1. Convert the following numbers to 12-bit binary numbers using 2's complement representation:
   a) $367_{10}$
   b) $-367_{10}$
   c) $-367_{8}$
   d) $367_{16}$

**Answer**

a) 000101101111
b) 111010010001
c) 111100001001
d) 0011 0110 0111
2. Convert the following 2’s complement binary numbers to decimal, octal, & hexadecimal:

   a) 1110011101
   b) 0110111100

   \[
   \begin{array}{cccc}
   \text{decimal} & \text{octal} & \text{hexadecimal} \\
   \hline
   \text{a) 1110011101} & -99 & 7635 & \text{F9D} \\
   \text{b) 0110111100} & 444 & 674 & \text{1BC} \\
   \end{array}
   \]

   1 point for each correct entry.
3. Perform the following calculations using 8-bit 2’s complement binary arithmetic: (8)

a) \[
\begin{array}{c}
11110111 \\
+ 11100111 \\
\end{array}
\]

**Answer**

11101010

b) \[
\begin{array}{c}
11110111 \\
- 11110011 \\
\end{array}
\]

**Answer**

00000100

c) \[
\begin{array}{c}
11110111 \\
\times \ 00001011 \\
\end{array}
\]

**Answer**

\[
\begin{array}{c}
11111111 1110111 \\
11111111 1110111 \\
00000000 000000 \\
+ 11111111 10111 \\
11111111 1001101 \\
\end{array}
\]

\(2\) points for each part.
4. Show that the expression $z + y' + yz'$ is equal to 1 using:

a) a truth table. (2)

b) the axioms and basic theorems. (4)

**Answer:**

a)

<table>
<thead>
<tr>
<th>$y$</th>
<th>$z$</th>
<th>$z + y' + yz'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b) $z + y' + yz'$

$= (y + y')z + y'(z + z') + yz'$

$= yz + y'z + y'z + y'z' + yz'$

$= y(z + z') + y'(z + z')$

$= y + y'$

$= 1$
5. Given $F'(x, y, z) = \Sigma(1, 3, 7)$, express the function $F$ using a truth table. (4)

**Answer:**

$F'$ is expressed as a sum of its 0-minterms. Therefore, $F$ is the sum of its 1-minterms = $\Sigma(0, 2, 4, 5, 6)$. Using three variables, the truth table is as follows:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>Minterms</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$m_0=x'y'z'$</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$m_1=x'yz$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$m_2=x'y'z$</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$m_3=x'y_z$</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$m_4=x'y'z$</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$m_5=x'y_z$</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$m_6=x'z$</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$m_7=xyz$</td>
<td>0</td>
</tr>
</tbody>
</table>
6. Convert the function $F = \Sigma(3, 4, 5)$ to its equivalent product-of-sum canonical form using the axioms and basic theorems. (4)

**Answer:**

\[ F = \Sigma(3, 4, 5) = m_3 + m_4 + m_5 \]
\[ = x'y'z + xy'z' + xy'z \]
\[ = (x' + x + x)(x' + x + y')(x' + x + z) \]
\[ (x' + y' + x)(x' + y' + y')(x' + y' + z) \]
\[ (x' + z' + x)(x' + z' + y')(x' + z' + z) \]
\[ (y + x + x)(y + x + y')(y + x + z) \]
\[ (y + x + x)(y + y' + y')(y + y' + z) \]
\[ (y + z' + x)(y + z' + y')(y + z' + z) \]
\[ (z + x + x)(z + x + y')(z + x + z) \]
\[ (z + y' + x)(z + y' + y')(z + y' + z) \]
\[ (z + z' + z)(z + z' + y')(z + z' + z) \]
\[ = (x' + y' + z) (x' + y' + z') (x + y + z) (x + y + z') (x + y' + z) \]
7. Convert the function \( F = x'y'z + xy'z' + xy'z \) to:

a) its standard form using the minimum number of operators. (2)

b) its nonstandard form using the minimum number of operators. (2)

**Answer:**

a) 
\[
F = x'y'z + xy'z' + xy'z \\
= (x' + x)y'z + xy'(z + z') \\
= y'z + xy'
\]

b) 
\[
F = x'y'z + xy'z' + xy'z \\
= (x' + x)y'z + xy'(z + z') \\
= y'z + xy' \\
= y'(x + z)
\]
8. Using 2-input AND gates, 2-input OR gates, and NOT gates, construct a circuit for the function \( F = x'y'z + x'y + yz' \) without simplifying the expression having the smallest delay from the primary input to the primary output and having the least number of transistors. What are the delay and number of transistors used? (6)

<table>
<thead>
<tr>
<th></th>
<th>Number of transistors</th>
<th>Gate Delay (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>AND</td>
<td>6</td>
<td>2.4</td>
</tr>
<tr>
<td>OR</td>
<td>6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Answer:**

![Circuit Diagram]

Delay: 8.2 ns
Number of transistors: 34

- 6 points for getting a circuit with both the delay and number of transistors correct.
- 4 points for getting a circuit with either the delay or number of transistor correct.
- 2 points for getting only a functional correct circuit but not the delay and number of transistors.