CS 130, Homework 3

Solutions

Problem 1

What problem is a z-buffer intended to solve?

The role of the z-buffer is to determine which of many objects that rasterize to a particular location on the screen should be visible at that location.

Problem 2

OpenGL provides direct support for transmitting triangles (GL_TRIANGLE) and lines (GL_LINE) to be rendered, but it also provides more complex options such as GL_TRIANGLE_STRIP and GL_LINE_LOOP, which do not provide functionality that cannot already be achieved with GL_TRIANGLE and GL_LINE. What role do these more complex options serve?

These more complex options reuse vertices. Since less data must be sent to the GPU, the transfer will be more efficient.

Problem 3

Express the (2D) operator $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$ as a composition of simpler operations: rotations, translations, scales.

Observe that the columns are already orthogonal. All that is required is to normalize them. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$. This is a combination of uniform scale by $\sqrt{2}$ and rotation by $\frac{\pi}{4}$. The order does not matter in this case.

Problem 4

Devise a transform, written as a product of homogeneous translation, rotation, and scale matrices, which will transform the points $(-1,-1)$, $(0,0)$, $(1,-1)$ into the points $(-1,-1)$, $(-2,2)$, $(1,1)$.

One solution is:

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 2\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

This corresponds to translating the midpoint of the first and last vertices at the origin (where it is after the transform). Next, apply a scale to get the lengths right. Finally, rotate it into place.

Problem 5

In the second lab, you drew lines with DDA. In doing this, you compared the slope of the line with 1. What is significant about 1? Why not 2, 3, or $\frac{1}{2}$?

If the slope is larger than 1, then y increases faster than x. If y is increased by more than one, gaps will result.