Systems and Internet
Infrastructure Security

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Static Analysis

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Outline

- Static Analysis Goals
- Static Analysis Concepts
- Abstract Interpretation
- Interprocedural Dataflow Analysis

Our Goal

- In this course, we want to develop techniques to detect vulnerabilities and fix them automatically
- What's a vulnerability?
- How to fix them?

- *Today we will start to develop some of the techniques that we will use*

Vulnerability

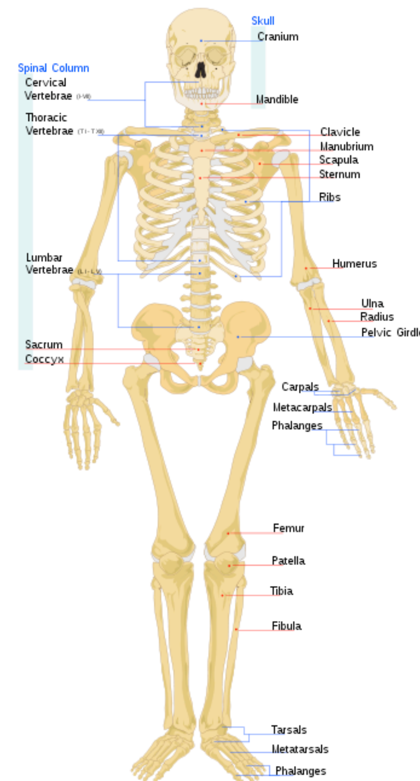
- How do you define computer ‘vulnerability’?
  - *Flaw*
  - *Accessible to adversary*
  - *Adversary has ability to exploit*

Vulnerability

- How do you define computer 'vulnerability'?
  - *Flaw – Can we find flaws in source code?*
  - *Accessible to adversary – Can we find what is accessible?*
  - *Adversary has ability to exploit – Can we find how to exploit?*

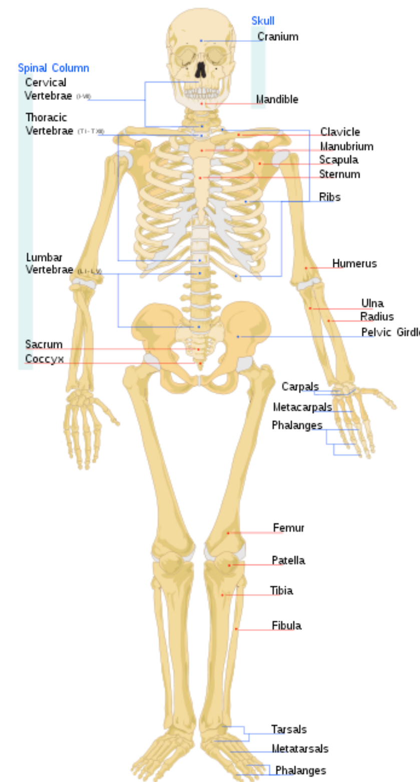
Anatomy of Control Flow Attacks

- Two steps
- First, the attacker changes the control flow of the program
  - In buffer overflow, overwrite the return address on the stack
  - What are the ways that this can be done?
- Second, the attacker uses this change to run code of their choice
  - In buffer overflow, inject code on stack
  - What are the ways that this can be done?



Anatomy of Control Flow Attacks

- Two steps
- First, the attacker changes the control flow of the program
  - In buffer overflow, overwrite the return address on the stack
  - *How can an adversary change control?*
- Second, the attacker uses this change to run code of their choice
  - In buffer overflow, inject code on stack
  - *How can we prevent this? ROP conclusions*



Static Analysis

- Explore all possible executions of a program
  - All possible inputs
  - All possible states

A Form of Testing

- Static analysis is an alternative to runtime testing
- Runtime
  - Select concrete inputs
  - Obtain a sequence of states given those inputs
  - Apply many concrete inputs (i.e., run many tests)
- Static
  - Select abstract inputs with common properties
  - Obtain sets of states created by executing abstract inputs
  - One run

Static Analysis

- Provides an approximation of behavior
- “Run in the aggregate”
  - Rather than executing on ordinary states
  - Finite-sized descriptors representing a collection of states
- “Run in non-standard way”
  - Run in fragments
  - Stitch them together to cover all paths
- Runtime testing is inherently incomplete, but static analysis can cover all paths

Static Analysis

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Static Analysis Example

- Descriptors represent the sign of a value
  - Positive, negative, zero, unknown
- For instruction, *c = a * b*
  - If *a* has a descriptor *pos*
  - And *b* has a descriptor *neg*
- What is the descriptor for *c* after that instruction?
- How might this help?

Descriptors

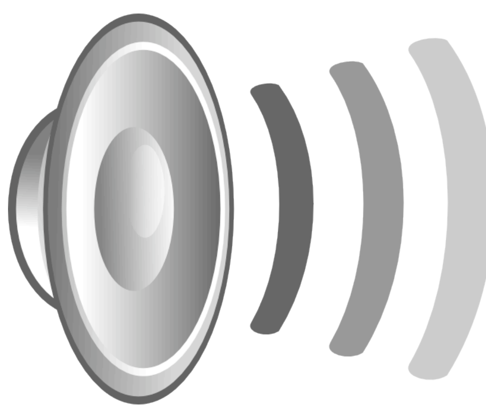
- Choose a set of descriptors that
  - Abstracts away details to make analysis tractable
  - Preserves enough information that key properties hold
    - Can determine interesting results
- Using *sign* as a descriptor
  - Abstracts away specific integer values (billions to four)
  - Guarantees when *a*b = 0* it will be zero in all executions
- Choosing descriptors is one key step in static analysis

Precision

- Abstraction loses some precision
- Enables run in aggregate, but may result in executions that are not possible in the program
  - *(a <= b)* when both are *pos*
  - If *b* is equal to *a* at that point, then false branch is never possible in concrete executions
- Results in false positives

Soundness

- The use of descriptors "over-approximates" a program's possible executions
- Abstraction must include all possible legal values
  - May include some values that are not actually possible
- The run-in-aggregate must preserve such abstractions
  - Thus, must propagate values that are not really possible



Implications of Soundness

- Enables proof that a class of vulnerabilities are completely absent
  - No false negatives in a sound analysis
- Comes at a price
  - Ensuring soundness can be complex, expensive, cautious
- Thus, unsound analyses have gained in popularity
  - Find bugs quickly and simply
  - Such analyses have both false positives and false negatives

What Is Static Analysis?

- Abstract Interpretation
  - Execute the system on a simpler data domain
    - Descriptors of the *abstract domain*
  - Rather than the *concrete domain*
- Elements in an abstract domain represent sets of concrete states
  - Execution mimics all concrete states at once
- Abstract domain provides an over-approximation of the concrete domain

Abstract Domain Example

- Use interval as abstract domain
  - *b* = [40, 41]
- *a* = 2**b*
  - a = [x, y]?
- What are the possible concrete values represented?
  - Which concrete states are possible?

Joins

- A join combines states from multiple paths
  - Approximates set-union as either path is possible
- Use Interval as abstract domain
  - *a* = [36, 39], *b* = [40, 41]
- *If (a >= 38) a=2*b; /* join */*
  - a = [x, y], b=[40, 41] – what are x and y?
- What's the impact of over-approximation?

Impact of Abstract Domain

- The choice of abstract domain must preserve the over-approximation to be sound (no false negatives)
- Integer arithmetic vs 2's-complement arithmetic
- *a = [126, 127], b = [10, 12]*
  - What is c = *a+b* in an 32-bit machine?
  - What is c = *a+b* in an 8-bit machine?

Successive Approximation

- The abstract execution of a system can often be cast as a problem of solving a set of equations by means of successive approximation.
- If constructed correctly, the execution of the system in the abstract domain over-approximates the semantics of the original system
    - Any behavior not exhibited by the abstract domain cannot be exhibited during concrete system execution.

Abstract Interpretation

- Patrick Cousot
  - Class slides/notes from MIT
  - http://web.mit.edu/afs/athena.mit.edu/course/16/16.399/www/

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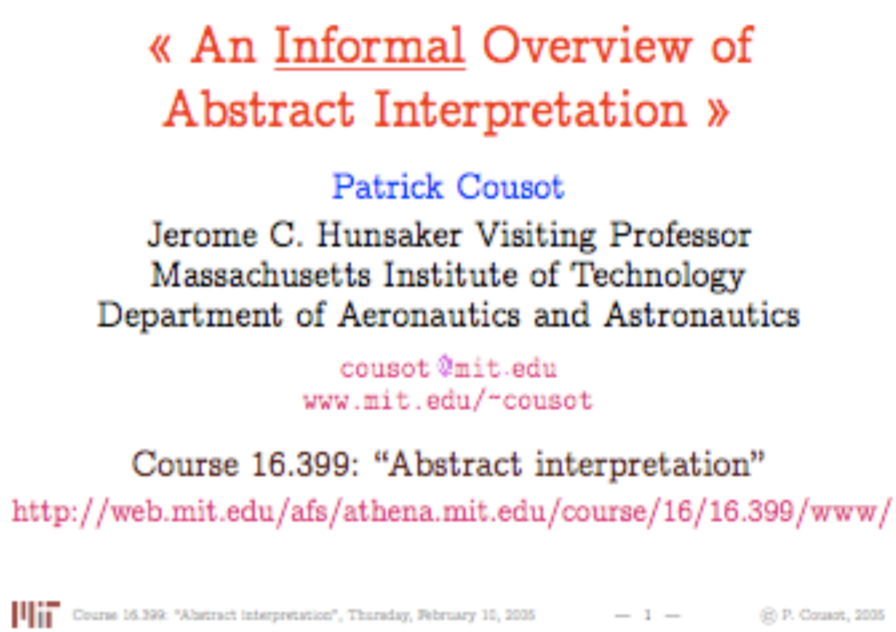

« An Informal Overview of Abstract Interpretation »

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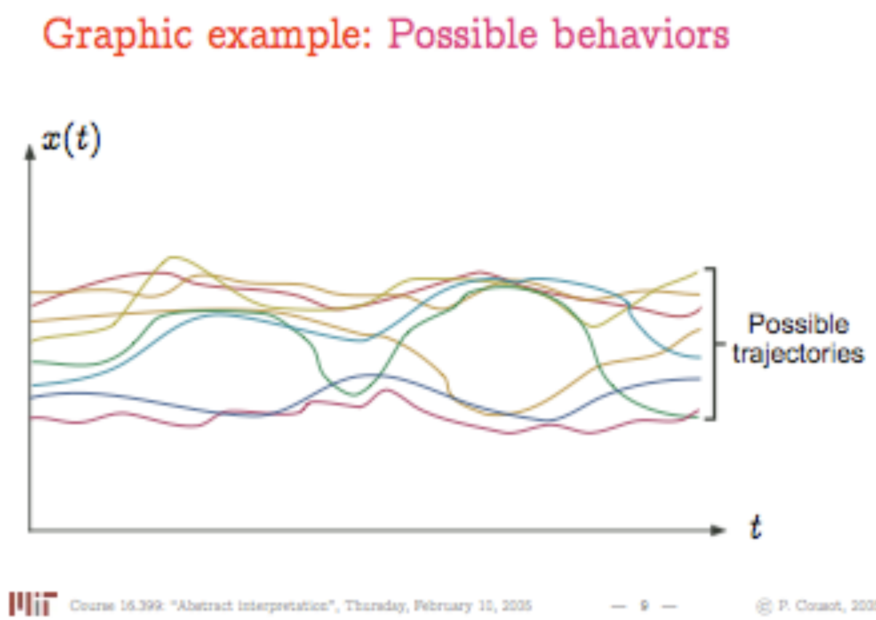
Abstract Interpretation



Abstract Interpretation

Undecidability

- The concrete mathematical semantics of a program is an "infinite" mathematical object, *not computable*;
- All non trivial questions on the concrete program semantics are *undecidable*.

Example: Kurt Gödel argument on termination

- Assume `termination(P)` would always terminates and returns `true` iff `P` always terminates on all input data;
- The following program yields a contradiction

$$\texttt{P} \equiv \texttt{while termination(P) do skip od.}$$

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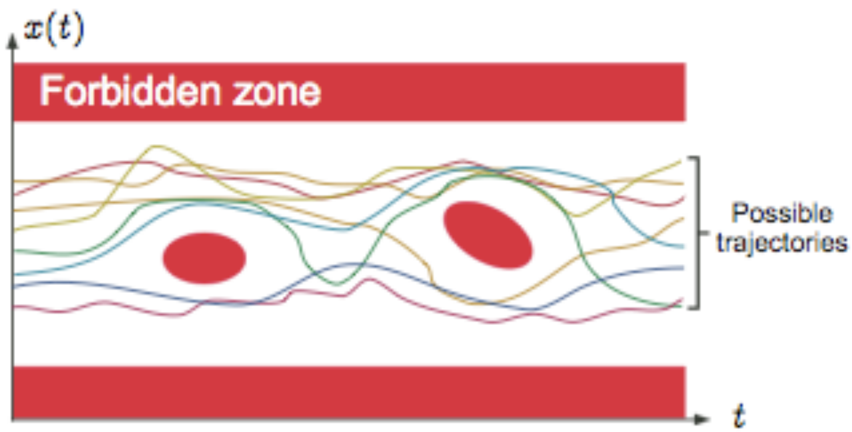
Abstract Interpretation

Graphic example: Safety properties

The *safety properties* of a program express that no possible execution in any possible execution environment can reach an erroneous state.

Graphic example: Safety property



Safety proofs

- A safety proof consists in proving that the intersection of the program concrete semantics and the forbidden zone is empty;
- Undecidable problem (the concrete semantics is not computable);
- Impossible to provide completely automatic answers with finite computer resources and neither human interaction nor uncertainty on the answer [2].

[2] e.g. probabilistic answer.

Abstract Interpretation

Abstract interpretation

– consists in considering an *abstract semantics*, that is to say a superset of the concrete semantics of the program;
– hence the abstract semantics covers all possible concrete cases;
– correct: if the abstract semantics is safe (does not intersect the forbidden zone) then so is the concrete semantics.

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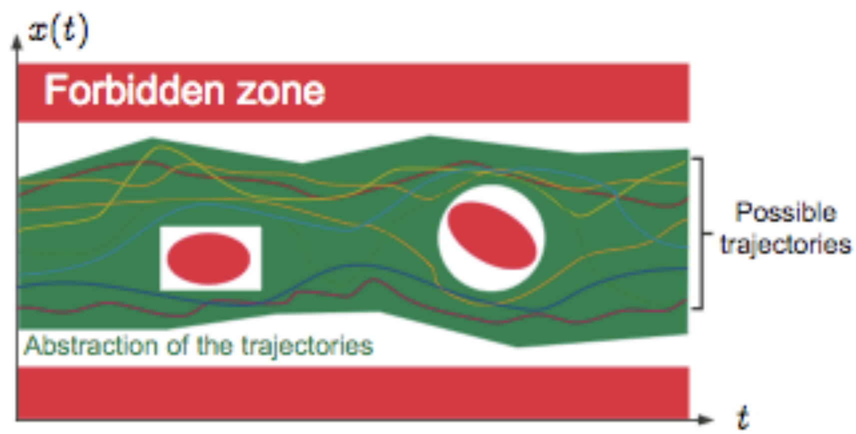
Graphic example: Abstract interpretation



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Abstract Interpretation

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Formal methods

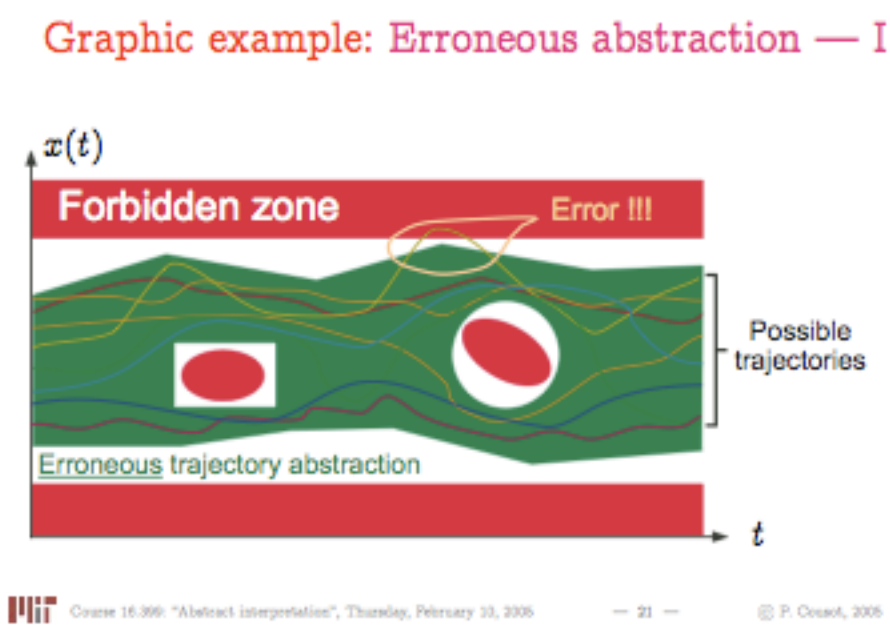
Formal methods are abstract interpretations, which differ in the way to obtain the abstract semantics:

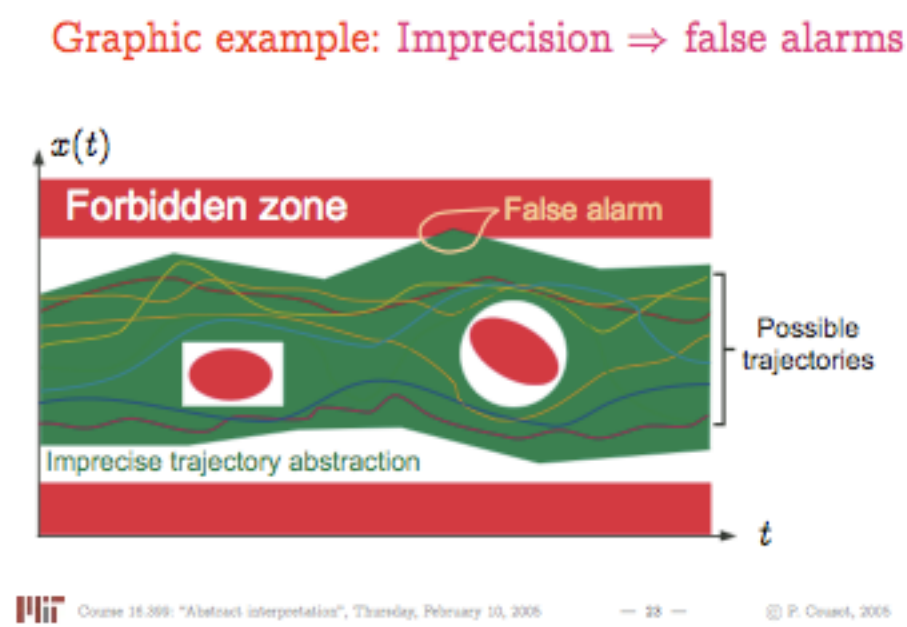
– "*model checking*":
  - the abstract semantics is given manually by the user;
  - in the form of a finitary model of the program execution;
  - can be computed automatically, by techniques relevant to static analysis.

– "*deductive methods*":
  - the abstract semantics is specified by verification conditions;
  - the user must provide the abstract semantics in the form of inductive arguments (e.g. invariants);
  - can be computed automatically by methods relevant to static analysis.

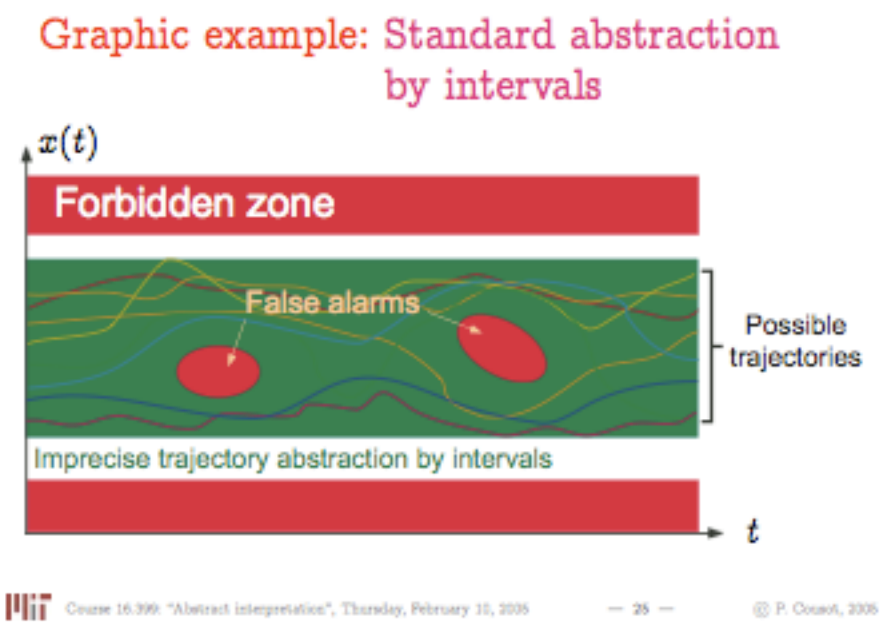
– "*static analysis*": the abstract semantics is computed automatically from the program text according to predefined abstractions (that can sometimes be tailored automatically/manually by the user).

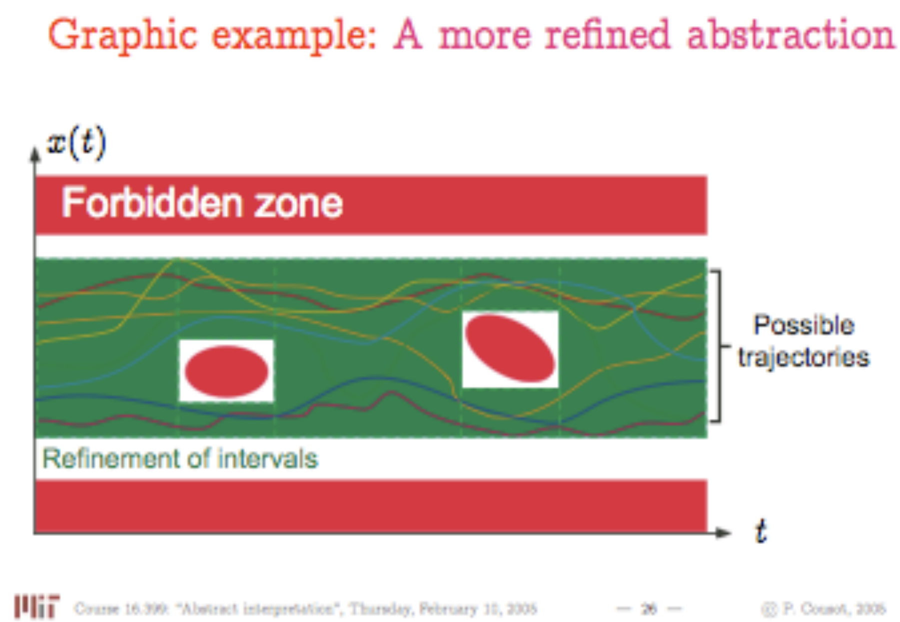
Abstract Interpretation





Abstract Interpretation





Abstract Interpretation

Abstraction by Galois connections

Abstracting sets (i.e. properties)

- Choose an abstract domain, replacing sets of objects (states, traces, ...) S by their abstraction $\alpha(S)$
- The abstraction function α maps a set of concrete objects to its abstract interpretation;
- The inverse concretization function γ maps an abstract set of objects to concrete ones;
- Forget no concrete objects: (abstraction from above) $S \subseteq \gamma(\alpha(S))$.

Abstract Interpretation

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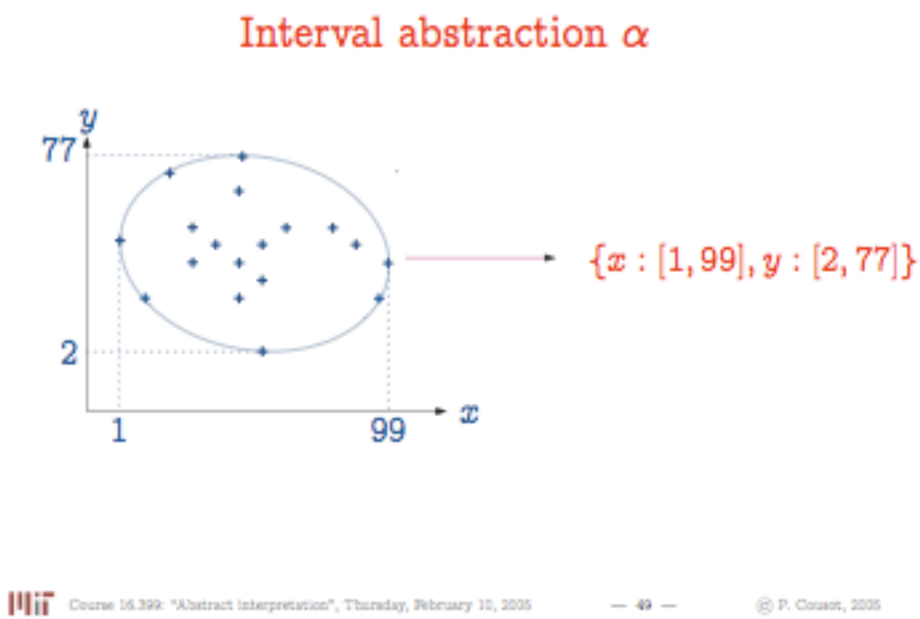

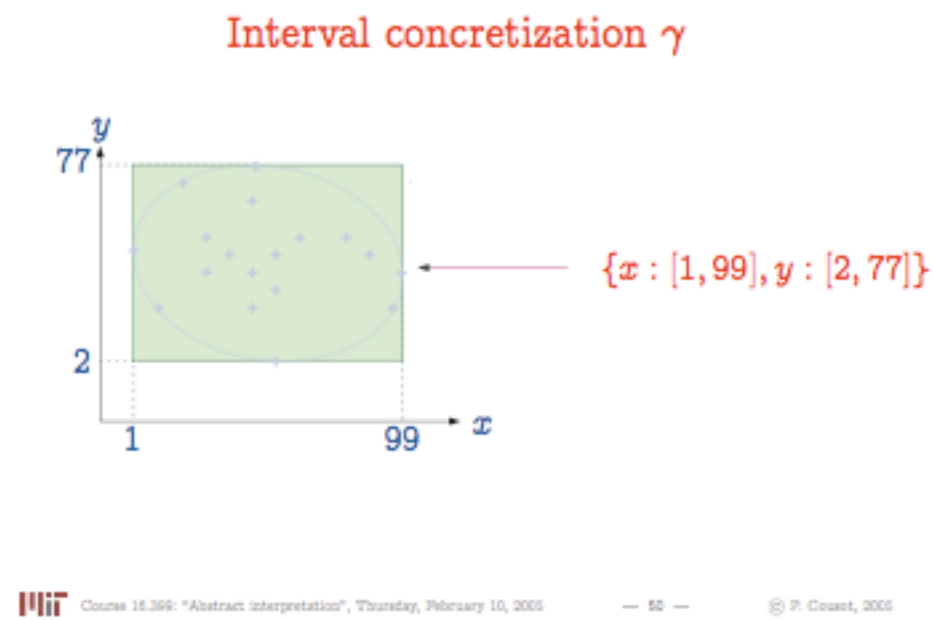

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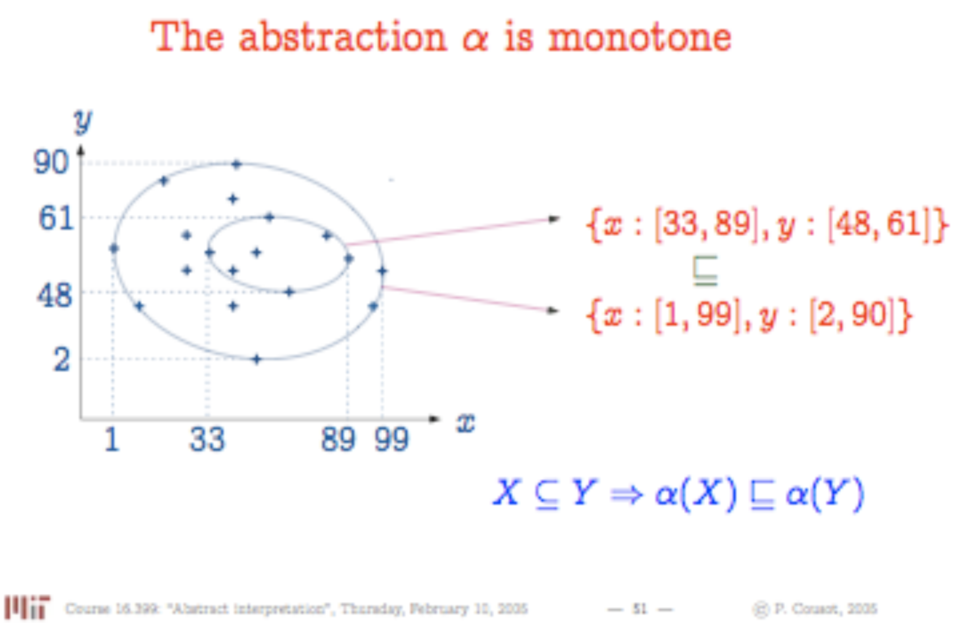

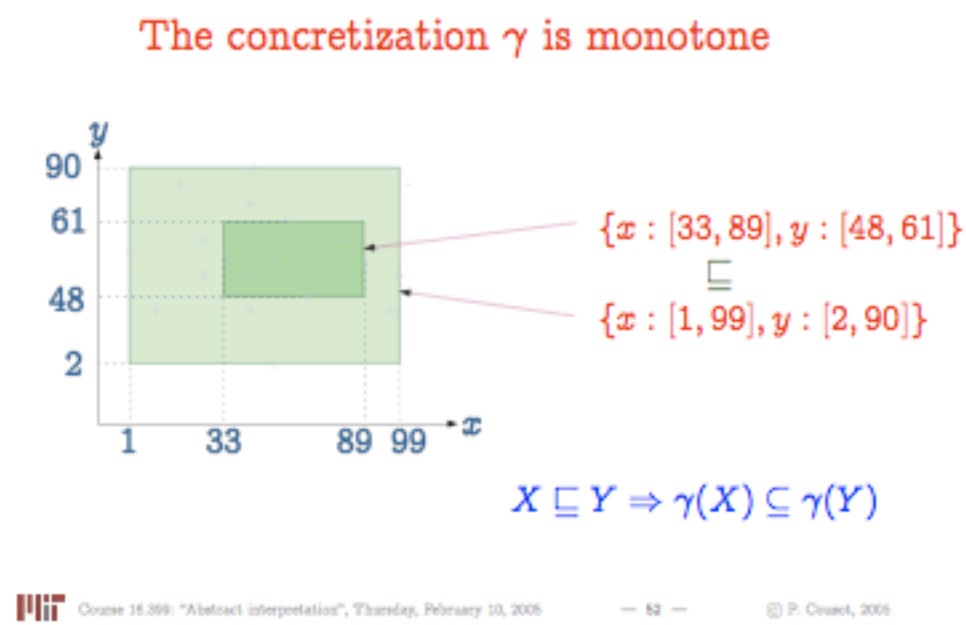

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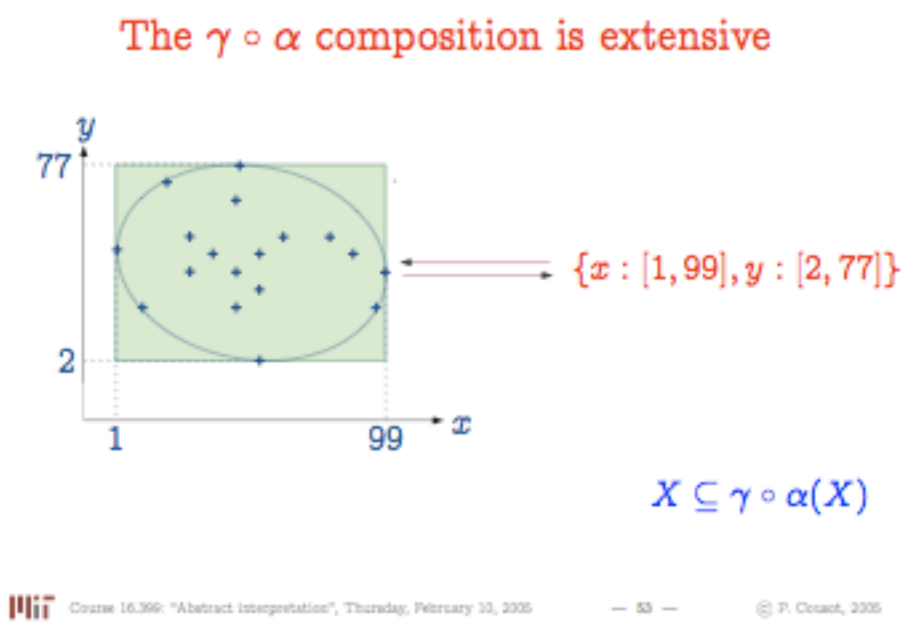

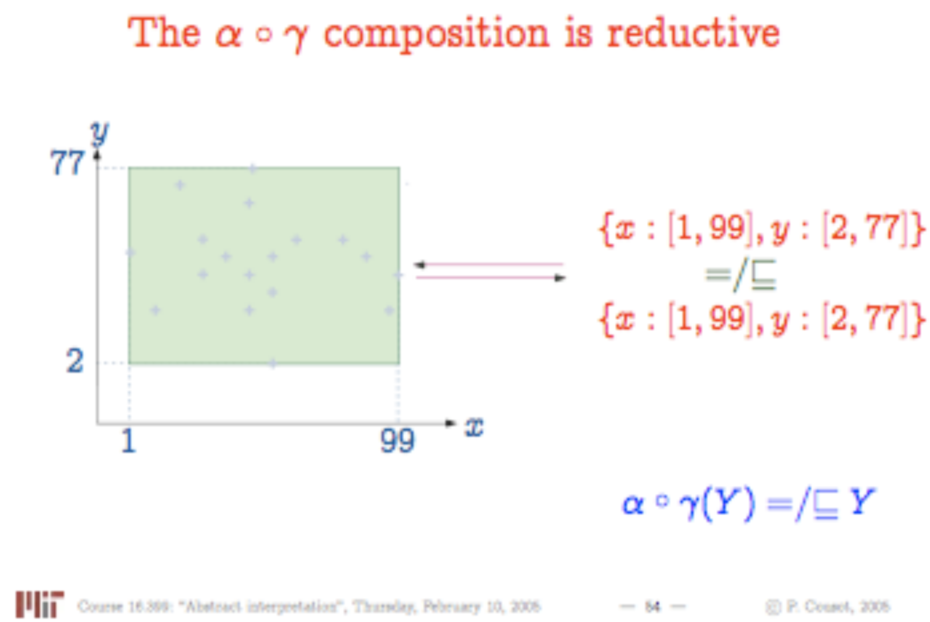

Abstract Interpretation

Galois connection

$$\langle \mathcal{D}, \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\longleftrightarrow}} \langle \overline{\mathcal{D}}, \sqsubseteq \rangle$$

iff $\forall x, y \in \mathcal{D} : x \subseteq y \Longrightarrow \alpha(x) \sqsubseteq \alpha(y)$

$\wedge\ \forall \overline{x}, \overline{y} \in \overline{\mathcal{D}} : \overline{x} \sqsubseteq \overline{y} \Longrightarrow \gamma(\overline{x}) \subseteq \gamma(\overline{y})$

$\wedge\ \forall x \in \mathcal{D} : x \subseteq \gamma(\alpha(x))$

$\wedge\ \forall \overline{y} \in \overline{\mathcal{D}} : \alpha(\gamma(\overline{y})) \sqsubseteq \overline{x}$

iff $\forall x \in \mathcal{D}, \overline{y} \in \overline{\mathcal{D}} : \alpha(x) \sqsubseteq y \Longleftrightarrow x \subseteq \gamma(y)$

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Lattices

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- A partially ordered set (poset) in which any two elements have a
  - Greatest lower bound (meet)
  - Least upper bound (join)
- Semilattice has one or the other (join or meet)
- Claim: any abstract interpretation must express at least a join semilattice

Lattices

Generalizing to complete lattices

- The reasoning on abstractions of concrete properties $\langle \wp(\Sigma), \subseteq, \emptyset, \Sigma, \cup, \cap, \neg \rangle$ to an abstract domain which, in case of best abstraction is a Moore family, whence a complete lattice, can be generalized to an arbitrary concrete complete lattice $\langle L, \sqsubseteq, \bot, \top, \sqcup, \sqcap \rangle$
- This allow a compositional approach where $\langle L, \sqsubseteq, \bot, \top, \sqcup, \sqcap \rangle$ is abstracted to $\langle A_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1 \rangle$ which itself can be further abstracted to $\langle A_2, \sqsubseteq_2, \bot_2, \top_2, \sqcup_2, \sqcap_2 \rangle, \ldots$

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Lattices

Why are abstract domains complete lattices in the presence of best abstractions?

- The abstractions start from the complete lattice of concrete properties $\langle \wp(\Sigma), \subseteq, \emptyset, \Sigma, \cup, \cap, \neg \rangle$ where objects in Σ represent program computations and the elements of $\wp(\Sigma)$ represent properties of these program computations
- We have defined abstract domains with best approximations in three equivalent different ways (more are considered in [3])
  - As a Moore family;
  - As a closure operator (which fixpoints form the abstract domain);
  - As the image of the concrete domain by a Galois surjection.

- In all cases, it follows that the abstract domain is a complete lattice, since we have seen that:
  - A Moore family of a complete lattice is a complete lattice;
  - The image of a complete lattice by an upper closure operator is a complete lattice (Ward);
  - The image of a complete latticc by the surjective abstraction of a Galois connection is a complete lattice.
- In general this property does not hold in absence of best abstraction or if arbitrary points are added to the abstract domain as shown next.

Reference

[3] P. Cousot and R. Cousot. Systematic design of program analysis frameworks. In *Conference Record of the Sixth Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, pages 269–282, San Antonio, Texas, 1979. ACM Press, New York, NY, U.S.A.

Lattices Too Limiting?

- Does the requirement for an abstract interpretation that is a lattice too restrictive?
  - How can we build a lattice for a set of values?
  - How do we combine two sets of values representing two properties into a lattice?
  - What are the pros/cons of these results?

Dataflow Analysis

- Interprocedural Control Flow Graph (ICFG)
  - Possible flow paths in system
- Join Semilattice for an Abstract Interpretation
  - How to combine values on joins
- Initial Configuration for the Abstract Interpretation
  - Starting values for system
- Dataflow Transfer Function over edges in ICFG
  - How values are changed by operations in system

Intraprocedural CFG

- Statements
  - Nodes
  - One successor and one predecessor
- Basic Blocks
  - Multiple successors to the join (multiple predecessors)
  - Examples?
- Unique Enter and Exit
  - All start nodes are successors of enter
  - All return nodes are predecessors of exit

Legal and Illegal Paths

- Interprocedurally, connect CFGs
  - Calls → Enter
  - Exit → Return-Site
- Want to represent only legal paths
  - In particular, calls must match returns
    - Will discuss the implications of this later
- Example…

Path Function Problem

- A path of length $j >= 1$ from node m to node n is a (non-empty) sequence of j edges,
- denoted by $[e_1, e_2, \ldots, e_j]$, such that
  - the source of e_1 is m,
  - the target of e_j is n,
  - and for all i, $1 <= i <= j-1$, the target of edge e_i is the source of edge e_{i+1}.

Intraprocedural Dataflow Analysis

- The path function pf_q for path $q = [e_1, e_2, \ldots, e_j]$ is the composition, in order, of q's transfer functions
  - $pf_q = M(e_j) \circ \ldots \circ M(e_2) \circ M(e_1)$
- In intraprocedural dataflow analysis, the goal is to determine, for each node *n*, the "join-over-all-paths" solution
  - $JOP_n = join(q \text{ in } Paths(enter, n))\ pf_q(v_0)$
    - *Paths(enter, n)* denotes the set of paths in the CFG from enter node to *n*
    - *v_0 is the possible memory configurations at the start of the procedure*
- Soundness depends on the abstract interpretation

Abstract Interpretation

- As discussed above, a sound *JOP_n* solution requires
  - A Galois connection is established between concrete states and abstract states
  - Each dataflow transfer function *M(e)* is shown to overapproximate the transfer function for the concrete semantics of e

Example

Interprocedural Dataflow Analysis

- Find join-over-all-valid-paths
- What is a valid path?
  - Is a matched or valid path
    - Where a valid path has an open call
    - Where a matched path has a matching return for each call
    - Or consists only of edges without calls and returns
- Be able to use the grammar on your own

Join Over All Valid Paths

- Solution is said to be "context-sensitive"
  - A context-sensitive analysis captures the fact that the results propagated back to each return site r should depend only on the memory configurations that arise at the call site that corresponds to r.
- Formal definition
  - $JOVP_n = join(q \text{ in } VPaths(enter_{main}, n))\ pf_q(v_0)$
- $VPaths(enter_{main}, n)$ denotes the set of valid paths from the main entry point to *n*

Summary

- To find and fix bugs, we need to understand how programs and systems work
  - Testing – time-consuming and incomplete
  - Validation – find all bugs
- Static analysis
  - Key concepts: concrete to abstract domains
  - Soundness – No false negatives
- OK, so what do you do with static analysis?
  - E.g., Interprocedural Dataflow Analysis