CS 130, Midterm 1

Solutions

This exam was originally given in two parts due to COVID-19. The first five problems were given in a timed 15-minute format. The rest of the exam was a 24-hour take-home exam. A full score was given for receiving 40 of the 50 possible points.

Problem 1 (2 points)

On the project, we discarded intersections closer than `small_t` when computing the closest intersection. Why?

When casting rays from a surface, such as for reflection rays and shadow rays, it is important that the ray not be considered to intersect the surface it starts on. This is done by discarding intersections closer than a tolerance `small_t`.

Problem 2 (2 points)

A raytracing scene has a yellow-colored sphere illuminated by a magenta-colored light. What color will the sphere appear in the image?

The sphere is color $(1, 1, 0)$ and the light is color $(1, 0, 1)$. These two vectors will be multiplied componentwise (whether ambient, specular, or diffuse), resulting in $(1, 0, 0)$, which is red. Depending on the lighting, this may be scaled down, resulting in a darker red. Note that a lighter red or pink is not possible.

Problem 3 (2 points)

What change would need to be made to your raytracer to implement antialiasing?

The ray tracer should be changed to cast multiple rays per pixel. This could be implemented reasonably in `Render_World::Render_Pixel` or `Render_World::Render`.

Problem 4 (2 points)

Given the intersection lists $A = (0, 1, 6, 6)$ and $B = (0, 7)$, compute the intersection lists for the four booleans: (a) A union B, (b) A intersect B, (c) $A - B$, and (d) $B - A$.

Observe that $A \subset B$, so that (a) $A \cup B = B = (0, 7)$, (b) $A \cap B = A = (0, 1, 6, 6)$, and (c) $A - B = \emptyset = ()$. Then, me must calculate (d) $B - A = (1, 6, 6, 7)$.

Problem 5 (2 points)

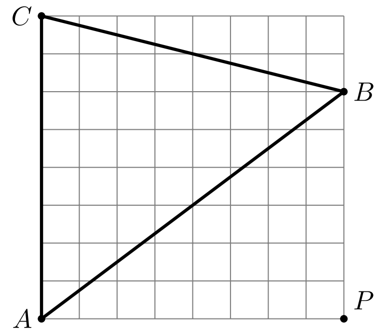
Why does a bump-mapped sphere cast a shadow that does not look bumpy?

Bump mapping only changes the color computation of the shader, not the physical geometry of the sphere. Since the sphere is geometrically smooth, so too are shadows and the silhouette. The intersection routines are not aware of the bump map.

This portion of the exam has 40 points. The iLearn portion has another 10 points. You need 40 total to get full credit. Additional points are extra credit.

Problem 6 (4 points)

Find the barycentric weights for the point P in the triangle below.



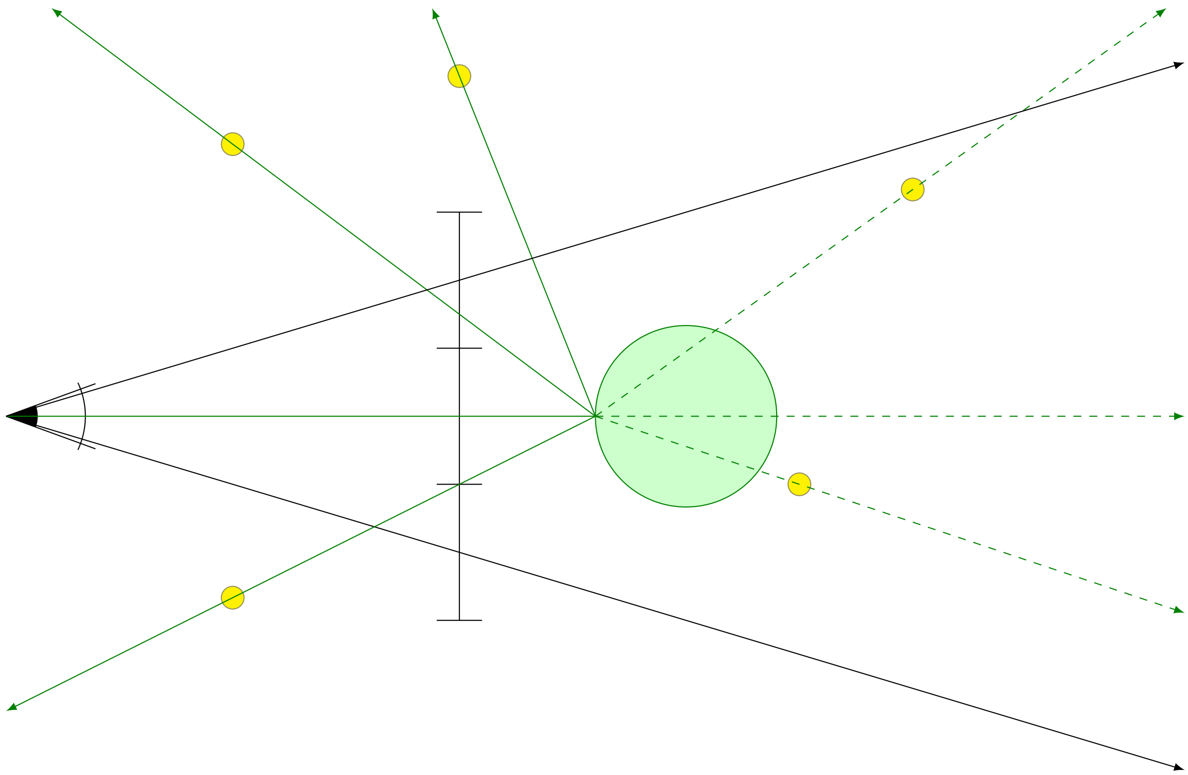
To do this, we need to compute the *signed* areas of the triangles, which we can then use to compute barycentric weights. Note that all of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute. We must also be careful about the orientations of the triangles. Triangles that are clockwise should have negative area.

$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 24 \qquad \text{area}(APC) = 32 \qquad \text{area}(ABP) = -24$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{24}{32} = \frac{3}{4} \qquad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{32}{32} = 1 \qquad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{-24}{32} = -\frac{3}{4}$$

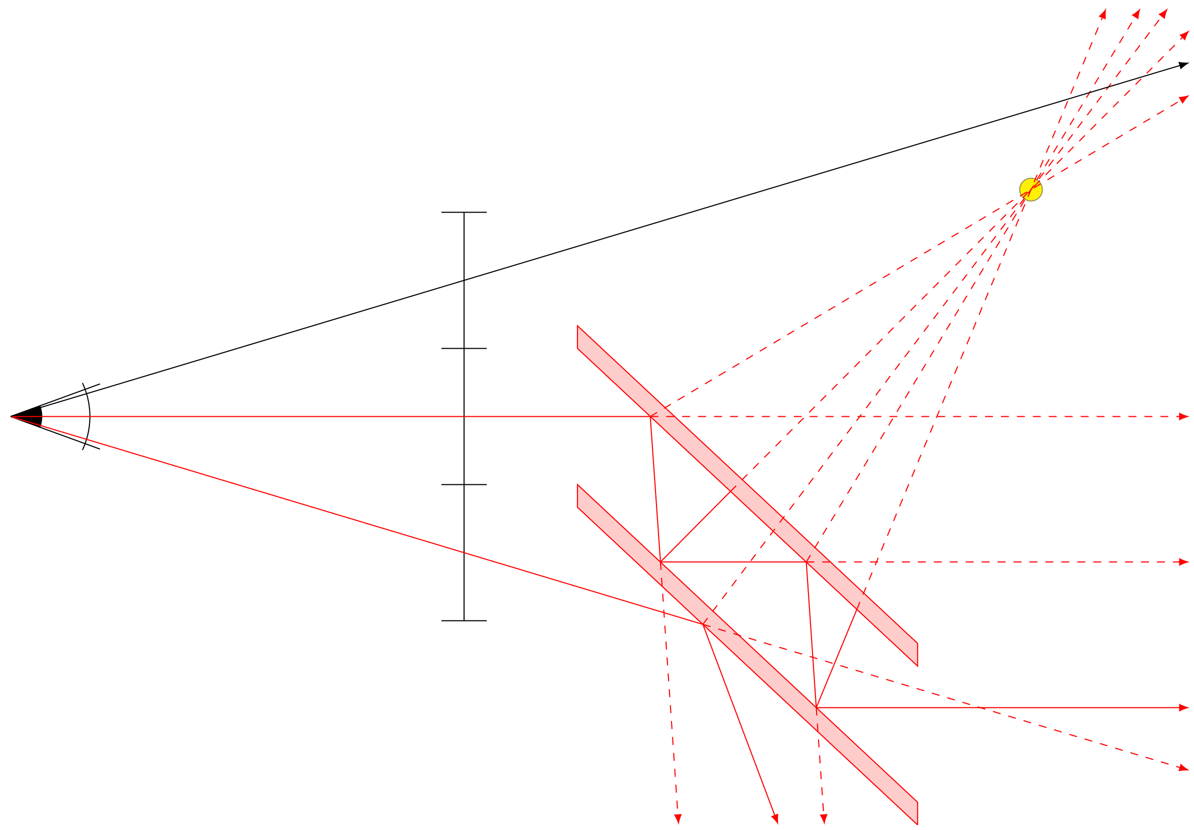
Problem 7 (8 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The green object is made of wood. The yellow circles are point lights. Draw all of the rays that would be cast while raytracing this scene.



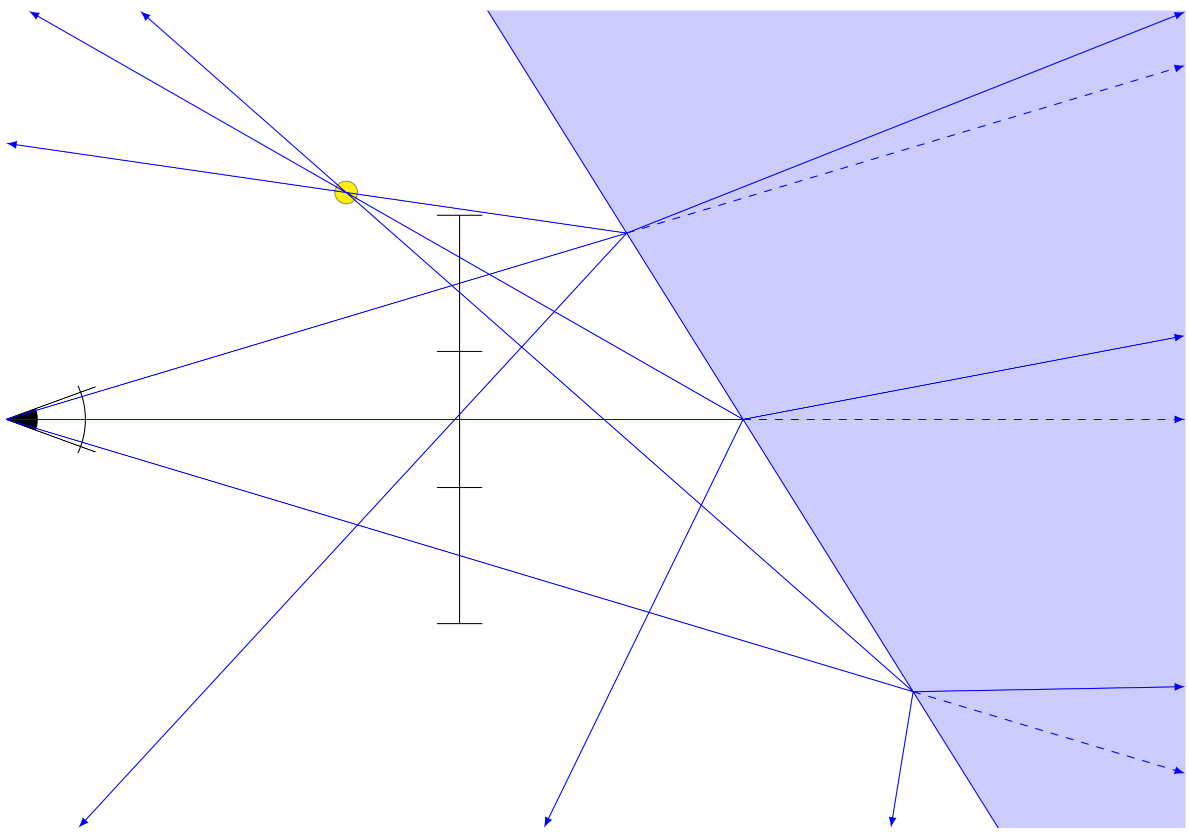
Problem 8 (8 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The red object is reflective. The yellow circle is a point light. Draw all of the rays that would be cast while raytracing this scene.



Problem 9 (8 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The blue object is made of glass (reflective and transparent). The yellow circle is a point light. Draw all of the rays that would be cast while raytracing this scene.



The remaining problems refer to the surface defined by the implicit function $f(x, y, z) = x^2 - y^2 - z$, which looks rather like a potato chip. You may assume that $f(x, y, z) > 0$ corresponds to *outside* of the surface. These problems also refer to the *ray* defined by $g(t) = \begin{pmatrix} 2t+1 \\ t+2 \\ -2t-2 \end{pmatrix}$, where $t \geq 0$. All of these problems are independent (you can solve them in any order, even if you have not solved earlier ones).

Problem 10 (4 points)

What are the direction and endpoint of the ray?

Endpoint is $g(0) = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$. Direction is $\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$. (Don't forget to normalize!)

Problem 11 (4 points)

Compute all intersection *locations* of the surface with the ray. [Hint: the answer should come out reasonably nicely; if it does not, check your math.]

Plugging $g(t)$ into $f(x, y, z)$ for the surface we get

$$\begin{aligned} 0 &= x^2 - y^2 - z \\ &= (2t+1)^2 - (t+2)^2 - (-2t-2) \\ &= (4t^2 + 4t + 1) - (t^2 + 4t + 4) + 2t + 2 \\ &= 3t^2 + 2t - 1 \\ &= (3t-1)(t+1) \\ t &= -1, \frac{1}{3} \end{aligned}$$

Since this is a ray and $t \geq 0$, we can exclude the first. This leaves $t = \frac{1}{3}$, which corresponds to $g(3) = \begin{pmatrix} \frac{5}{3} \\ \frac{7}{3} \\ -\frac{8}{3} \end{pmatrix}$.

Problem 12 (4 points)

What is the normal direction at an arbitrary point (x, y, z) lying on the surface of the surface? Don't worry about whether the normal points inwards or outwards.

$$
\begin{aligned}
f &= x^2 - y^2 - z \\
\nabla f &= \begin{pmatrix} 2x \\ -2y \\ -1 \end{pmatrix} \\
\|\nabla f\| &= \sqrt{4x^2 + 4y^2 + 1} \\
n = \frac{\nabla f}{\|\nabla f\|} &= \frac{1}{\sqrt{4x^2 + 4y^2 + 1}} \begin{pmatrix} 2x \\ -2y \\ -1 \end{pmatrix}
\end{aligned}
$$

The normal direction can also be worked out using a parametric representation such as $g(x, y) = (x, y, x^2 - y^2)$. In this case,

$$
\frac{\partial g}{\partial x} \times \frac{\partial g}{\partial y} = \begin{pmatrix} 1 \\ 0 \\ 2x \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -2y \end{pmatrix} = \begin{pmatrix} -2x \\ 2y \\ 1 \end{pmatrix}
$$

Normalizing leads to the same result, up to sign.