CS 130, Midterm 1

Solutions

Problem 1 (4 points)

For what values of x,y,z is the code fragment

```
glBegin(GL_POINTS);
glVertex3f(x,y,z);
glEnd();
```

equivalent to this fragment?

```
glPushMatrix();
glRotatef(180,0,1,0);
glScalef(1,2,1);
glTranslatef(0,1,1);
glBegin(GL_POINTS);
glVertex3f(1,2,3);
glEnd();
glPopMatrix();
```

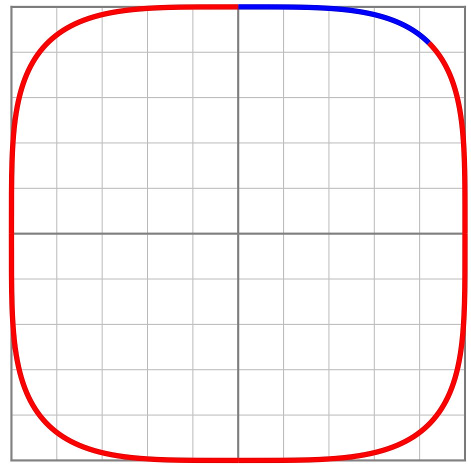
Recall that the operations are specified to OpenGL in reverse order.

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \overset{\text{translate}}{\Longrightarrow} \begin{pmatrix}1\\3\\4\end{pmatrix} \overset{\text{scale}}{\Longrightarrow} \begin{pmatrix}1\\6\\4\end{pmatrix} \overset{\text{rotate}}{\Longrightarrow} \begin{pmatrix}-1\\6\\-4\end{pmatrix}$$

Thus, $x = -1$, $y = 6$, and $z = -4$.

Problem 2 (4 points)

Construct an extension of the midpoint algorithm that rasterizes the boundary of the object described by the equation $x^4 + y^4 = r^4$, sketched at right.

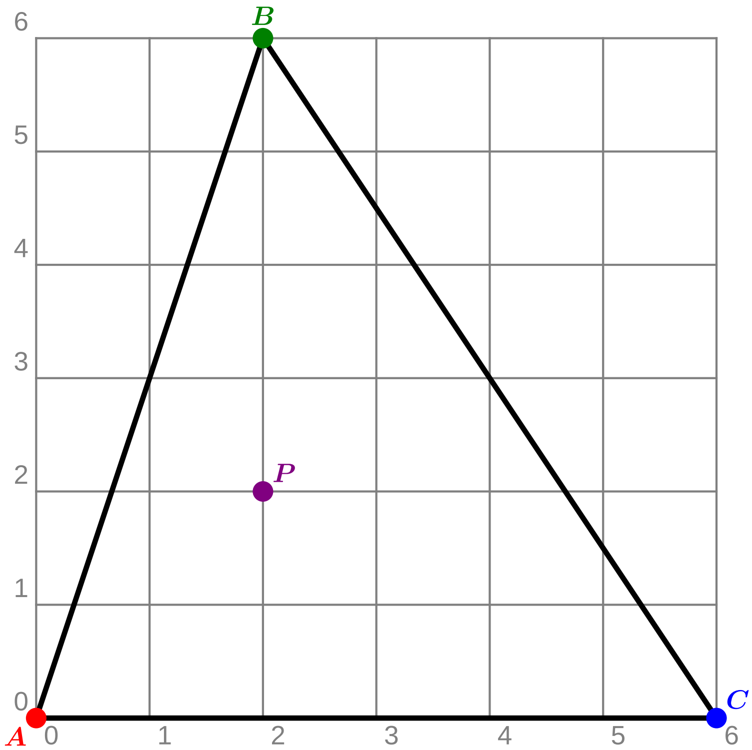


- **Your solution should be similar to C++ in syntax.**
- **The signature of the function you are implementing is** `void Rasterize_Shape(int r);`
- **You may call** `void Set_Pixel(int x,int y);` **to turn on pixels. It is okay to call it with positive or negative values.**
- **Your code does not need to be incremental, but it should not use floating point (directly or indirectly).**
- **It is sufficient to rasterize the part shown in blue. ($0 \leq x \leq y$).**

```
void Rasterize_Shape(int r)
{
  int x = 0, y = r;
  while(x <= y)
  {
    Set_Pixel(x,y);
    x++;
    if(16*x*x*x*x+(2*y-1)*(2*y-1)*(2*y-1)*(2*y-1)>16*r*r*r*r)
        y--;
  }
}
```

Problem 3 (4 points)

The triangle at right is to be rasterized. The colors of the vertices are A = red = $(1, 0, 0)$, B = green = $(0, 1, 0)$ and, C = blue = $(0, 0, 1)$. Compute the color of the point P.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights.

$$\text{area}(ABC) = 18$$
$$\text{area}(PBC) = 8$$
$$\text{area}(APC) = 6$$
$$\text{area}(ABP) = 4$$
$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{4}{9}$$
$$\beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{3}{9}$$
$$\gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{2}{9}$$
$$C_P = \alpha C_A + \beta C_B + \gamma C_C = \left(\frac{4}{9}, \frac{3}{9}, \frac{2}{9}\right)$$

Problem 4 (4 points)

What initial values of `x,y,z` in `barB` will make the routines `barA` and `barB` equivalent?

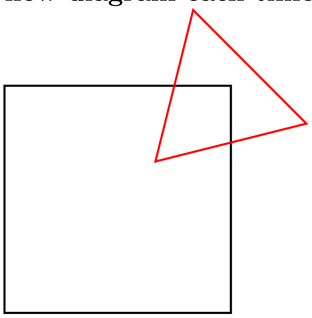
```
void barA(float a, float b, float c, int n)
{
  for(int i = 0 ; i < n ; i++)
  {
    float z = a * i * i + b * i + c;
    foo(z);
  }
}

void barB(float a, float b, float c, int n)
{
  float x = ???;
  float y = ???;
  float z = ???;
  for(int i = 0 ; i < n ; i++)
  {
    foo(z);
    z += y;
    y += x;
  }
}
```

Note that in `barA`, the first three calls to `foo` are `foo(c)`, `foo(a+b+c)`, and `foo(4*a+2*b+c)`. In `barB`, the first three calls to `foo` are `foo(z)`, `foo(z+y)`, and `foo(z+2*y+x)`. Since the first calls are the same, `z=c`. Since the second calls are the same, `y=a+b`. To make the third calls the same, we need `x=2*a`. This is the simplest way to find the initial values.

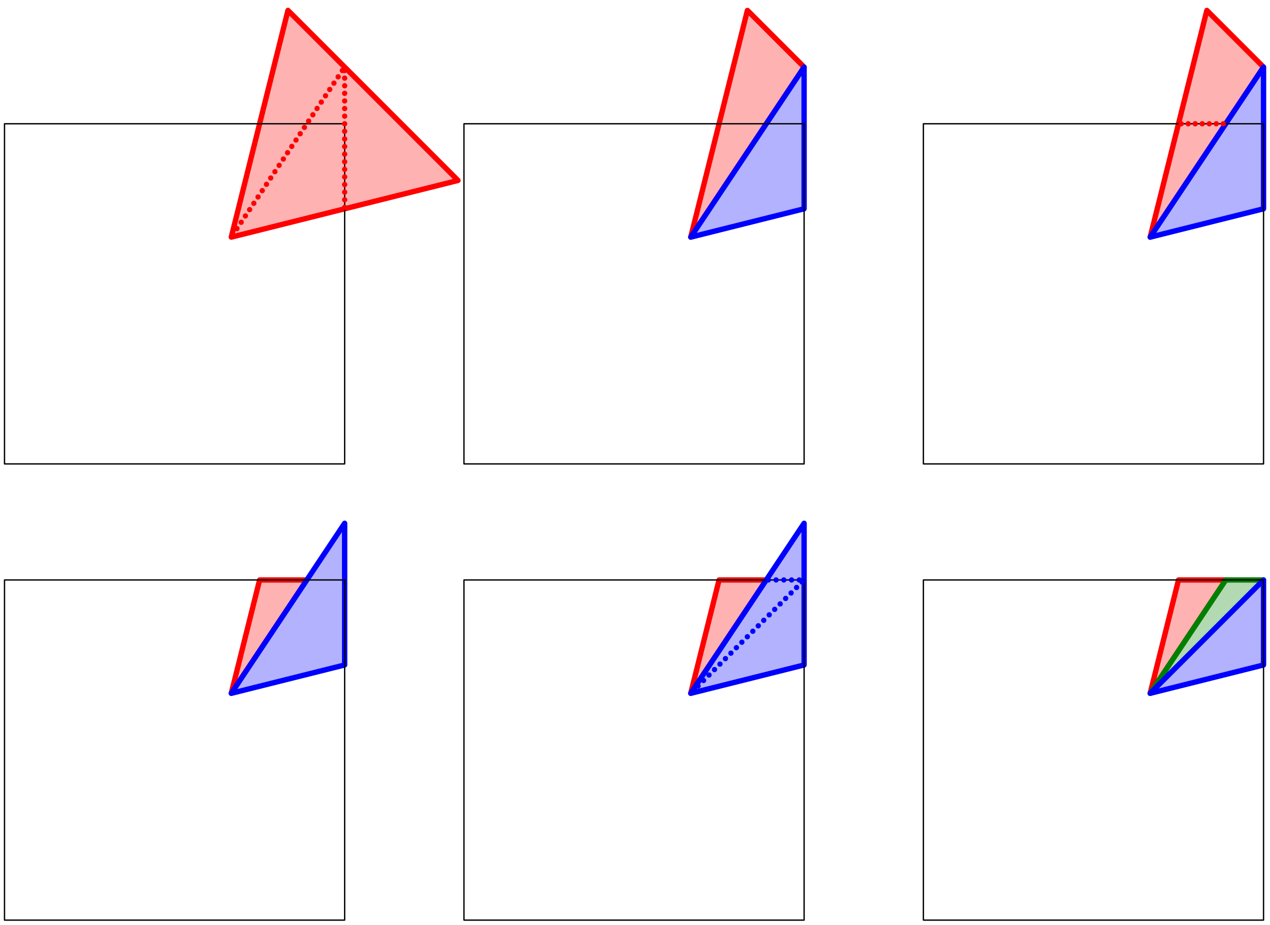
```
void barB(float a, float b, float c, int n)
{
  float x = 2 * a;
  float y = a + b;
  float z = c;
  for(int i = 0 ; i < n ; i++)
  {
    foo(z);
    z += y;
    y += x;
  }
}
```

Problem 5 (4 points)

Show step by step how OpenGL would clip the triangle below against the square. (Draw a new diagram each time you clip a triangle.)



Here is one way it may occur.



Problem 6 (1 point each part)

Let $f(t) = \langle 4t, -t+1, t-1, t+1 \rangle$ define a curve in homogeneous coordinates for $t \in [0,1]$.
(a) Express $f(t)$ in non-homogeneous 3D coordinates.
(b) Using (a), show that this corresponds to a line. (This is not at all obvious from the homogeneous representation.)

(a)

$$f(t) = \begin{pmatrix} \frac{4t}{t+1} \\ \frac{-t+1}{t+1} \\ \frac{t-1}{t+1} \end{pmatrix}$$

(b) One way to do this is to show that it can be expressed in the form

$$f(t) = \begin{pmatrix} \frac{4t}{t+1} \\ \frac{-t+1}{t+1} \\ \frac{t-1}{t+1} \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + s \begin{pmatrix} d \\ e \\ f \end{pmatrix} = g(s)$$

$$f(0) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{aligned} f(t) &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{4t}{t+1} \\ \frac{-t+1}{t+1} - 1 \\ \frac{t-1}{t+1} + 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{4t}{t+1} \\ \frac{-2t}{t+1} \\ \frac{2t}{t+1} \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \frac{2t}{t+1} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \end{aligned}$$

Problem 7 (1 point each part)

Let $f(t) = \langle 4t, -t+1, t-1, t+1 \rangle$ define a curve in homogeneous coordinates for $t \in [0, 1]$.
(a) What are the values of $f(t)$ at its endpoints (express your answer in non-homogeneous 3D coordinates)?
(b) Construct (in non-homogeneous 3D coordinates) an equation $g(s) = \vec{u} + \vec{v}s$ for this line by using the endpoints you found in (a). You may assume that the curve is in fact a line, even if you did not show this.

(a)

$$f(t) = \begin{pmatrix} 4t \\ -t+1 \\ t-1 \\ t+1 \end{pmatrix}$$

$$f(0) = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$f(1) = \begin{pmatrix} 4 \\ 0 \\ 0 \\ 2 \end{pmatrix} \equiv \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

(b)

$$\begin{aligned} g(s) &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s\left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right) \\ &= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \end{aligned}$$

Problem 8 (1 point each part)

The clipping region in homogeneous equations (3D) is given by $-w \leq x \leq w$, $-w \leq y \leq w$, and $-w \leq z \leq w$. Let $f(t) = \langle 4t, -t+1, t-1, t+1 \rangle$ define a curve in homogeneous coordinates for $t \in [0,1]$.

(a) Clip the equation for the line $f(t)$ in homogeneous coordinates against the clipping constraint $-w \leq x \leq w$. (Find the value of t and homogeneous coordinates for the clipping point.)

(b) What is the clipping constraint corresponding to clip the equation $-w \leq x \leq w$ in non-homogeneous coordinates?

(a)

$$\begin{aligned} -w &= x \\ -(t+1) &= 4t \\ -1 &= 5t \\ t &= -\frac{1}{5} \end{aligned}$$

This is not an intersection, since $t \notin [0,1]$.

$$\begin{aligned} w &= x \\ t+1 &= 4t \\ 1 &= 3t \\ t &= \frac{1}{3} \\ f\left(\frac{1}{3}\right) &= \begin{pmatrix} \frac{4}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \\ \frac{4}{3} \end{pmatrix} \end{aligned}$$

(b)

$$\begin{aligned} -w \leq x &\leq w \\ -1 \leq \frac{x}{w} &\leq 1 \\ -1 \leq x' &\leq 1 \end{aligned}$$