X

gravity pressure viscosity

F=ma

$$m \frac{Du}{Dt} = f_p + f_g + f_v$$

velocity

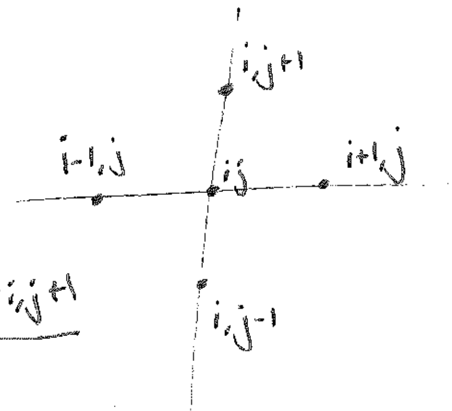
density rho ρ

pressure p

$f_g = mg$

$f_p = -V \nabla p$

Viscosity



me: $u_{i,j}$

ave: $\dfrac{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}}{4}$

diff ave-me $\dfrac{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j}}{4}$

force of viscosity proportional to this

factor: $\dfrac{4\mu}{\Delta x^2} \Rightarrow \mu \dfrac{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j}}{\Delta x^2} = \mu \nabla^2 u$

Laplacian

$f_v = \mu \nabla^2 u \, V$

volume

$$m \frac{Du}{Dt} = -V \nabla p + mg + \mu \nabla^2 u \, V \qquad \rho = \frac{M}{V}$$

$$\rho \frac{Du}{Dt} = -\nabla p + \rho g + \mu \nabla^2 u$$ ← Navier-Stokes

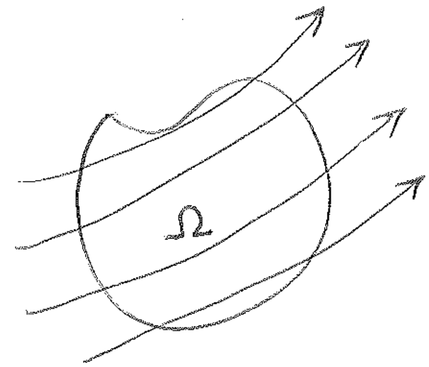
Incompressibility

$0 = \text{mass leaving region}$

$$= \int_{\partial\Omega} \rho \, dA \, \Delta t \, u \cdot n$$

volume = $dA \, \Delta t \, u \cdot n$

$\partial\Omega$ ↑ boundary



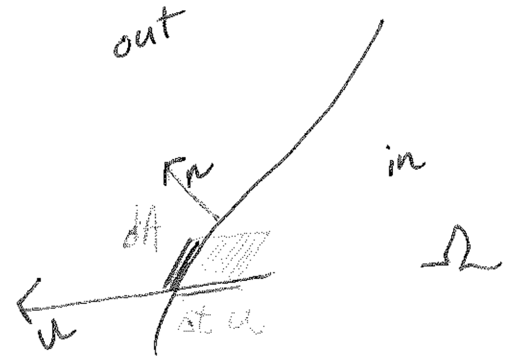
$d\vec{S} = dA \, \vec{n}$

$$= \int_{\partial\Omega} \rho \Delta t \, \vec{u} \cdot d\vec{S}$$

$\boxed{\text{divergence theorem}}$

$$= \int_{\Omega} \rho \Delta t \, \nabla \cdot u \, dV$$

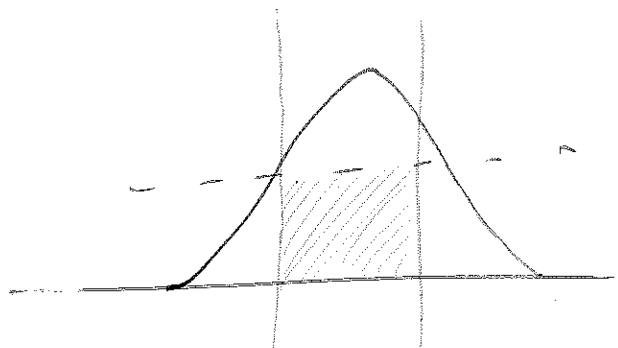
↑ divergence

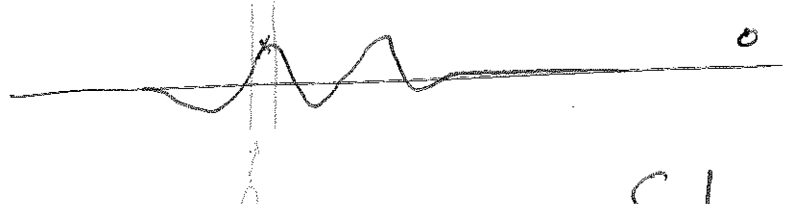


time interval Δt

for any region

$$\Rightarrow \underbrace{\rho \Delta t}_{\neq 0} \nabla \cdot u = 0 \quad \Rightarrow \quad \boxed{\nabla \cdot \vec{u} = 0}$$





$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$\frac{Du}{Dt} = \frac{d}{dt}\vec{u}(\vec{x}(t),t) = \frac{\partial\vec{u}}{\partial t} + \frac{\partial\vec{u}}{\partial\vec{x}}\cdot\frac{d\vec{x}}{dt} = \frac{\partial u}{\partial t} + \nabla\vec{u}\cdot\vec{u}$$

$$= \frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}$$

means the same

$$\frac{\partial u}{\partial t} + (u\cdot\nabla)u = -\frac{1}{\rho}\nabla p + g + \frac{\mu}{\rho}\nabla^2 u \quad \Big| \quad \nabla\cdot u = 0$$

$\cancel{u\cdot\nabla u}$ wrong!

$$\frac{\partial u}{\partial t} = A + B \qquad \frac{u^{n+1}-u^n}{\Delta t} = A + B$$

Splitting

$$\frac{u^{n+1}-u^*}{\Delta t} = A \qquad \frac{u^*-u^n}{\Delta t} = B$$

$$\frac{\partial u}{\partial t} = A \qquad \frac{\partial u}{\partial t} = B$$

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coupled

$$\frac{u^{n+1}-u^n}{\Delta t} = A(u^n) + B(u^{n+1})$$

(1) $$\frac{u^{n+1}-u^*}{\Delta t} = A(u^n) \qquad \frac{u^*-u^n}{\Delta t} = B(u^{n+1})$$

(2) $$\frac{u^{n+1}-u^*}{\Delta t} = B(u^{n+1}) \qquad \frac{u^*-u^n}{\Delta t} = A(u^n)$$

do second — do first

decoupled