CS 230, Quiz 6

Solutions

You will have 10 minutes to complete this quiz. No books, notes, or other aids are permitted.

Problem 1 (4 points)

Discretize the differential equation $z' = \cos(z)$ using (1) forward Euler, (2) backward Euler, (3) midpoint rule, and (4) trapezoid rule.

$$(1) \qquad \frac{z^{n+1} - z^n}{\Delta t} = \cos(z^n)$$

$$(2) \qquad \frac{z^{n+1} - z^n}{\Delta t} = \cos(z^{n+1})$$

$$(3) \qquad \frac{z^{n+1} - z^n}{\Delta t} = \cos\left(\frac{z^{n+1} + z^n}{2}\right)$$

$$(4) \qquad \frac{z^{n+1} - z^n}{\Delta t} = \frac{\cos(z^{n+1}) + \cos(z^n)}{2}$$

Problem 2 (4 points)

For each of the discretizations below, identify the differential equation that it solves.

$$(1) \qquad \frac{u^{n+1} - u^n}{\Delta t} = 4 - c\frac{u^{n+1} + u^n}{2}$$

$$u' = 4 - cu$$

$$(2) \qquad \frac{u_i^{n+1} - u_i^n}{\Delta t} = 4 - c\frac{u_{i+1}^n - u_i^n}{\Delta x}$$

$$u_t = 4 - cu_x$$

$$(3) \qquad \frac{u_i^{n+1} - u_i^n}{\Delta t} = 4 - c\frac{u_{i+1}^n - u_{i-1}^n}{\Delta x}$$

$$u_t = 4 - 2cu_x$$

$$(4) \qquad \frac{u_i^{n+1} - u_i^n}{\Delta t} = 4 - c\frac{\frac{u_{i+1}^n - u_i^n}{\Delta x} - \frac{u_i^n - u_{i-1}^n}{\Delta x}}{\Delta x}$$

$$u_t = 4 - cu_{xx}$$