# Ink Segmentation 

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## Outline

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## Motivation

- Segment pen strokes automatically into the intended lines and arcs

- The outcome will be the input of higher level sketch parser and symbol recognizer
- Determine which bumps and bends are intended, and which are accidents
- Match the drawer's intent not the ink


## Motivation (cont)

- Consider the shape alone is not enough
- Slow the pen when making intentional discontinuities in a shape




## Contribution

- Use pen speed and curvature to figure out the segments
- Accuracy between 92\% to 96\%


## Background - Pen Speed

- Arc length of each coordinate point

$$
d_{i}=\sum_{j=1}^{i}\left|\vec{P}_{j}-\vec{P}_{j-1}\right|
$$

- Speed

$$
s_{i}=\frac{d_{i+1}-d_{i-1}}{t_{i+1}-t_{i-1}}
$$

- Smooth

$$
S_{i}=\left(S_{i-1}+S_{i}+S_{i+1}\right) / 3
$$

## Background - Curvature

- Derivative of the tangent angle with respect to arc length

$$
C=\frac{\partial \theta}{\partial s}
$$



## Background - Least Squares Line Fit

- $y=A x+B$
- Regression equation

$$
\left[\begin{array}{ll}
n & \sum x_{i} \\
\sum x_{i} & \sum x_{i}^{2}
\end{array}\right]\left[\begin{array}{l}
A \\
B
\end{array}\right]=\left[\begin{array}{c}
\sum y_{i} \\
\sum x_{i} y_{i}
\end{array}\right]
$$

- Total Square Error/Error of Fit

$$
\sum_{i=1}^{n}\left(A x_{i}+B-y_{i}\right)^{2}
$$

## Background - Least Squares Line Fit (Cont)



## Background - Least Squares Circle Fit

- $\mathrm{X}^{2}+\mathrm{y}^{2}+2 \mathrm{ax}+2 \mathrm{by}+\mathrm{c}=0$
- $r=\sqrt{a^{2}+b^{2}-c}$
- Regression equation

$$
\left[\begin{array}{lll}
2 \sum x_{i}^{2} & 2 \sum x_{i} y_{i} & \sum x_{i} \\
2 \sum x_{i} y_{i} & 2 \sum y_{i}^{2} & \sum y_{i} \\
2 \sum x_{i} & 2 \sum y_{i} & n
\end{array}\right]\left[\begin{array}{l}
a \\
b \\
c
\end{array}\right]=\left[\begin{array}{l}
\sum-\left(x_{i}^{2}+y_{i}^{2}\right) x_{i} \\
\sum-\left(x_{i}^{2}+y_{i}^{2}\right) y_{i} \\
\sum-\left(x_{i}^{2}+y_{i}^{2}\right)
\end{array}\right]
$$

- Total Square Error/Error of Fit

$$
\sum_{i=1}^{n}\left(x_{i}^{2}+y_{i}^{2}+2 a x_{i}+2 b y_{i}+c\right)^{2}
$$

## Background - Least Squares Circle Fit (cont)



## Background - Least Squares Circle Fit (cont)



## How It Works

- Compute Pen Speed and Curvature
- Select candidate/initial segment points
- Points that are both a minima of speed ( $<25 \%$ of average speed) and maxima of curvature
- Discard the point
- If it is within 7 data points of a subsequent segment point
- If the first segment contains < 15 data points
- If the first or last segment is much shorter than its immediate neighbors


## How It Works (cont)

- Fit primitives to the segments
- Construct both line and circle fit for the segment between each pair of consecutive segment points
- Pick one with the smallest Error of Fit
- If the fit is an arc, it also has to be at least one tenth of a circle ( $36^{\circ}$ )


## How It Works (cont)

- Merge
- If a segment is shorter than $20 \%$ of the length of its adjacent segment, or, if adjacent segments are of the same type, the program tries to merge them
- If error of fit of new segment is < $10 \%$ of the sum of fit errors of the original 2 segments, new segment is used


## How It Works (cont)

- Split
- If neither line nor arc fits the ink, the program tries to split them by including a segment point based on a change in the sign of the curvature
- All points of curvature sign changing in a segment are considered, the minimum that is $<65 \%$ of the original fit error will be retained
- If there is no point whose curvature changing sign, it will try to include the points whose speed are less then $130 \%$ of the average speed


## How It Works (cont)

- Summary
- Determine initial set of segment points
- Fit line/arc to the segments
- Filter noise at the start and end segments
- Merge segments
- Split segments
- Merge segments again


## Conclusion

- Use pen speed to segment pen stroke into lines and arcs
- Work better with curvature
- Achieve accuracy of 92\% - 96\%


## Reference

- Thomas F. Stahovich. Segmenting Hand Drawn Curves Using Pen Speed. 2005
- Thomas F. Stahovich. Segmentation of Pen Strokes Using Pen Speed. 2004
- Chris Calhoun, Thomas F. Stahovich, Tolga Kurtoglu, Levent Burak Kara. Recognizing Multi-Stroke Symbols

