Intermediate Data Structures & Algorithms – CS 141 (Discussion)

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Some Exercises

• **Upper bound**: Any sum is at most the number of terms times the maximum term.
  
  • *Example*: $F(n) = 1 + 4 + 9 + \cdots + n^2$ is at most $n \times n^2 = n^3$
  
• **Lower bound**: if the terms are non-negative, any sum is at least half the number of terms times the median term.

  • *Example*: $F(n) = 1 + 4 + 9 + \cdots + n^2$ is at least $\left(\frac{n}{2}\right) \times \left(\frac{n}{2}\right)^2 = \frac{n^3}{8}$
Some Exercises

• All logarithmic functions $\log_a n$ belongs to the same class $\theta(\log n)$ no matter what the logarithm’s base $a > 1$ is.
Some Exercises

• **Note:** Exponential functions $a^n$ have different orders of growth for different $a$'s:
  • *Example:* $3^n \notin O(2^n)$

• Determine the asymptotic value of function:
  \[ f(n) = 7n^52^n + 3^n \]

• Asymptotic value for function:
  \[ f(n) = 8^\log_2 n^3 + 7n^2 + 4^{n+1} \log^2 n^5 + 2^{n+5}n^7 \]
Some Exercises

- The Fibonacci numbers $F_0, F_1, F_2, ...$ are defined by the rule:

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$$

In this problem we will confirm that this sequence grows exponentially fast and obtain some bounds on its growth.

1. Use induction to prove that $F_n \geq 2^{0.5n}$ for $n \geq 6$
2. Find a constant $c < 1$ such that $F_n \leq 2^{cn}$ for all $n \geq 0$. Show that your answer is correct.
Some Exercises

• Prove or Disprove:
  • \( f(n) + g(n) = \theta(\min(f(n), g(n))) \)
  • \( f(n) = \theta(f\left(\frac{n}{2}\right)) \)