

CS 230, Quiz 8

Solutions

You will have 8 minutes to complete this quiz. There are more problems on the back. No books, notes, or other aids are permitted.

A rigid body has the following properties:

$$m = 4 \quad \hat{\mathbf{I}} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \boldsymbol{\omega} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Problem 1

Compute the momentum \mathbf{p} .

$$\mathbf{p} = m\mathbf{v} = \begin{pmatrix} -4 \\ 0 \\ -8 \end{pmatrix}$$

Problem 2

Compute the inertia \mathbf{I} .

$$\mathbf{I} = \mathbf{R}\hat{\mathbf{I}}\mathbf{R}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & -4 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Problem 3

Compute the angular momentum \mathbf{L} .

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

Problem 4

Compute the orientation rate of change $\dot{\mathbf{R}}$.

$$\dot{\mathbf{R}} = \omega^* \mathbf{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$