CS 230, Quiz 7

Solutions

You will have 6 minutes to complete this quiz. No books, notes, or other aids are permitted.

One technique that processors use to compute $z = \sqrt{a}$ is to first compute $x = \frac{1}{\sqrt{a}}$ and then multiply $z = ax$. Let $f(x) = \frac{1}{x^2} - a$.

Problem 1 (1 Point)

Show that $f(x) = 0$.

$$f(x) = \frac{1}{x^2} - a = \frac{1}{\left(\frac{1}{\sqrt{a}}\right)^2} - a = \frac{1}{\frac{1}{a}} - a = a - a = 0$$

Problem 2 (4 Points)

Given an estimate $\bar{x}$ to x (so that $f(\bar{x}) \approx 0$), use Newton's method to derive an update rule to compute a better estimate $\hat{x}$ from the original estimate $\bar{x}$.

$$\begin{aligned}
\hat{x} &= \bar{x} + y \\
0 = f(\hat{x}) &= f(\bar{x} + y) \approx f(\bar{x}) + f'(\bar{x})y \\
y &= -\frac{f(\bar{x})}{f'(\bar{x})} \\
\hat{x} &= \bar{x} - \frac{f(\bar{x})}{f'(\bar{x})} \\
f(x) &= \frac{1}{x^2} - a \\
f'(x) &= -\frac{2}{x^3} \\
\hat{x} &= \bar{x} - \frac{\frac{1}{\bar{x}^2} - a}{-\frac{2}{\bar{x}^3}} = \bar{x} + \frac{\bar{x} - a\bar{x}^3}{2} = \frac{1}{2}\bar{x}(3 - a\bar{x}^2)
\end{aligned}$$