CS 130, Homework 4

Solutions

Problem 1

When computing the eye coordinate system, we used $\mathbf{w} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$, $\mathbf{u} = \frac{\mathbf{t}\times\mathbf{w}}{\|\mathbf{t}\times\mathbf{w}\|}$, and $\mathbf{v} = \mathbf{w} \times \mathbf{u}$. Why did we not normalize v?

$\mathbf{v}$ is automatically normalized. Recall that $\|\mathbf{v}\| = \|\mathbf{w} \times \mathbf{u}\| = \|\mathbf{w}\|\|\mathbf{u}\| \sin\theta = 1$, since we have $\|\mathbf{w}\| = \|\mathbf{u}\| = 1$ and $\theta = \frac{\pi}{2}$.

Problem 2

Label each table entry below "Y" or "N" to indicate whether the operation preserves each feature.

	rotation	**nonuniform scale**	**translation**	**shear**	**projection**	**affine**
parallel lines						
angles						
distances						
intersections						
lines						
circles						
conic sections[1]						

	rotation	nonuniform scale	translation	shear	projection	affine
parallel lines	Y	Y	Y	Y	N	Y
angles	Y	N	Y	N	N	N
distances	Y	N	Y	N	N	N
intersections	Y	Y	Y	Y	Y	Y
lines	Y	Y	Y	Y	Y	Y
circles	Y	N	Y	N	N	N
conic sections	Y	Y	Y	Y	Y	Y

Note that any operation which preserves lengths must necessarily preserve angles as well (think of a triangle); scaling demonstrates that the converse is not true.

The observation that projections preserve conic sections is very useful in some contexts. In particular, projections are a very powerful tool in studying problems involving intersections, conic sections, and lines. A projection can be used to make one or more pairs of lines parallel, or it may be used to turn a parabola, ellipse, or hyperbola into a circle. This may result in a simpler problem.

Problem 3

Are projective transformations linear? Why? Justify your answer.

[1] You will probably need to research this.

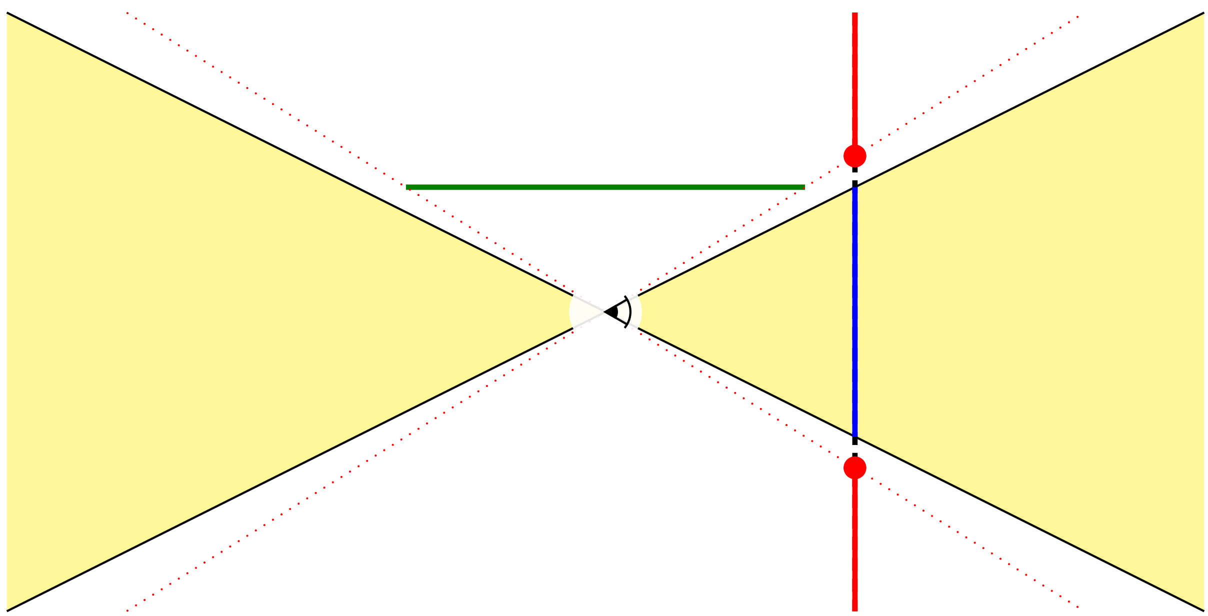
Both "Yes" and "No" may be appropriate, depending on the interpretation. Giving a suitable justification is crucial. As mappings from 3-vectors to 3-vectors, the projective transforms involve division by the last component and are nonlinear. As mappings from 4-vectors to 4-vectors, the transformation is linear and can be represented by a matrix.

Problem 4

Explain why clipping triangles after switching from homogeneous 4-vectors to 3-vectors can give you incorrect results. Give a concrete example exhibiting the failure and explain in detail why your example causes this failure. (You can demonstrate the problem in a lower dimension if you like, such as a triangle in 2D or even a line segment in 1D). Repeating the explanation from the book (or an explanation found online) is not an adequate answer. You must demonstrate that you understand the source of the problem.

In the 1D illustration below (2D with homogeneous coordinates), we are projecting the green segment into the screen (vertical line). The blue region is the viewing frustum. (Note that there are two of them.) The yellow region is inside. We can easily see that the green segment should be eliminated.

Once the w component is divided off, we get the two red dots. The segment is projected into the red segments. The two segments are "connected through infinity," which lies outside the view as it should. However, if we connect the dots with a segment, it will cross through the view and completely fill it. Clipping at this stage would only remove the ends of the segment, leaving it covering the whole screen.



Problem 5

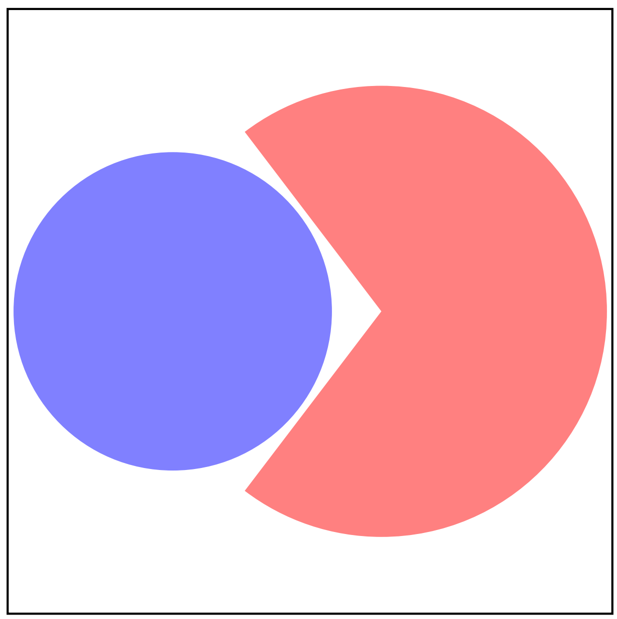
Propose a texture mapping for the entire surface of the cone defined by $x^2+y^2 \leq z^2$, $0 \leq z \leq 1$. The texture coordinates should lie in $[0,1] \times [0,1]$, and the entire cone (both curved and flat parts) should be covered. No two points on the cone should map to the same place in the texture image. In addition to providing the mapping, draw a square (representing the texture image) and sketch the portion of it that is being used for the curved part and the flat part. (You do not have to utilize the whole texture image.)

One nice mapping is obtained by removing the bottom piece and flattening out the cone as if it were made of paper. The two mappings are:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} as\cos\left(\frac{r}{s}\,\mathrm{atan2}(y,x)\right) + p_x \\ as\cos\left(\frac{r}{s}\,\mathrm{atan2}(y,x)\right) + p_y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ax + q_x \\ ay + q_y \end{pmatrix}$$

The first mapping is for the curved part (shown red below), and the second is for the circle (shown blue below). Here, $r = 1$ is the radius, $s = \sqrt{2}$ is the diagonal of the cone (distance from tip to bottom edge), a is a scale needed to make the mapping fit in the page, and p_x, p_y, q_x, and q_y are translations. Shown below is with $a = 0.265$, $q_x = 0.273$, $p_x = 0.618$, $p_y = q_y = 0.5$.



Another reasonable mapping would be to map the flat part onto a circle using the same mapping as for the base, but translated to a different part of the texture. Many other reasonable mappings are possible.