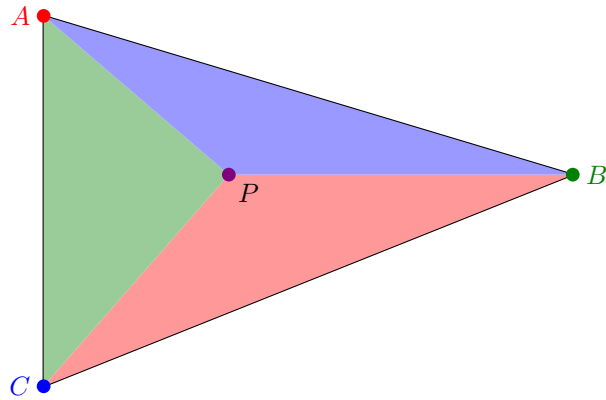
- Weights form isocontours:



- Note that  $\alpha < 0$  or  $\alpha > 1$  lies outside the triangle
- Compute using distance to edge:



- $\alpha = \frac{\text{len}(PF)}{\text{len}(AE)} = \frac{\frac{1}{2}\text{len}(PF)\text{len}(BC)}{\frac{1}{2}\text{len}(AE)\text{len}(BC)} = \frac{\text{area}(PBC)}{\text{area}(ABC)}$
- Similarly:  $\beta = \frac{\text{area}(APC)}{\text{area}(ABC)}$ ,  $\gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)}$



- Pattern of areas
- Since  $\text{area}(PBC) + \text{area}(APC) + \text{area}(ABP) = \text{area}(ABC)$ , we have  $\alpha + \beta + \gamma = 1$
- Barycentric interpolation is okay for  $z$ -values
- Barycentric interpolation is okay for colors in orthographic case
- Barycentric interpolation does not work for colors in the projective case

### 3. Inside/outside tests

- $\alpha < 0$  or  $\alpha > 1$  lies outside the triangle (Same for  $\beta < 0$  or  $\beta > 1$ ,  $\gamma < 0$  or  $\gamma > 1$ )
- Inside the triangle if  $0 \leq \alpha \leq 1$  and  $0 \leq \beta \leq 1$  and  $0 \leq \gamma \leq 1$ .
- Sufficient to check  $\alpha, \beta, \gamma \geq 0$
- For example if  $\alpha \geq 0$  and  $\beta \geq 0$  then  $\gamma = 1 - \alpha - \beta \leq 1 - \beta \leq 1$ .
- Since we need the weights to compute the depth values when doing  $z$ -buffering, we might as well also use them to determine inside/outside.