Normal Forms

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1 Basics

Definition 1.1 Funtional Dependency *given a relation R,* $X \rightarrow Y$ *is a FD iff* $t_1 \in R \wedge t_2 \in R \wedge \pi_X(t_1) = \pi_X(t_2) \rightarrow \pi_Y(t_1) = \pi_Y(t_2)$

Definition 1.2 Legal Instances *a instance* r *of* R *is legal wrt to* f *if it complies with* f*. A instance* r *violates a* f *if the functional dependency does not hold in* r

Definition 1.3 Valid Relations/ R Holds f *a relation* R *is valid wrt to* f *if all the possible instances of* r *complies with the* f

You can not prove that a functional dependency is valid given one instance. You can check if the FD is illegal or violated for some instance.

Definition 1.4 Superkey/Key $X \subseteq R \wedge X \rightarrow R$ *the* X *is a superkey. A key is a minimal superkey*

Definition 1.5 Implication *A FD* f *is implied by a set* F *iff* f *holds whenever* F *holds*

Definition 1.6 Closure *we define the closure of* F *(denoted by* F^+*) as the set of all the FD implied by* F

Definition 1.7 Armstrong Rules *the following rules can be used to find* F^+

- *Reflexibity:* $Y \subset X$ *then* $X \rightarrow Y$ *(trivial)*
- *Augmentation:* $X \rightarrow Y$ *then* $XZ \rightarrow YZ$
- *Transitivity:* $X \rightarrow Y$ *and* $Y \rightarrow Z$ *then* $X \rightarrow Z$

This rules are sound and complete.

- Union: $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$
- Decomposition: $X \rightarrow YZ$ then $X \rightarrow Y$ and $X \rightarrow Z$

Definition 1.8 Attribute Closure *we define the attribute closure of X wrt F (denoted by X_F^+) as the set of all the attributes such $X \rightarrow A$ in F^+*

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closure = X;
repeat until there is no change: {
  if there is an FD U -> V in F such that U in closure,
      then set closure = closure + V
}
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Definition 1.9 BCNF *R is in BCNF if for all $X \rightarrow A$ in F^+*

- *$X \subset A$ is trivial*
- *X contains a key for R*

Definition 1.10 3NF *R is in 3NF if for all $X \rightarrow A$ in F^+*

- *$X \subset A$ is trivial*
- *X contains a key for R*
- *A is part of some key for R*

$BCNF \subseteq 3NF \subseteq 2NF \subseteq 1NF$

Determining the keys is NP complete

Definition 1.11 Decomposition *given a relation R a decomposition consist in replace R with two relations R_1 and R_2 such that $R_1 \cup R_2 = R$ and $R_1 \cap R_2 = \emptyset$*

Definition 1.12 Lossless Join *a descomposition of R in X, Y is lossless wrt F if $\pi_(X)(R) \bowtie \pi_Y(R) = R$*

Theorem 1.13 *Let R be a relation and F a set of FD over R. Then R_1, R_2 is a lossless decomposition if F^+ contains either $R_1 \cap R_2 \rightarrow R1$ or $R_1 \cap R_2 \rightarrow R2$.*

Definition 1.14 Dependency Preserving *A decomposition R in X, Y is dependency preserving wrt F if we can check each original dependency in X or Y. F $((F_X^+ \cup F_Y^+) = F^+)$*

BCNF decomposition If $X \rightarrow A$ cause problems, create $R - A$ y XA