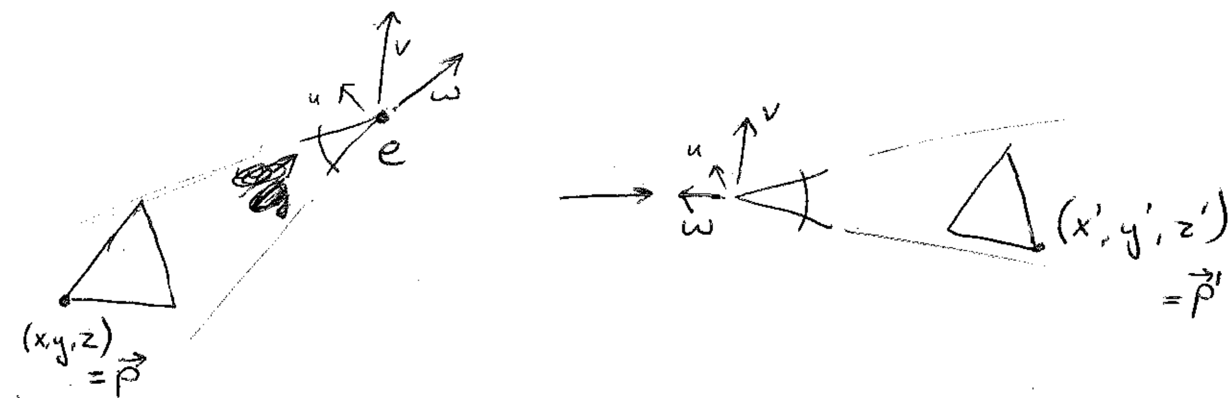
Camera transform



back — right — up

$$\omega = \frac{g}{\|g\|} \qquad u = \frac{t \times \omega}{\|t \times \omega\|} \qquad v = \omega \times u \qquad M = \begin{pmatrix} \vec{\omega} & \vec{u} & \vec{v} \end{pmatrix}$$

matrix (orthogonal)

$$\vec{p} = \vec{e} + x'\vec{\omega} + y'\vec{u} + z'\vec{v} = \vec{e} + M\vec{p}'$$

$$M\vec{p}' = \vec{p} - \vec{e}$$

$$p' = M^T(\vec{p} - \vec{e})$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \left(\begin{array}{c|c} M^T & -M^T\vec{e} \\ \hline \vec{0}^T & 1 \end{array}\right) \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

perspective

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{d}{z}\cancel{g}x \\ \frac{d}{z}\, y \\ ? \\ 1 \end{pmatrix} \equiv \begin{pmatrix} x \\ y \\ ? \\ \frac{z}{\cancel{d}n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ * & * & * & * \\ 0 & 0 & \frac{1}{\cancel{d}n} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \omega$$

what about z? $d = n$

$$\begin{pmatrix} * \\ * \\ n \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} * \\ * \\ \omega n \\ \omega \end{pmatrix} \qquad an + b = \omega n \qquad \frac{n}{d} = \omega \qquad \rightarrow \qquad an + b = \frac{n^2}{d} = n$$

$$\begin{pmatrix} * \\ * \\ \cancel{z} f \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} * \\ * \\ \omega' f \\ \omega' \end{pmatrix} \qquad af + b = \omega' f \qquad \frac{\cancel{a}f}{d} = \omega' \qquad af + b = \frac{f^2}{d}$$

$$a(n-f) = \frac{1}{d}(n^2 - f^2)$$

$$a = \frac{n+f}{\cancel{d}n}$$

$$\cancel{b = \frac{n}{d}(n - ad)} \qquad b = n - an = -f$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{n+f}{n} & -f \\ 0 & 0 & \frac{1}{n} & 0 \end{pmatrix}$$