x x x x

$$\frac{\partial u}{\partial t} + (u\cdot\nabla)u = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 u + g$$ *Navier - Stokes*

$\nabla\cdot u = 0$ *incompressibility*

$$\frac{\partial u}{\partial t} = B(u) + C(u) \qquad \frac{u^{n+1} - u^n}{\Delta t} = B(u^n) + C(u^{n+1})$$

$$\frac{u^{n+1} - u^*}{\Delta t} = C(u^{n+1}) \qquad \frac{u^* - u^n}{\Delta t} = B(u^n)$$

$$\frac{u^{n+1} - u^n}{\Delta t} + (u^n\cdot\nabla)u^n = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 u^{n+1} + g$$

$$\frac{u^* - u^n}{\Delta t} + (u^n\cdot\nabla)u^n = g \qquad \frac{u^{n+1} - u^*}{\Delta t} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 u^{n+1}$$

$$\boxed{\nabla\cdot u^n = 0 \qquad \nabla\cdot u^{n+1} = 0}$$

$$\frac{\cancel{\nabla\cdot u^{n+1}} - \nabla\cdot u^*}{\Delta t} = -\nabla\cdot\left(\frac{1}{\rho}\nabla p\right) + \frac{\mu}{\rho}\underbrace{\nabla\cdot(\nabla^2 u^{n+1})}_{=\nabla^2(\cancel{\nabla\cdot u^{n+1}})}$$

$$\nabla^2 p = \frac{\rho}{\Delta t}\nabla\cdot u^*$$

$\Rightarrow$ solve for p

$$\frac{u^{n+1} - u^*}{\Delta t} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 u^{n+1}$$

<u>split</u>

$$\frac{\hat{u} - u^*}{\Delta t} = -\frac{1}{\rho}\nabla p \qquad \frac{u^{n+1} - \hat{u}}{\Delta t} = \frac{\mu}{\rho}\nabla^2 u^{n+1}$$

p is known
$\Rightarrow$ compute $\hat{u}$

heat equation
$\Rightarrow$ compute u^{n+1}

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① advection
② gravity
$\Rightarrow u^*$

③ Poisson equation for p $\Rightarrow p$

④ Apply pressure $\Rightarrow \hat{u}$

⑤ Heat equation $\Rightarrow u^{n+1}$

Navier-Stokes = advection + poisson equation + heat equation

$$\frac{\partial u}{\partial t} = a \nabla^2 u \qquad \text{heat eqn.}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = a \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} \qquad \text{FTCS}$$

$$u_i^{n+1} = \underbrace{\frac{2\Delta t a}{\Delta x^2}}_{\beta} \left(\frac{u_{n+1}^n + u_{n-1}^n}{2} \right) + u_i^n \left(1 - \frac{2\Delta t a}{\Delta x^2} \right)$$

$$u_i^{n+1} = (1-\beta) u_i^n + \beta \left(\frac{u_{i+1}^n + u_{i-1}^n}{2} \right)$$

$\downarrow$

1 0 1 0 1 0 1 0 ...

$1-\beta$ β $1-\beta$ β $1-\beta$ β $1-\beta$ ...

$$\beta = \frac{2\Delta t a}{\Delta x^2} < 1$$

$$\boxed{\Delta t < \frac{\Delta x^2}{2a}}$$

$$-1 < \beta - (1-\beta) < 1$$

$$2\beta - 1 < 1$$

$$2\beta < 2 \qquad \boxed{\beta < 1}$$

$$-1 < 2\beta - 1$$

$$\boxed{0 < \beta}$$