CS 231

Inverse Kinematics
Intro to Motion Capture

3D characters

Representation

1) Skeleton
   Origin (root)
   Joint centers/
bones lengths

2) Keyframes
   Pos/Rot Root (x)
   Joint Angles (q)
Kinematics – study of static movement

Forward: \( x = f(\alpha, \beta) \)
- easy to compute

Inverse: \( \alpha, \beta = f^{-1}(x) \)
- intuitive to control
- positioning hand/feet
- higher-level goals

Forward Kinematics

\[
x = l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3)
y = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3)
\]
Inverse Kinematics (IK)

What is it?

Solving IK

Kinematic equations

\[
\begin{align*}
    x &= l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3) \\
    y &= l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3)
\end{align*}
\]

Number of equation : 2
Unknown variables : 3

\textit{Infinite number of solutions !}
Redundancy

System DOF > End Effector DOF

\[ x = l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3) \]
\[ y = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3) \]

System DOF = 3
End Effector DOF = 2

Redundancy in humans?

A redundant system has infinite number of solutions

Human skeleton has 70 DOF
Super redundant

How to solve highly redundant system?
Some intuition: Naturalness

Based on observation of natural human posture

Neurophysiological/ biomech experiments

Example
Hybrid/analytical solution for human arm
Comfort pose
Planning

Inverse Kinematics Solvers

Analytical - only for simplest systems
Optimization - requires performance criteria
Hybrid - combine heuristic and optimization
Nonlinear Optimization

Optimization Problem

\{ \text{minimize } \text{Goal}(\theta) \}

Available Optimization Packages
MATLAB,
Numerical Recipes
Etc...

Goal Potential Function

\[ \|p_g - p_e\|^2 \]
Position Goal

\theta_1 \quad \theta_2 \quad \theta_3
Base

Goal
End Effector
distance
Jacobian

\[ x = f(\theta) \quad \dot{x} = J\dot{\theta} \quad J = \frac{\partial f}{\partial \theta} \]

\[ \dot{\theta} = J^{-1}\dot{x} \]

Jacobian Matrix

Linearity relates end-effector change to joint angle change

Differential Kinematics

3-link arm example

\[
\begin{bmatrix}
    x \\
y
\end{bmatrix} = \begin{bmatrix}
f_1(\theta) \\
f_2(\theta)
\end{bmatrix} = \begin{bmatrix}
l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 \\
l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \dot{x} \\
    \dot{y}
\end{bmatrix} = \begin{bmatrix}
    \dot{\theta}_1 \\
    \dot{\theta}_2 \\
    \dot{\theta}_3
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
\frac{\partial f_1(\theta)}{\partial \theta_1} & \frac{\partial f_1(\theta)}{\partial \theta_2} & \frac{\partial f_1(\theta)}{\partial \theta_3} \\
\frac{\partial f_2(\theta)}{\partial \theta_1} & \frac{\partial f_2(\theta)}{\partial \theta_2} & \frac{\partial f_2(\theta)}{\partial \theta_3}
\end{bmatrix} = \begin{bmatrix}
-l_1 \sin \theta_1 & -l_2 \sin \theta_2 & -l_3 \sin \theta_3 \\
l_1 \cos \theta_1 & l_2 \cos \theta_2 & l_3 \cos \theta_3
\end{bmatrix}
\]
Differential Kinematics

**J is not always invertible**

Remedy : Pseudo Inverse

\[
J^+ = J^T (JJ^T)^{-1}
\]

\[
\dot{\theta} = J^+ \dot{x}
\]

**Problems with IK**

Will \( G(\theta) = 0 \) always obtainable?

No : Unreachable Workspace

Will the solution be always found?

No : Local Minima/Singular Configuration

Will the solution be always unique?

No : Redundancy
Constrained optimization with fewer variables
-one-variable parameterization for arms and legs

Lee and Chen ’99
Hybrid Solutions

\[ \cos(\phi) = \frac{r^2 - r_1^2 - r_2^2}{2r_1r_2} \]

Still must choose “swivel angle”

Hybrid solution: IKAN

“Inverse Kinematics using ANalytical Methods” IK for an anthropomorphic arm or leg

Combination of analytic and numerical methods to solve generalized inverse kinematics problems

Results faster and more reliable than conventional inverse jacobian and optimization based techniques.

Download IKAN from SourceForge
Motion Capture Animation Technologies
What is motion capture?

Recording data from the real world and using it to animate

Usually, one of a few specific technologies used to capture a human actor in order to animate a character

Got its start in biomechanics

Motivation
Why use motion capture?

Captures realistic movements, can be specific to individuals, good for athletics and physical activities

Can capture a lot of data reasonably quickly

Can capture on-the-fly for real-time performance and interactivity

Where is motion capture used most?

Many games, capture celebrities, lifelike motion
What are the issues?

- Recording the data accurately and quickly
  - types of technologies
  - running a motion capture shoot
- Applying the data to the character
  - filtering the data
  - fitting to a skeleton
- Editing motion data post-mortem
  - changing and concatenating
  - generalized motion from a database

Types of technologies - Optical Systems

Vicon (also Motion Analysis)
Types of technologies - Optical Systems

10-24+ cameras
1000 frames/sec at 1280x1024 resolution (!)
4mm size marker @ 10m within < 1mm
Cost: $30-150K+

Plus: High data rate
Best motion in the industry!

Minus: Expense
Occlusion
Relies on software for data resolution
Alternative- Rent a day in the studio: $10-20K

Types of technologies - Active Optical Systems

Ascension's Reactor (Optotrak, PhaseSpace)
Types of technologies - Active Optical Systems

- 540 cameras
- 900 measures/second (up to 60 frames/sec)
- Capture region: 3x4 meters
- Cost: ~$70K
- Plus: Minimal data loss after occlusion re-identifies
- No environment restrictions
- Reasonably priced?
- Minus: Low capture rate, small region

Types of technologies - Electromagnetic Systems

- Ascension (Polhemus) – tethered and tetherless
Types of technologies - Electromagnetic Systems

18 sensors six dof's
120 frames/sec
<1 cm/~1 degree accuracy
3m circular capture area
Cost: ~$50K

Plus: No occlusion
   Orientation information recorded
Minus: Environment restrictions
   Can be bulky

Types of technologies - Mechanical Systems

Exoskeletons
Joint sensors plus a gyroscope
Types of technologies - Mechanical Systems

500 samples/second
No range limits/Wired system
Cost: ~40K

Plus: Fits a rigid body skeleton well
   High data rate
Minus: Tethered
   Not accurate in body location/orientation

Types of technologies - Mechanical Systems

Monkeys and other input devices
Types of technologies - Kinect

RGB camera plus range scanner

- 60? samples/second
- Small range limits
- Cost: $150

**Plus:** Very Inexpensive
  - Depth information
**Minus:** Noisy
  - Limits on good tools
Marker placement

On limbs vs joints – neither is ideal
Over tight clothing or thin skin
In repeatable 'landmarks'
Using standard marker sets
Marker placement

Magnetic

Optical

Using marker data - mapping

Problems: no perfect match, joint-center and rigid-body assumptions, limits on ranges of motion, aesthetic and production requirements
Using marker data - skeleton estimation

Sources of noise

Outliers

Joints are approximated as pivots

Simplifications (like a rigid back)

Markers move on the skin, clothing

Errors may accumulate
Filtering

- Gets rid of outliers
- Smoothes data
- But removes important details!
- All data is filtered

Some notes about IK and mocap

- Full-body IK starting from root places feet on the ground – assumes body root is correct, (foot slide becomes a new problem)
- Frame-to-frame coherence achieved by initializing frame with previous
Known problems with the data

Correcting errors

Modifying data to fit your character

Edit the data to do something new

Connecting data sequences

Generalize a pool of data