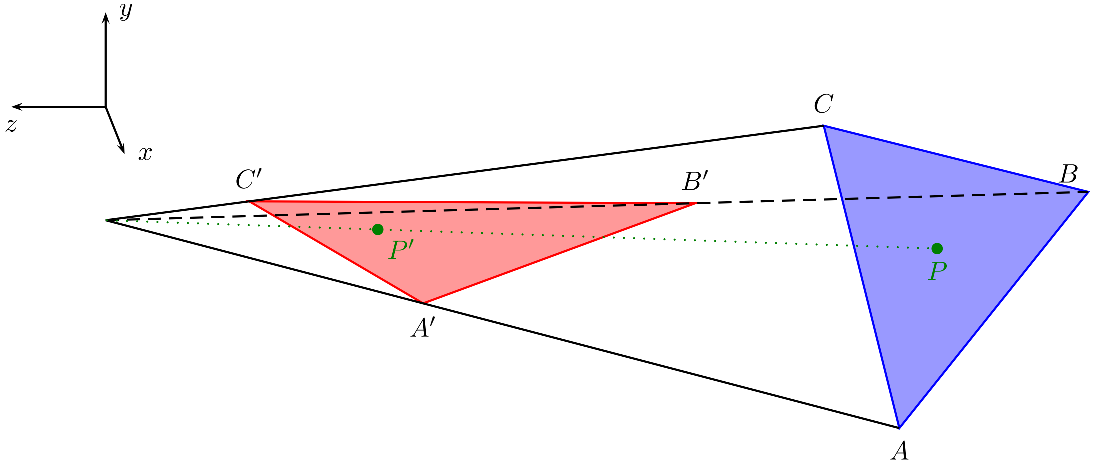
Perspective Correct Interpolation

CS 130

1. Viewing frustum in camera space. Camera is at the origin.



2. Transform from A, B, C, P to A', B', C', P' by homogeneous matrix $\mathbf{M}$.

$$\begin{pmatrix} A'w_a \\ w_a \end{pmatrix} = \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} B'w_b \\ w_b \end{pmatrix} = \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} C'w_c \\ w_c \end{pmatrix} = \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} P'w_p \\ w_p \end{pmatrix} = \mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix}$$

3. The real barycentric weights are α, β, γ. Because of the projection, they appear to be α', β', γ'.

$$P = \alpha A + \beta B + \gamma C$$
$$P' = \alpha' A' + \beta' B' + \gamma' C'$$

4. While rasterizing, we can compute α', β', γ' directly, but we will need the real weights α, β, γ to correctly interpolate color.

5. Noting $\alpha + \beta + \gamma = 1$,

$$
\begin{aligned}
\begin{pmatrix} P \\ 1 \end{pmatrix} &= \alpha \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} C \\ 1 \end{pmatrix} \\
\mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix} &= \alpha \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix} \\
P' w_p &= \alpha A' w_a + \beta B' w_b + \gamma C' w_c \\
w_p &= \alpha w_a + \beta w_b + \gamma w_c \\
P' &= \frac{\alpha A' w_a + \beta B' w_b + \gamma C' w_c}{\alpha w_a + \beta w_b + \gamma w_c} \\
P' &= \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c} A' + \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c} B' + \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c} C' \\
\alpha' &= \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c} \\
\beta' &= \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c} \\
\gamma' &= \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c}
\end{aligned}
$$

6. This is the wrong way around. We have α' but need α.

$$
\begin{aligned}
k &= \frac{1}{\alpha w_a + \beta w_b + \gamma w_c} \\
\alpha' &= \alpha w_a k \\
\beta' &= \beta w_b k \\
\gamma' &= \gamma w_c k \\
\alpha &= \frac{\alpha'}{w_a k} \\
\beta &= \frac{\beta'}{w_b k} \\
\gamma &= \frac{\gamma'}{w_c k} \\
1 &= \alpha + \beta + \gamma = \frac{\alpha'}{w_a k} + \frac{\beta'}{w_b k} + \frac{\gamma'}{w_c k} \\
k &= \frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c} \\
\alpha &= \frac{\frac{\alpha'}{w_a}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}} \\
\beta &= \frac{\frac{\beta'}{w_b}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}} \\
\gamma &= \frac{\frac{\gamma'}{w_c}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}}
\end{aligned}
$$

7. Can now use α, β, γ to interpolate colors.