CS 231

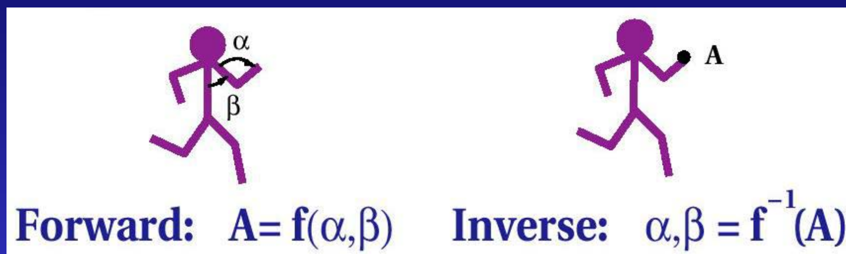
Kinematics and IK

Project Proposals (3pages)

Short Abstract: What's the idea?
Goals: What do you want to learn?
Detailed summary: be specific!

- What are minimum requirements for completion?
- How will you realize the system? (what software/libs, what end-product, animations)
- Possible additions (time permitting)

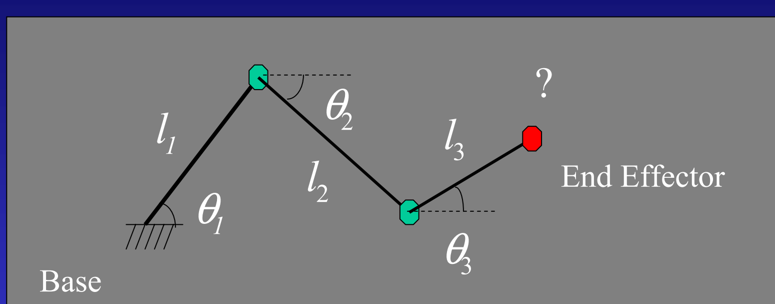
Kinematics – study of static movement



easy to compute

intuitive to control
positioning hand/feet
higher-level goals

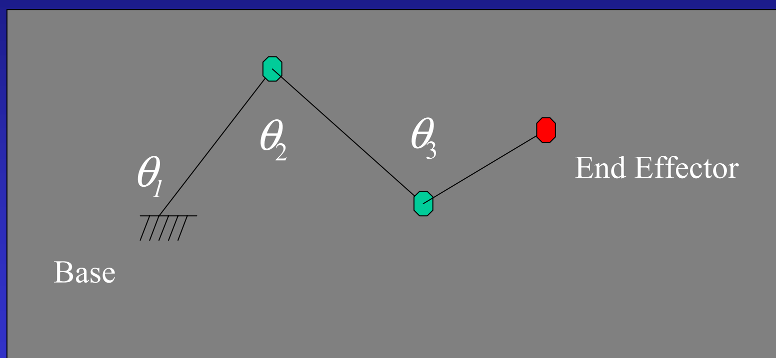
Start with Kinematics



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3)$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3)$$

Inverse Kinematics

- **What is Inverse Kinematics?**



Solving IK

- **Kinematic equations**

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3)$$

$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3)$$

Number of equation : 2

Unknown variables : 3

→ ***Infinite number of solutions !***

Redundancy

System DOF > End Effector DOF

Our example

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + l_3 \cos(\theta_3)$$

$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + l_3 \sin(\theta_3)$$

- **System DOF = 3**
- **End Effector DOF = 2**

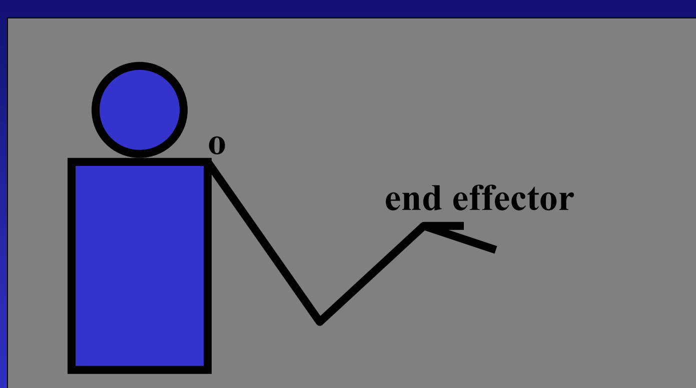
Redundancy in humans?

- **A redundant system has infinite number of solutions**
- **Human skeleton has 70 DOF**
  - **Ultra-super redundant**
- **How to solve highly redundant system?**

Some intuition: Naturalness

- Based on observation of natural human posture
- Neurophysiological/biomech experiments
- Example
  - Hybrid/analytical solution for human arm
  - Comfort pose
  - Planning

Inverse Kinematics - solvers



analytical - only for simplest systems
optimization - requires performance criteria
hybrid - combine heuristic and optimization

Nonlinear Optimization

- **Optimization Problem**

$$\left\{ \text{minimize } Goal(\boldsymbol{\theta}) \right.$$

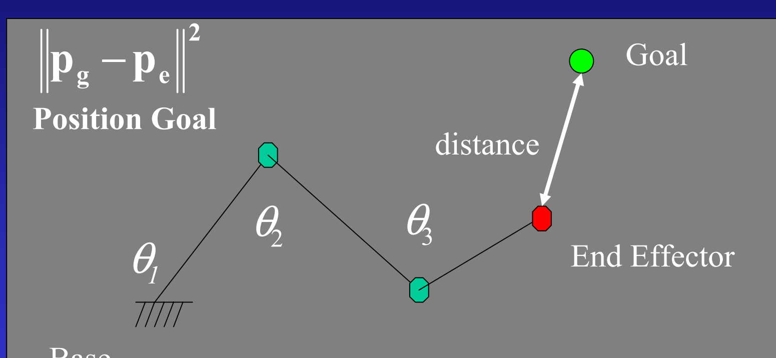
Available Optimization Packages

MATLAB,

Numerical Recipes

Etc...

Goal Potential Function



Jacobian

$$\mathbf{x} = f(\boldsymbol{\theta}) \qquad \dot{\mathbf{x}} = \mathbf{J}\dot{\boldsymbol{\theta}} \qquad \mathbf{J} \equiv \frac{\partial f}{\partial \boldsymbol{\theta}}$$

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^{-1}\dot{\mathbf{x}}$$

- **J : Jacobian Matrix**
  - **Linearly relates end-effector change to joint angle change**

Differential Kinematics

- **Our Example**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_1(\boldsymbol{\theta}) \\ f_2(\boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 \\ l_1 \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 \end{bmatrix} \qquad \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_2} & \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_3} \\ \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_2} & \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 & -l_2 \sin\theta_2 & -l_3 \sin\theta_3 \\ l_1 \cos\theta_1 & l_2 \cos\theta_2 & l_3 \cos\theta_3 \end{bmatrix}$$

Differential Kinematics

- **Is J always invertible? No!**
  - **Remedy : Pseudo Inverse**

$$\mathbf{J}^{+} = \mathbf{J}^{T}(\mathbf{J}\mathbf{J}^{T})^{-1}$$

$$\dot{\theta} = \mathbf{J}^{+}\dot{\mathbf{x}}$$

Calculating Pseudo Inverse?

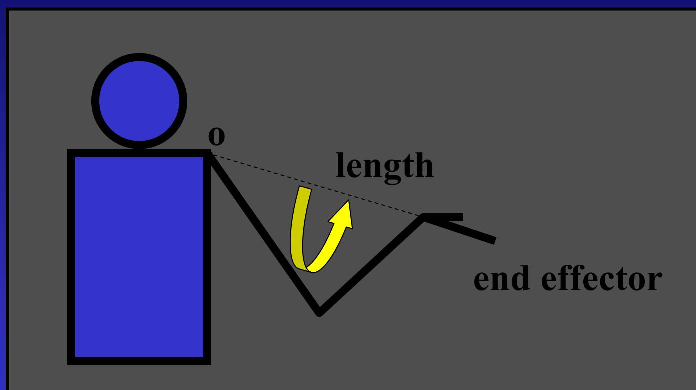
- **Gaussian Elimination**

  (use MATLAB, etc.)

Problems with IK

- Will G(θ) be always zero?
  - No : Unreachable Workspace
- Will the solution be always found?
  - No : Local Minima/Singular Configuration
- Will the solution be always unique?
  - No : Redundancy

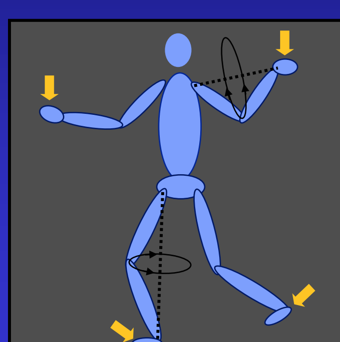
Hybrid Solutions



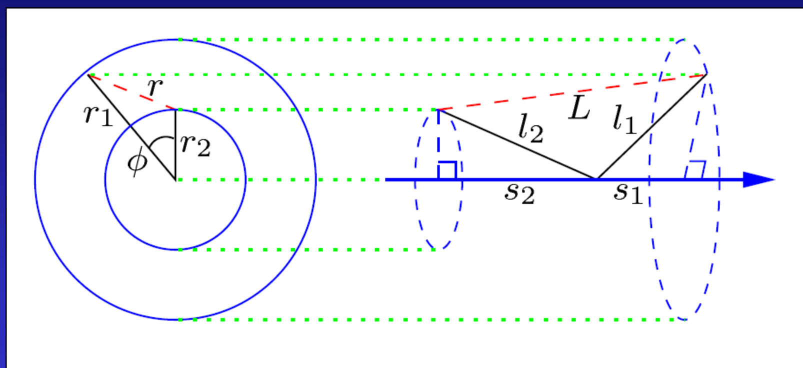
- Apply to arms and legs
- Lee and Chen '99

Inverse Kinematics (Lee and Shin 99)

Constrained optimization with fewer variables
-one-variable parameterization for arms and legs

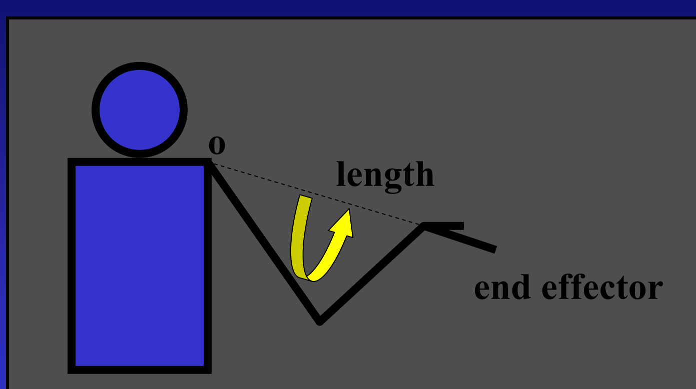


Hybrid Solutions



"Elbow Circle"
[Korein and Badler 82]

Hybrid Solutions



- $\cos(\theta_{elbow}) = (L^2 - L_1^2 - L_2^2) / 2 L_1 L_2$
- Still must choose "swivel angle"

Hybrid solution: IKAN

- "Inverse Kinematics using ANalytical Methods" IK for an anthropomorphic arm or leg
- Combination of analytic and numerical methods to solve generalized inverse kinematics problems
- Results faster and more reliable than conventional inverse jacobian and optimization based techniques.
- See Journal - Graphical Models on Real-time IK techniques
- Download IKAN from SourceForge