Line Rasterization

DDA algorithm for lines

Parametric Lines: the DDA algorithm
(digital differential analyzer)

$$Y_{i+1} = m\,x_{i+1} + B$$

$$= m(x_i + \Delta x) + B \qquad \Delta x = (x_{i+1} - x_i)$$

$$= y_i + m(\Delta x)$$ **<- must round to find int**

If we increment by 1 pixel in X, we turn on
[xi, Round(yi)] or same for Y if m > 1

Scan conversion for lines

DDA includes Round( ); and this is fairly slow

For Fast Lines, we want to do only integer math +,-

We do this using the **Midpoint Algorithm**

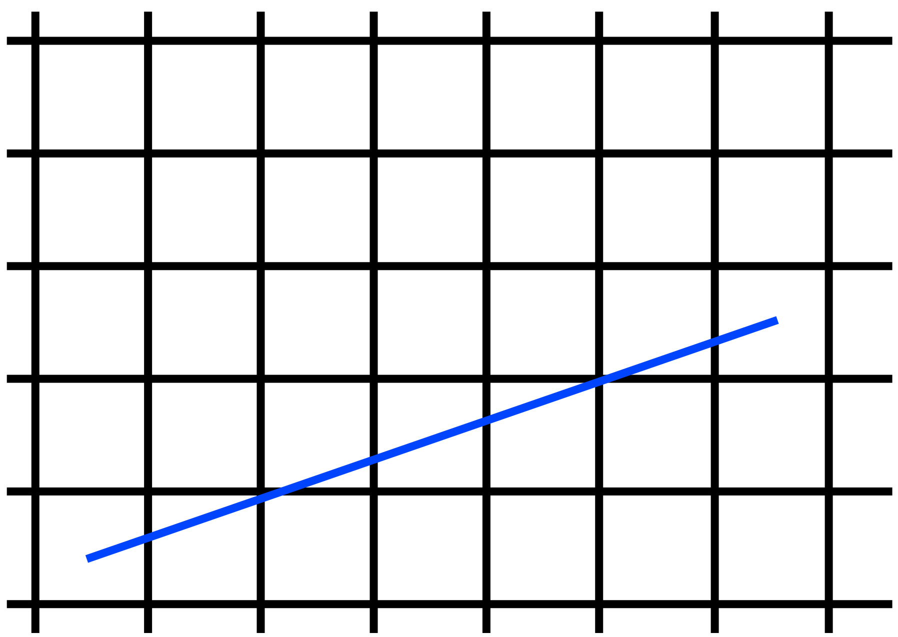
To do this, lets look at lines with y-intercept B
and with slope between 0 and 1:

$$y = (dy/dx)x + B \quad \Longrightarrow$$

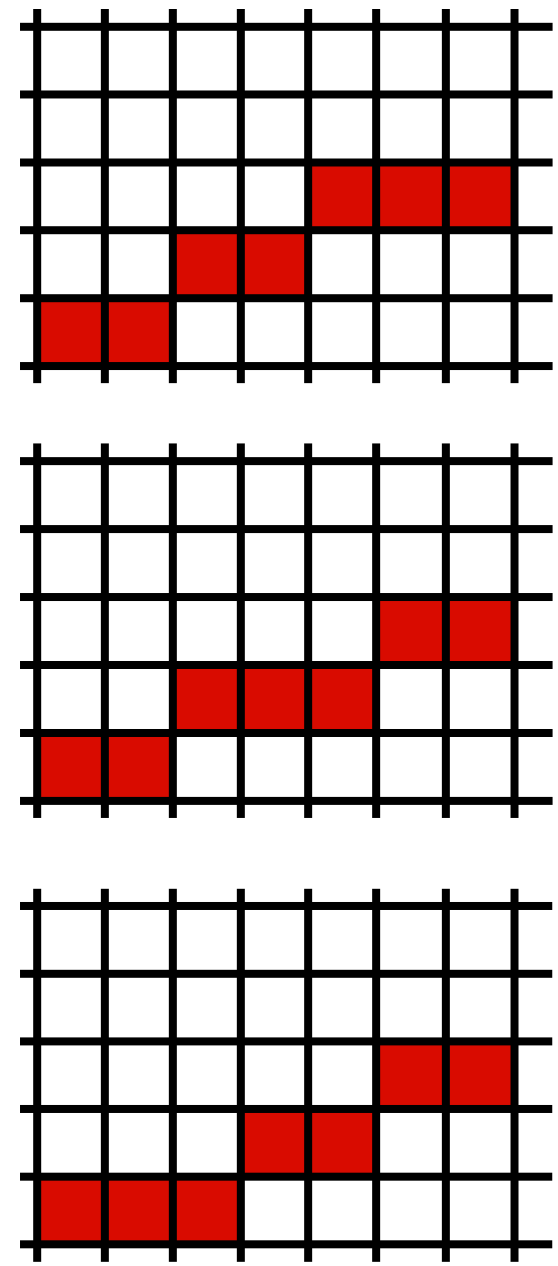
$$f(x,y) = (dy)x - (dx)y + B(dx) = 0$$

Removes the division => slope treated as 2 integers

Which pixels should be used to approximate a line?



Draw the thinnest possible line that has no gaps



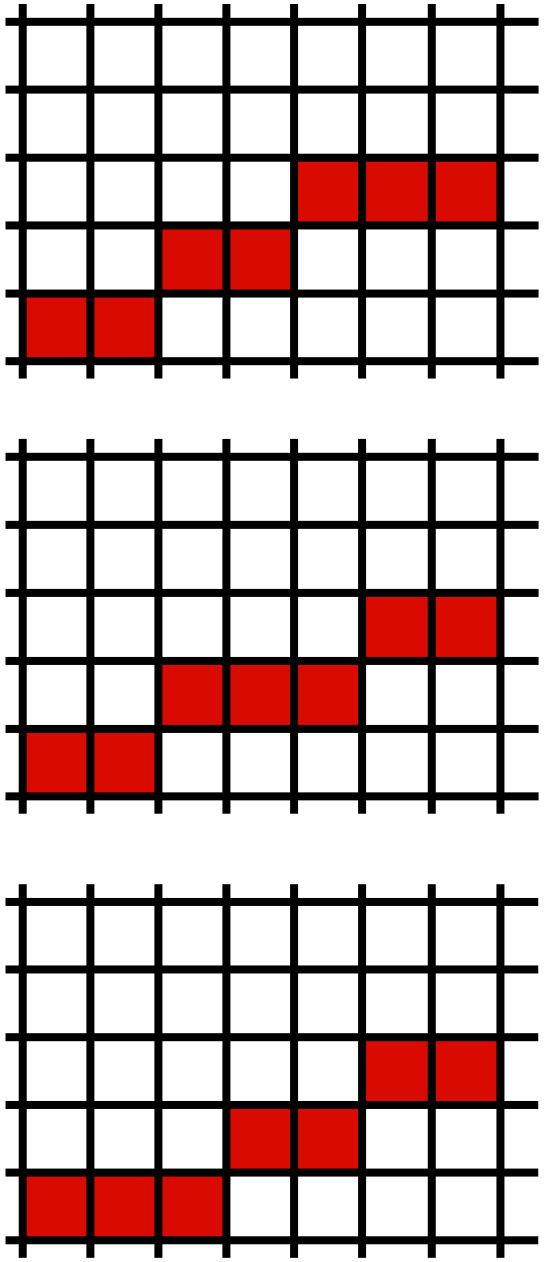
Line drawing algorithm

(case: $0 < m <= 1$)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (<condition>) then
        y = y+1
```

- move from left to right
- choose between

$(x+1,y)$ and $(x+1,y+1)$



Line drawing algorithm

(case: 0 < m <= 1)

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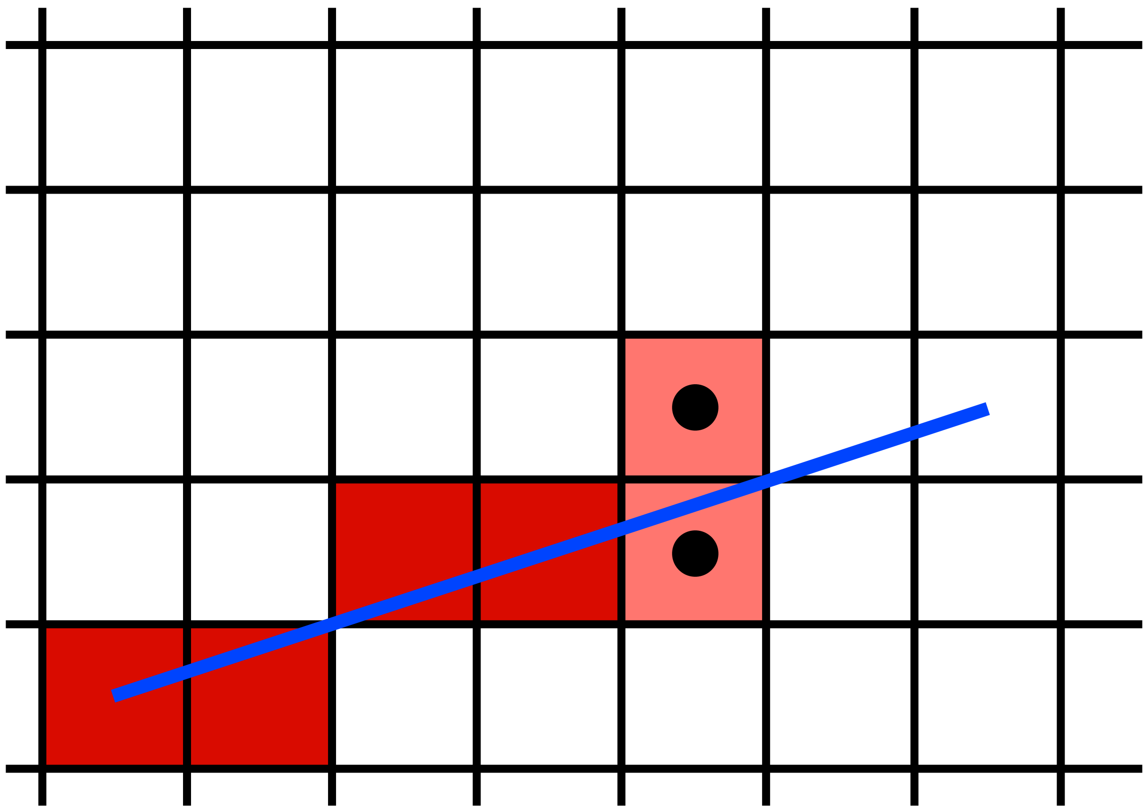
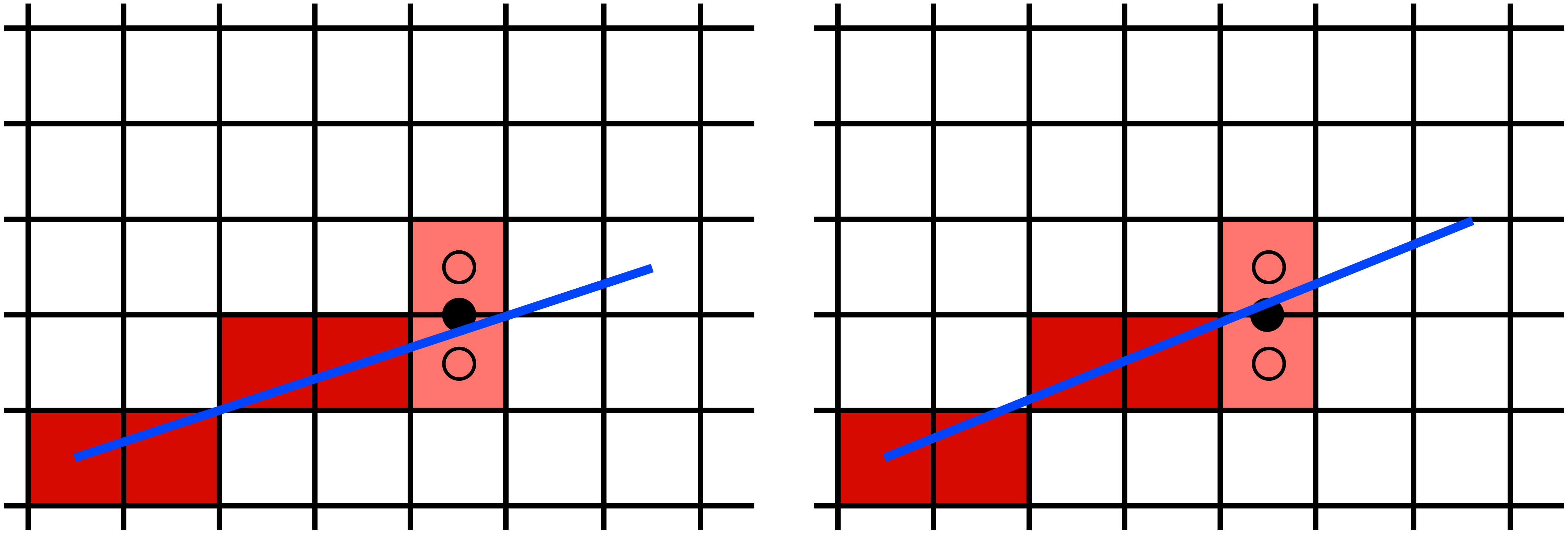
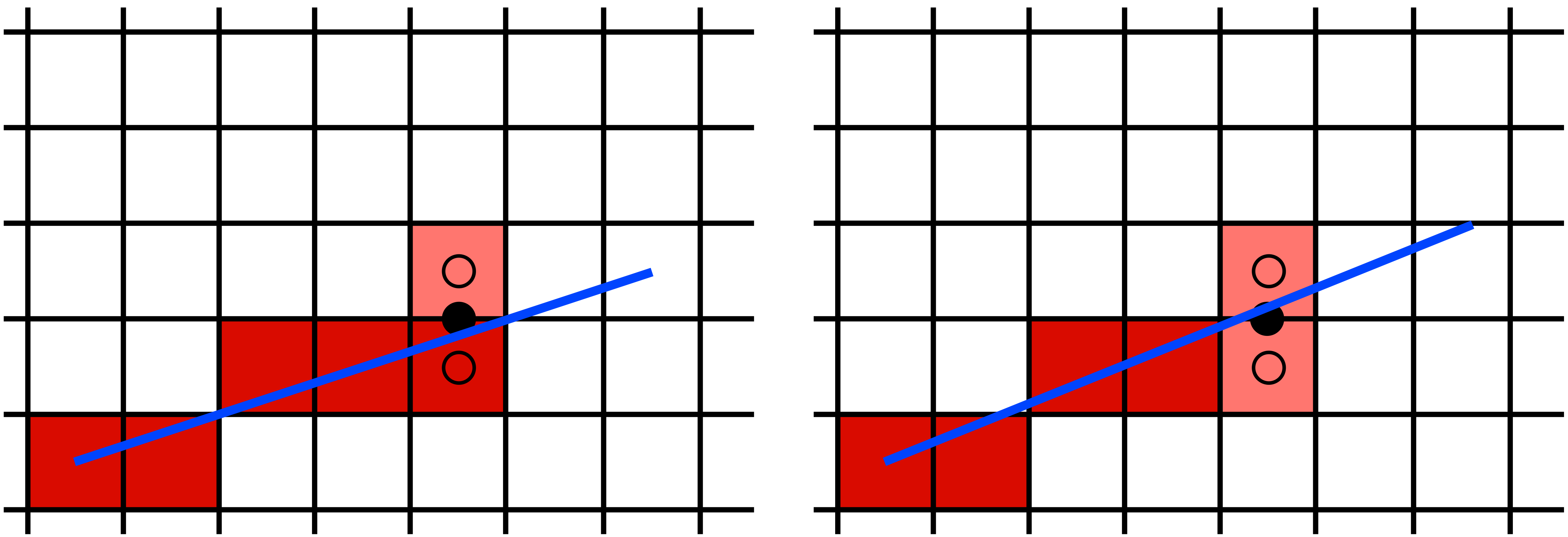


- move from left to right
- choose between (x+1,y) and (x+1,y+1)

Use the midpoint between the two pixels to choose



Use the midpoint between the two pixels to choose



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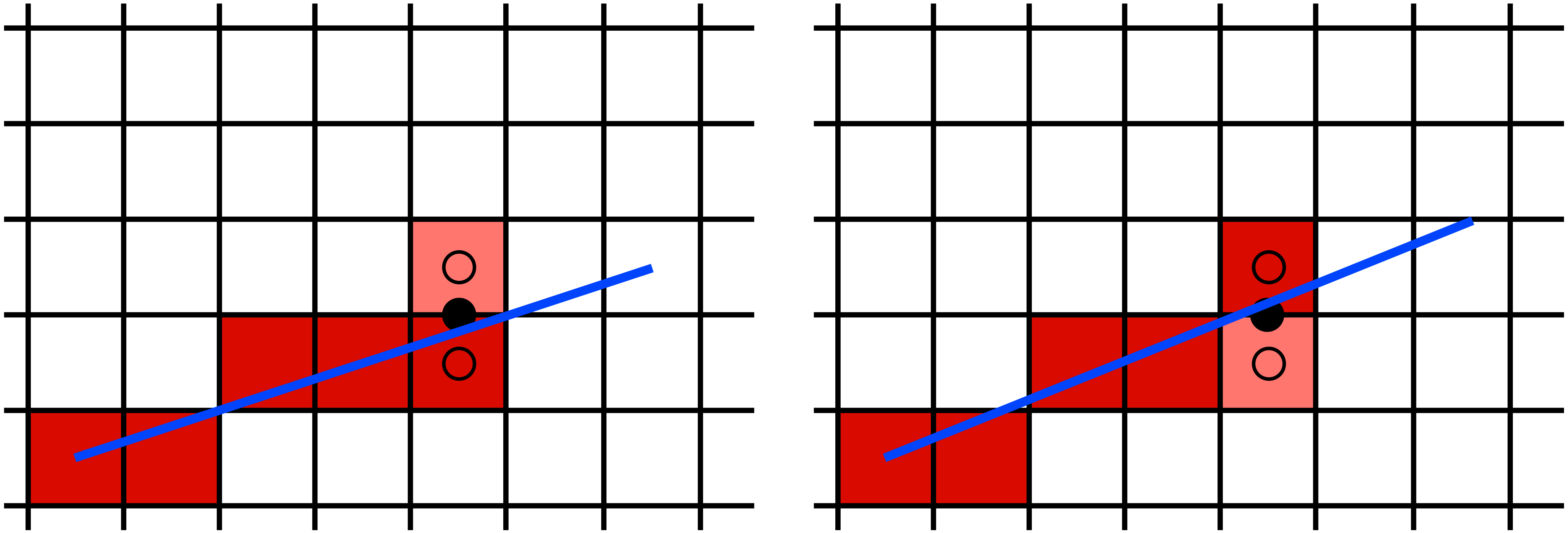

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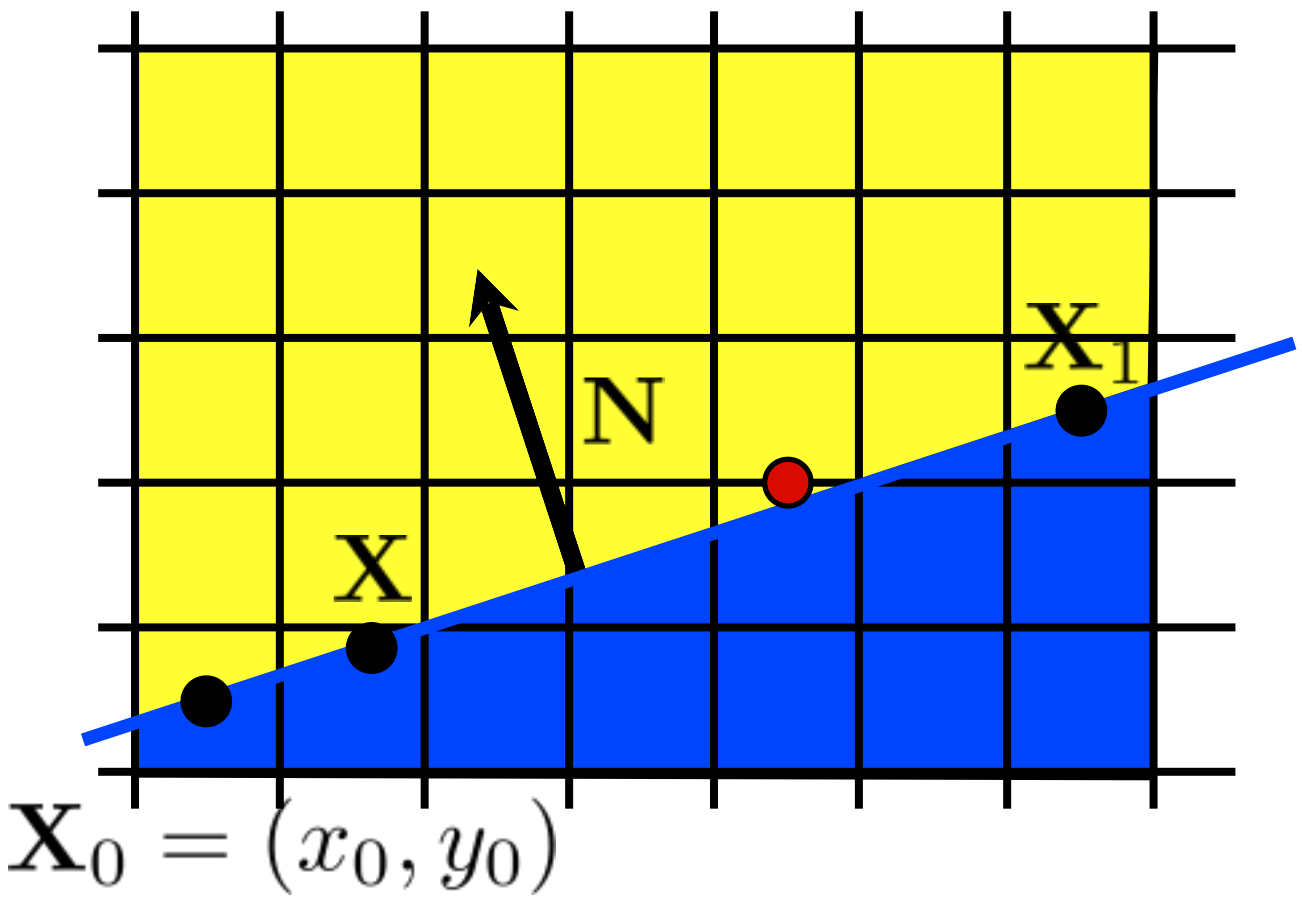

implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

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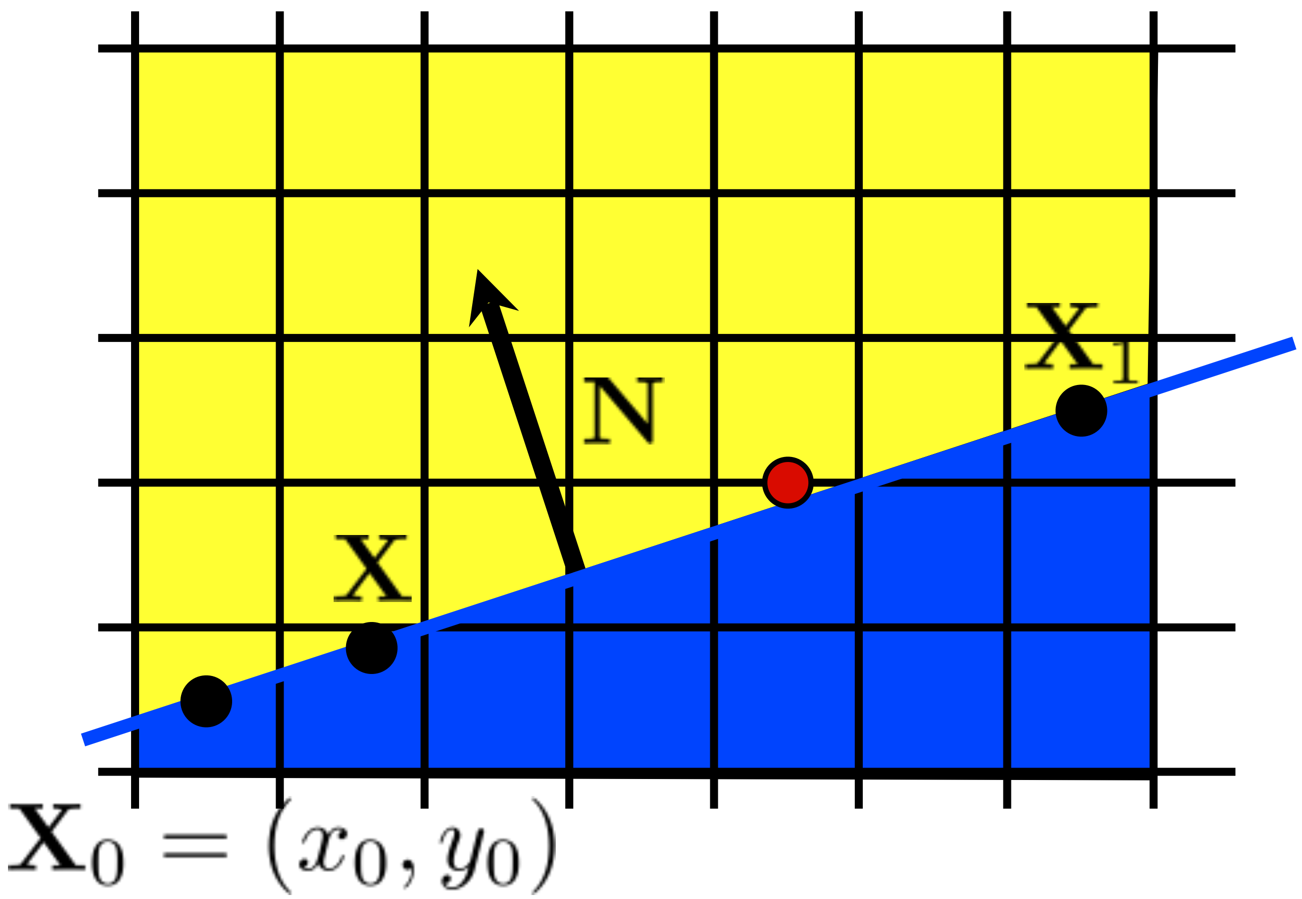
evaluate f at midpoint:

$$f(x, y + \frac{1}{2}) \ ? \ 0$$

Use the midpoint between the two pixels to choose



implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

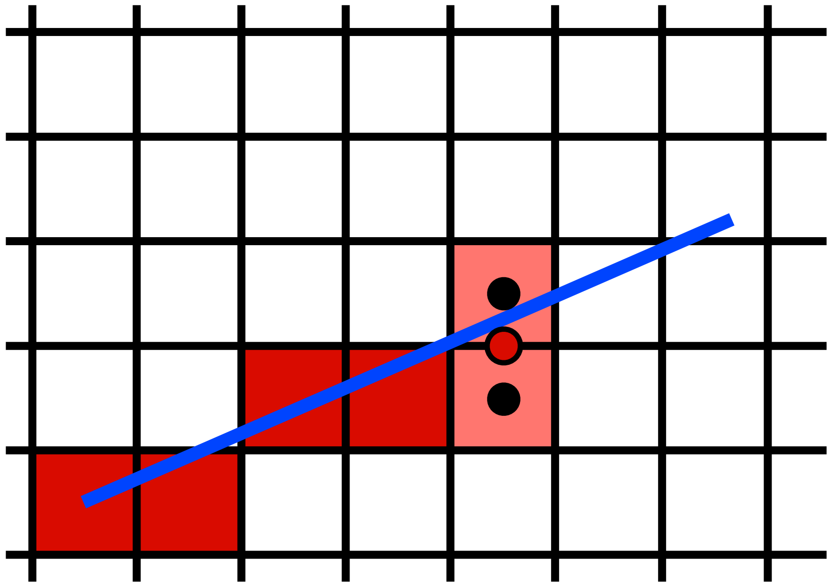
evaluate f at midpoint:

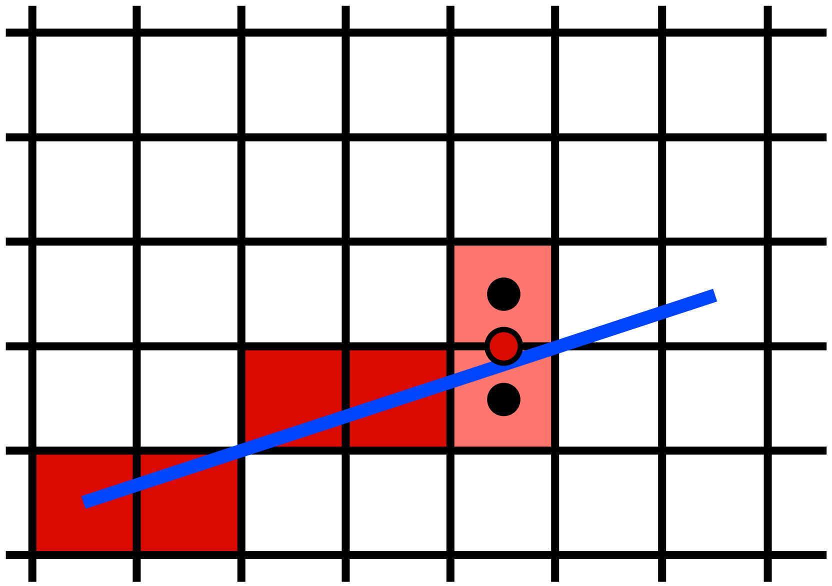
$$f(x, y + \frac{1}{2}) \; > \; 0$$

Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if ( f(x+1, y+1/2) < 0) then
        y = y+1
```

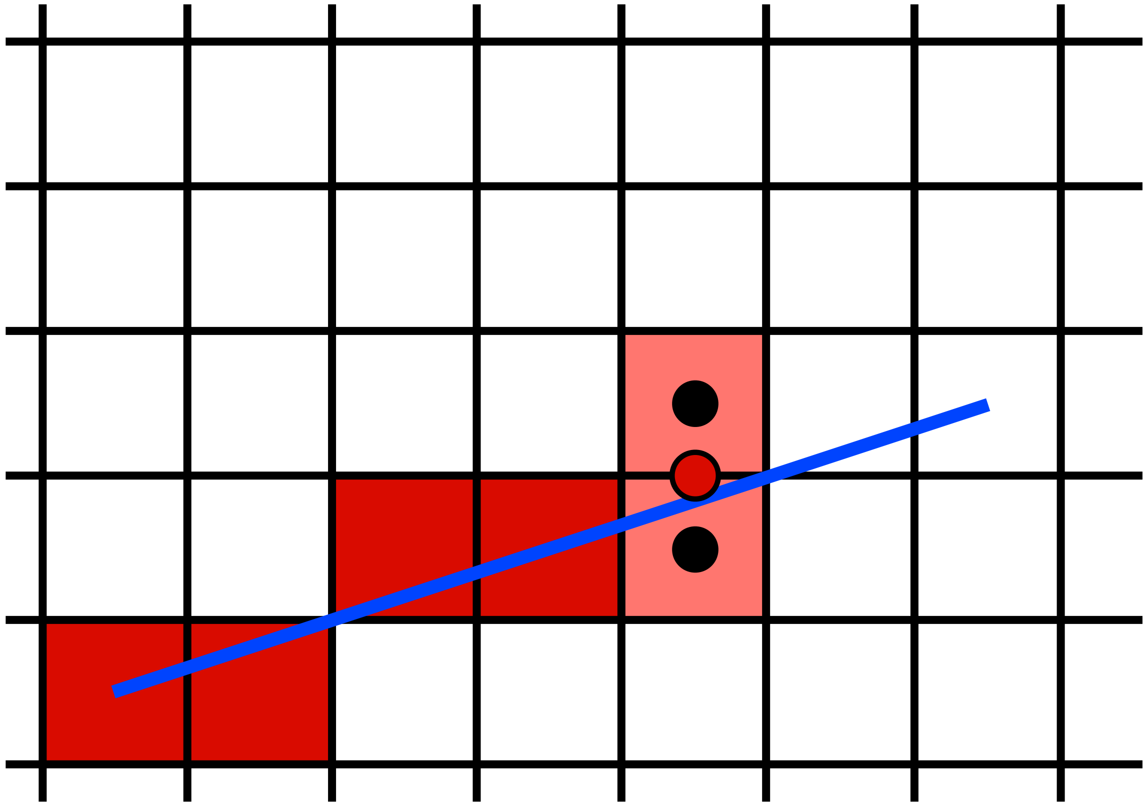




We can make the Midpoint Algorithm more efficient

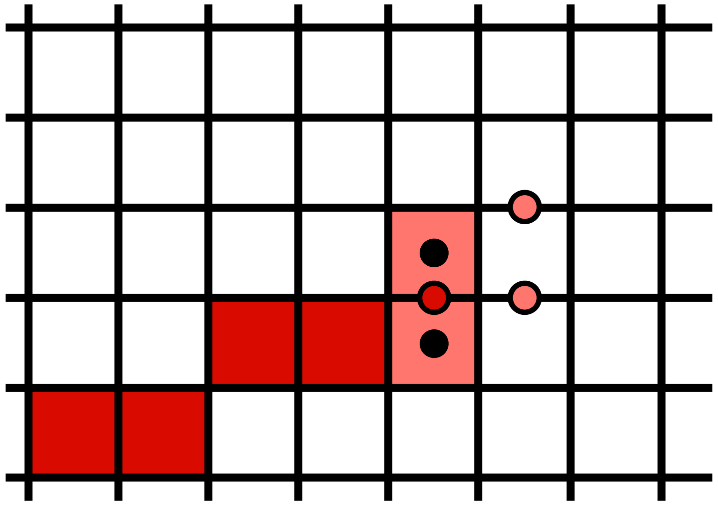
```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if ( f(x+1, y+1/2) < 0 ) then
        y = y+1
```

$f(x+1, y+\frac{1}{2}) < 0$



We can make the Midpoint Algorithm more efficient

by making it incremental!



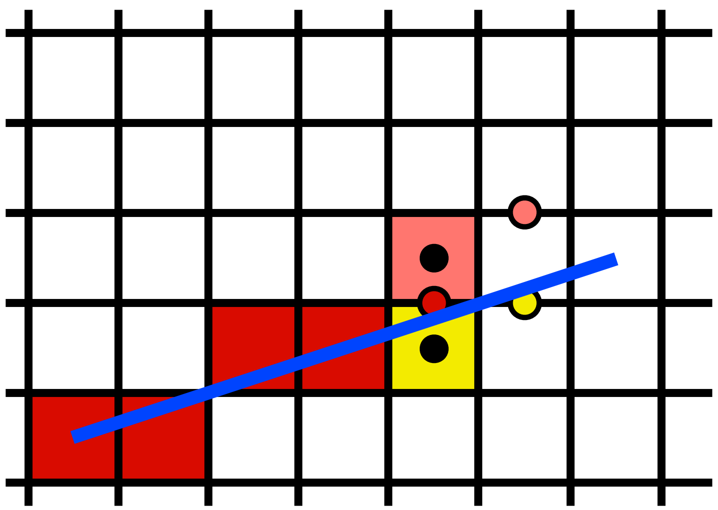
$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

$$f(x+1, y) = f(x, y) + (y_0 - y_1)$$

$$f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) > 0$$



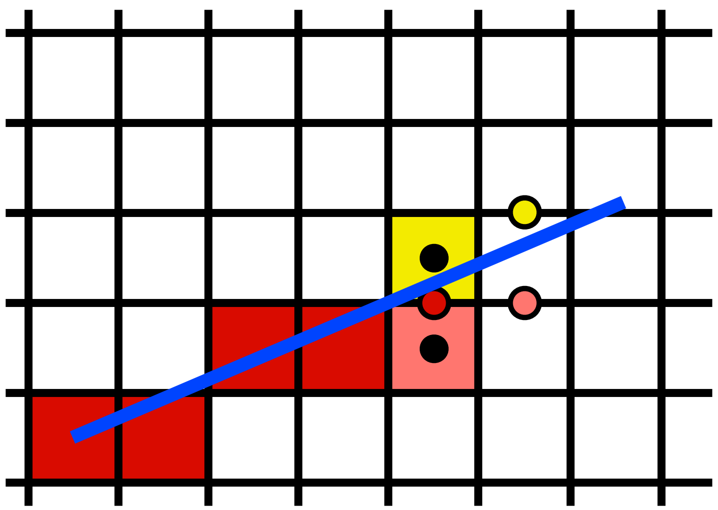
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0 = 0$$

$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) < 0$$



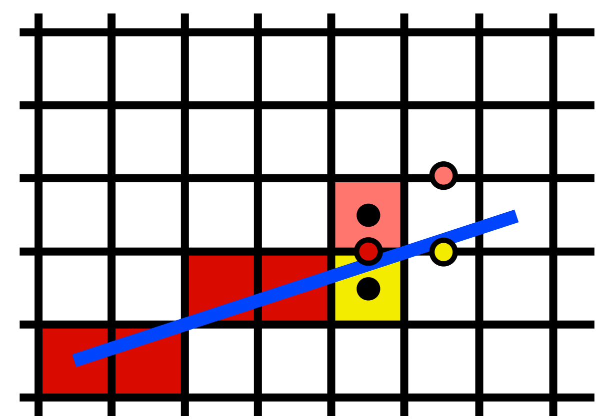
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

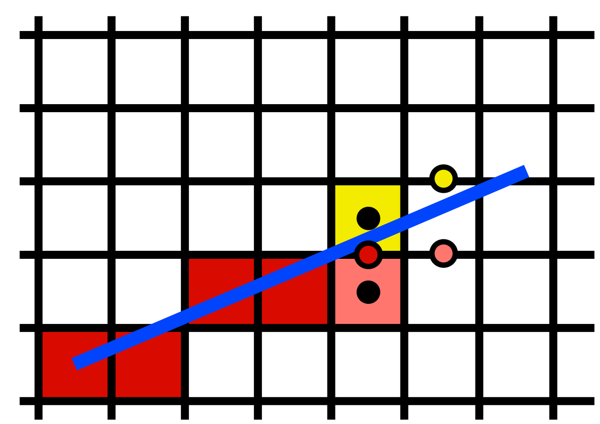
$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

```
y = y0
d = f(x0+1,y0+1/2)
for x = x0 to x1 do
    draw(x,y)
    if (d<0) then
        y = y+1
        d = d+(y0-y1)+(x1-x0)
    else
        d = d+(y0-y1)
```

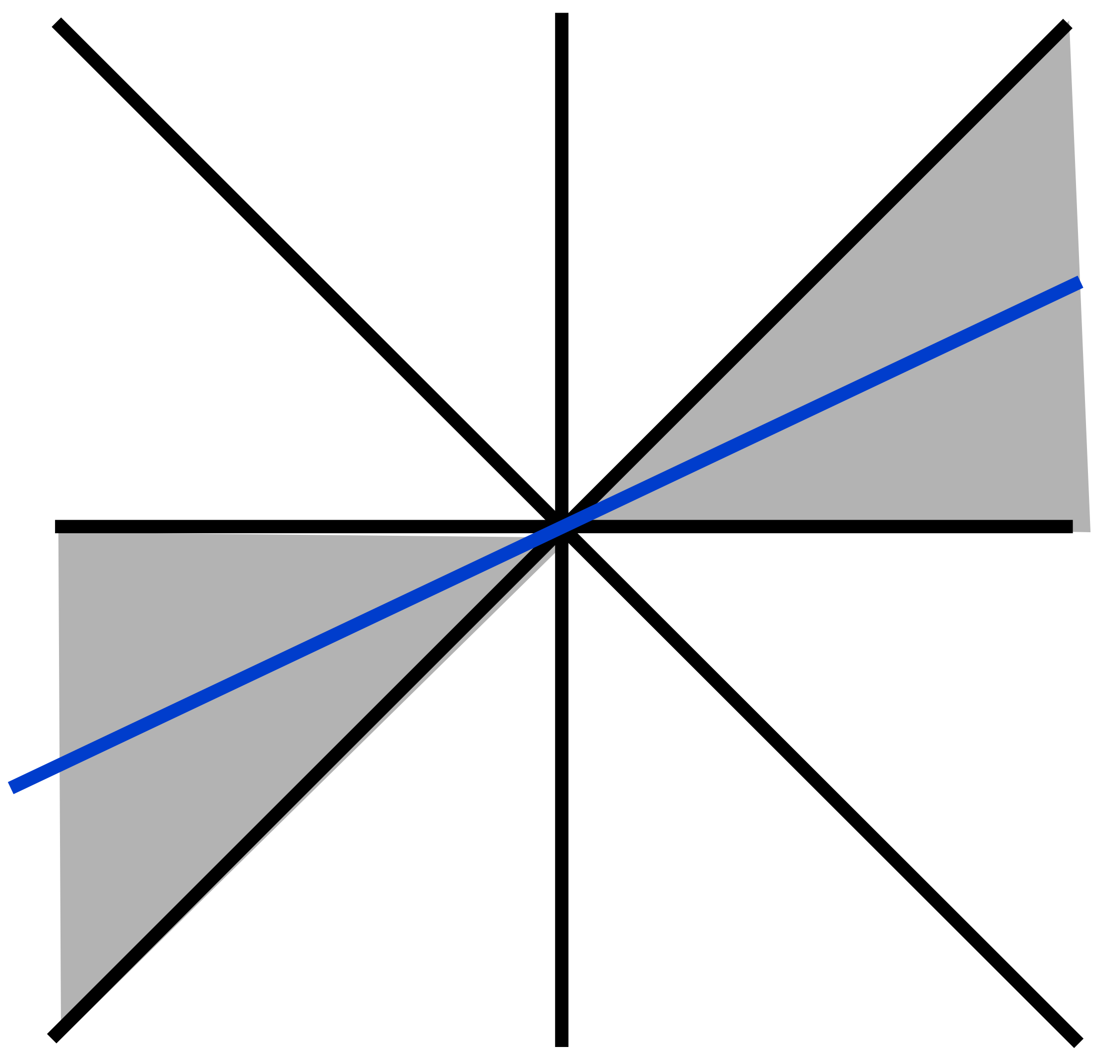


$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$

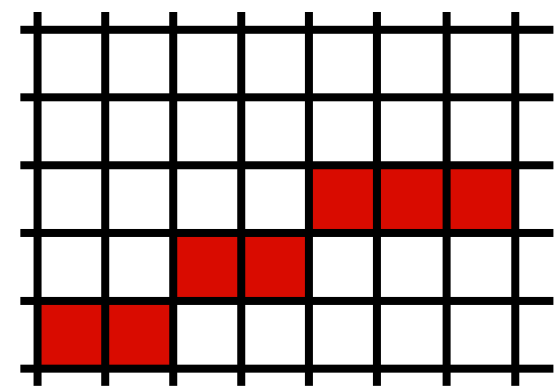


$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

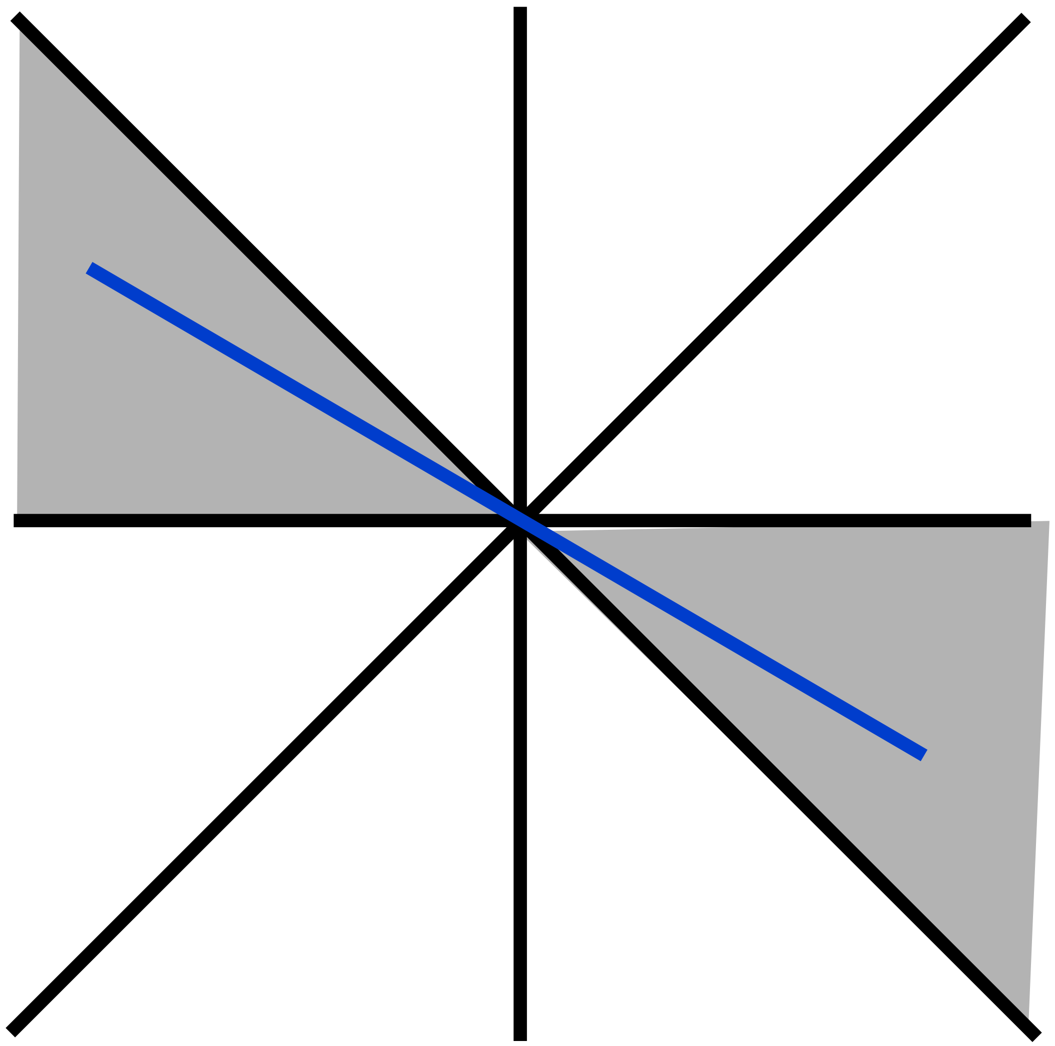
Adapt Midpoint Algorithm for other cases



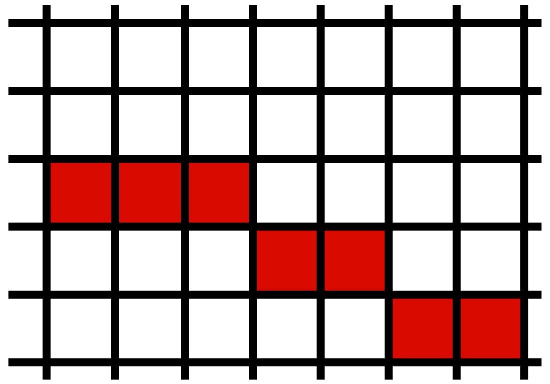
case: 0 < m <= 1



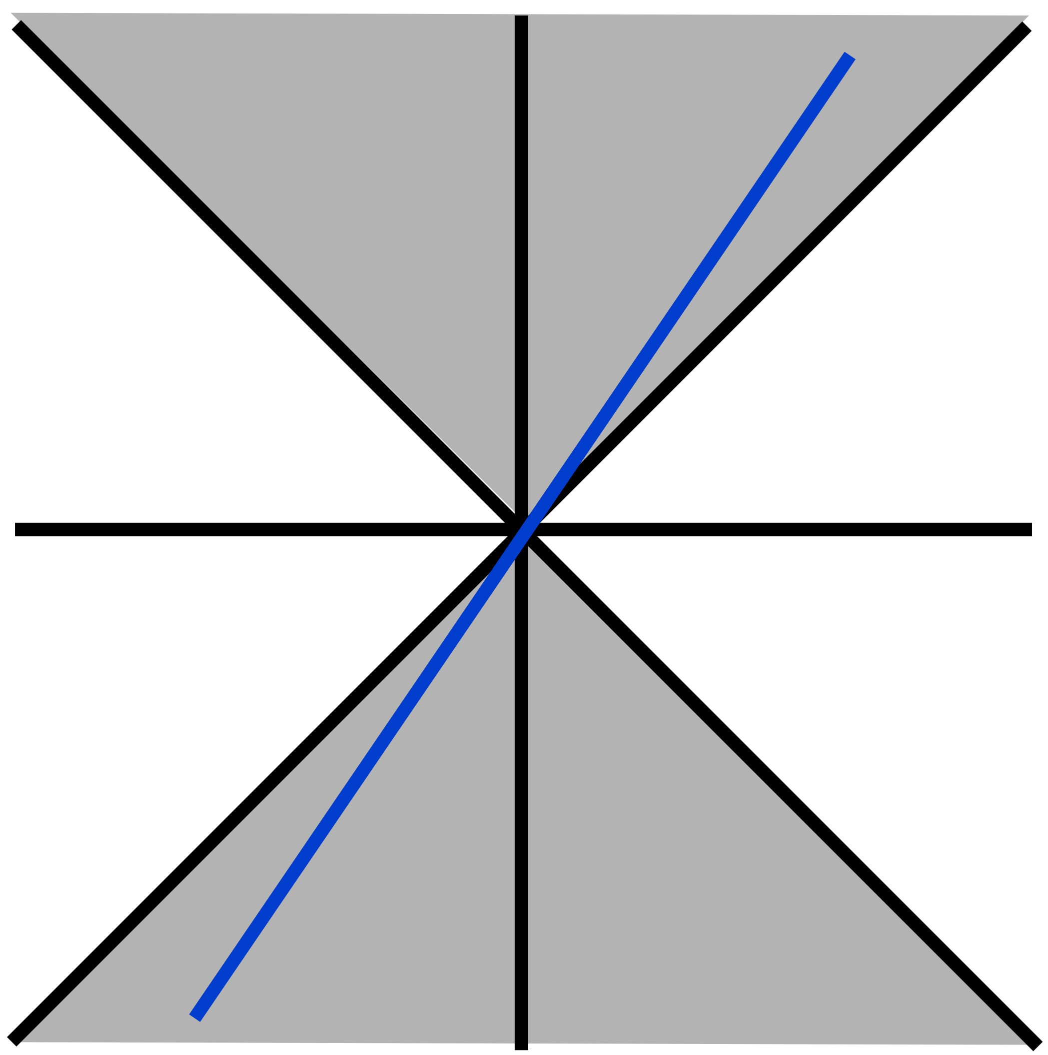
Adapt Midpoint Algorithm for other cases



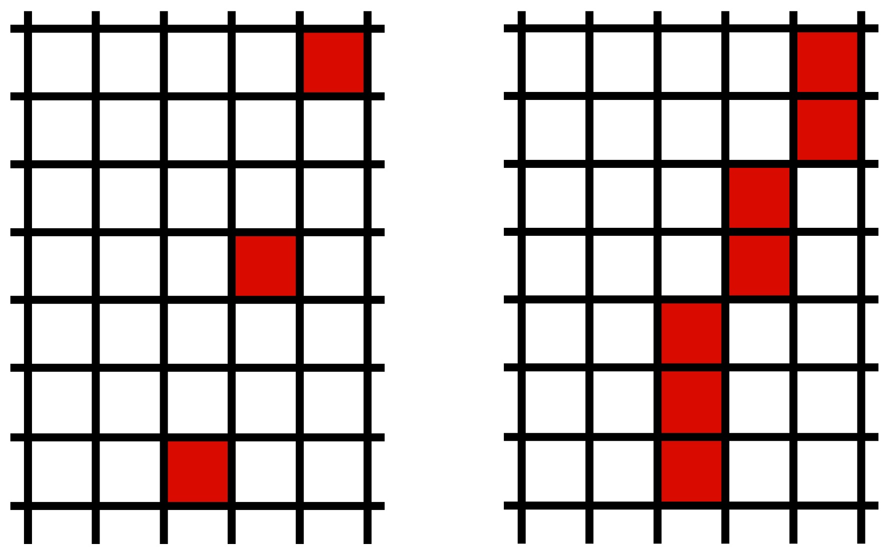
case: -1 <= m < 0



Adapt Midpoint Algorithm for other cases



case: 1 <= m
or m <= -1



Line drawing references

- The algorithm we just described is the *Midpoint Algorithm* (Pitteway, 1967), (van Aken and Novak, 1985)
  - Handles floating point coordinates
- Draws the same lines as the *Bresenham Line Algorithm* (Bresenham, 1965)
  - Simpler, cheaper
  - Integer coordinates only