Finding biclusters by random projections

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What is a bicluster

- Given a matrix over a finite alphabet
- Remove some of the columns and some of the rows
- Each row of what is left read the same string (row-wise)

Can you find the largest bicluster?

ABAACDADBBAABCDBBBCCABCBBAABBBDCDDCBCCAADAAB
CCACBDABDCADBBDDBAABBBBACAAACABDDCCDADDBDDBD
BBBBCCCDCDCCCDACDADABCABCACDADCBBDCDBACDDBBC
CCBCBDCCBCCAABDCBABCDBBAAAAACCDDDCCDBDBADDBD
CCCCBDCDACBCACDABCDCCBBADAABCCDDDCCDBDCDBDBD
CDCDABCACABDABACCDABCCCBACBACBBAADAAACACCBCC
CCDCBDCBACDBDCDDBCCAABBCDABCCCDDDCCDCDCACDBD
CCDCBDCCACBBBADDBCADABBABDCDCADDDCCDBDDDCDBD
CCAACACACABDDCDBDACDDCDAADCCAAACDBDBBBBABDDA
CCBCBDCCACACCDDABADBCBBABBDCCADDDCCDCDBCADBD

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This is one … is it the largest?

ABAAC**D**ADBBAABC**D**B**B**BCCA**B**CBB**AA**BBBDC**D**D**C**BCCAADAAB
CCACB**D**ABDCADBB**D**D**B**AABB**B**BAC**AA**ACABD**D**C**C**DADDBDDBD
BBBBCCCDCDCCCDACDADABCABCACDADCBBDCDBACDDBBC
CCBCB**D**CCBCCAAB**D**C**B**ABCD**B**BAA**AA**ACCDD**D**C**C**DBDBADDBD
CCCCB**D**CDACBCAC**D**A**B**CDCC**B**BAD**AA**BCCDD**D**C**C**DBDCDBDBD
CDCDABCACABDABACCDABCCCBACBACBBAADAAACACCBCC
CCDCBDCBACDBDCDDBCCAABBCDABCCCDDDCCDCDCACDBD
CCDCBDCCACBBBADDBCADABBABDCDCADDDCCDBDDDCDBD
CCAACACACABDDCDBDACDDCDAADCCAAACDBDBBBBABDDA
CCBCBDCCACACCDDABADBCBBABBDCCADDDCCDCDBCADBD

Area=4×8=32

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One more time …

ABAACDADBBAABCDBBBCCABCBBAABBBDCDDCBCCAADAAB
CCACBDABDCADBBDDBAABBBBACAAACABDDCCDADDBDDBD
BBBBCCCDCDCCCDACDADABCABCACDADCBBDCDBACDDBBC
CCBCBDCCBCCAABDCBABCDBBAAAAACCDDDCCDBDBADDBD
CCCCBDCDACBCACDABCDCCBBADAABCCDDDCCDBDCDBDBD
CDCDABCACABDABACCDABCCCBACBACBBAADAAACACCBCC
CCDCBDCBACDBDCDDBCCAABBCDABCCCDDDCCDCDCACDBD
CCDCBDCCACBBBADDBCADABBABDCDCADDDCCDBDDDCDBD
CCAACACACABDDCDBDACDDCDAADCCAAACDBDBBBBABDDA
CCBCBDCCACACCDDABADBCBBABBDCCADDDCCDCDBCADBD

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Was this the one you found?

ABAACDADBBAABCDBBBCCABCBBAABBBDCDDCBCCAADAAB
CCA**CBD**ABD**C**ADBB**D**D**B**AABB**BB**ACAAA**C**AB**DDCCD**A**D**DBD**DBD**
BBBBCCCDCDCCCDACDADABCABCACDADCBBDCDBACDDBBC
CCB**CBD**CCB**C**CAAB**D**C**B**ABCD**BB**AAAAA**C**CD**DDCCD**B**D**BAD**DBD**
CCC**CBD**CDA**C**BCAC**D**A**B**CDCC**BB**ADAAB**C**CD**DDCCD**B**D**CDB**DBD**
CDCDABCACABDABACCDABCCCBACBACBBAADAAACACCBCC
CCD**CBD**CBA**C**DBDC**D**D**B**CCAA**BB**CDABC**C**CD**DDCCD**C**D**CAC**DBD**
CCD**CBD**CCA**C**BBBA**D**D**B**CADA**BB**ABDCD**C**AD**DDCCD**B**D**DDC**DBD**
CCAACACACABDDCDBDACDDCDAADCCAAACDBDBBBBABDDA
CCB**CBD**CCA**C**ACCD**D**A**B**ADBC**BB**ABBDC**C**AD**DDCCD**C**D**BCA**DBD**

Area=6×20=120

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The general problem

- Biclustering is the problem of finding a partition of the vectors and a subset of the dimensions such that the projections along those directions of the vectors in each cluster are close to one another

- The problem requires to cluster the vectors and the dimensions simultaneously, thus the name “biclustering”

Questions

- How difficult is the problem of finding large biclusters?

- How to find them efficiently?

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Applications

- Collaborative filtering and recommender systems
- Finding web communities
- Discovery association rules in databases
- Gene expression analysis
- …

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Related works

- Hartigan, ‘72
- Aggarwal *et al.*, SIGMOD’99
- Cheng & Church, ISMB’00
- Wang *et al.*, SIGMOD’02
- Ben-Dor *et al.*, RECOMB’02
- Tanay *et al.*, ISMB’02
- Procopiuc *et al.*, SIGMOD’02
- Murali & Kasif, PSB’03
- Sheng *et al.*, ECCB’03
- Mishra *et al.*, COLT’03

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Problem definition

LARGEST_BICLUSTER(***f***) problem

- **Instance:** A matrix $X \in \Sigma^{n \times m}$
- **Question:** Find a row selection *R* and a column selection *C* such that the rows of $X_{(R,\,C)}$ read the same string and $f(X_{(R,\,C)})$ is maximized

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Examples of objective functions

$$f_1\left(X_{(R,C)}\right) = |R| + |C|$$

$$f_2\left(X_{(R,C)}\right) = |R| \text{ provided that } |C| = |R|$$

$$f_3\left(X_{(R,C)}\right) = |R||C|$$

f_1: Maximum Vertex Biclique – polytime
f_2: Balanced Biclique – hard
f_3: Maximum Edge Biclique - hard

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Randomized search

Assume $X \in \Sigma^{n \times m}$ contains a maximal bicluster (R^*, C^*). Assume we know $|R^*| = r^*$ and $|C^*| = c^*$.

Observation:

- If we knew R^*, then C^* could be obtained
- If we knew C^*, then R^* could be obtained

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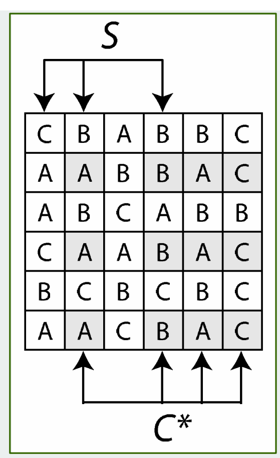
Randomized search (step 1)

Select a random subset *S* of size *k* uniformly from the set of columns *{1, 2, …, m}*



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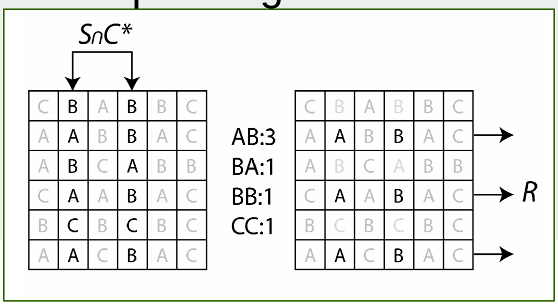
Randomized search (step 2)

For all the subset of *S*, find the string *w* that appears at least $\hat{r}$ times in each subset of *S* and record the corresponding rows *R* in which *w* occurs



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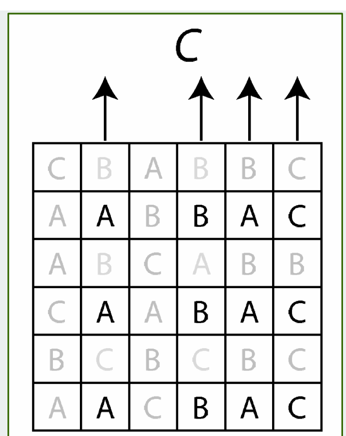
Randomized search (step 3)

Select the set of clean columns *C* with size at least $\hat{c}$ corresponding to each *R*

- A column *j* is *clean* with respect to *R* if the symbols in the *j*-th column of *X* restricted to the rows *R*, are identical



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Randomized search (step 4)

Save the solutions and repeat step 1 to 4 for *t* iterations

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Parameters

- Projection size *k* (k_{min})
- Column threshold $\hat{c}$
- Row threshold $\hat{r}$
- Number of iterations *t*

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Selecting the projection size *k*

- Occurrences of substrings are
  - Gaussian distributed for strings shorter than $\log_a m$
  - Poisson distributed for strings longer than $\log_a m$

  where $a = |\Sigma|$
- Choose $k = \log_a m$
- Choose $k_{min} = 1$ or $k_{min} = k$

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Selecting the number of iterations *t*

- We can miss a solution in two cases
  - *S* completely misses *C**
  - when *S* overlaps *C**, and the string *w* selected by the algorithm also appears in a row outside *R**

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Selecting the number of iterations *t*

- The probability of missing the solution in one iteration is

$$\alpha(n,m,k,r^*,c^*,a) = \Pr\{S \cap C^* = \emptyset\} + \sum_{i=1}^{k} \Pr\{|S \cap C^*| = i \text{ and } |R| > r^*\}$$
$$= \Pr\{S \cap C^* = \emptyset\} + \sum_{i=1}^{k} \Pr\{|R| > r^* \text{ given } |S \cap C^*| = i\} \Pr\{|S \cap C^*| = i\}$$

which is

$$\alpha(n,m,k,r^*,c^*,a) = \left(\binom{m-c^*}{k} + \sum_{i=1}^{k}\left(1-\left(1-\frac{1}{a^i}\right)^{n-r^*}\right)\binom{c^*}{i}\binom{m-c^*}{k-i}\right) \Big/ \binom{m}{k}$$

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Selecting the number of iterations t

- Given the probability of missing the solution in *t* iterations to be smaller than *e*

$$t \geq \frac{\log \epsilon}{\log \alpha(n,m,k,r^*,c^*,a)}$$

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Selecting the number of iterations *t*

ϵ	$a=2,k=8$	$a=4,k=4$	$a=8,k=3$	$a=16,k=2$	$a=32,k=2$
0.005	18794	1342	306	179	99
0.05	10626	759	173	101	56
0.1	8168	583	133	78	43
0.2	5709	408	93	54	30
0.3	4271	305	70	41	23
0.4	3250	232	53	31	17
0.5	2459	176	40	23	13
0.6	1812	129	29	17	10
0.7	1265	90	21	12	7
0.8	792	57	13	8	4
0.9	374	27	6	4	2

Table 1: The estimated number of iterations for a matrix 256×256 with a submatrix 64×64, for different choices of ϵ, alphabet size a, and projection size k (sampling columns)

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Simulations

- Generate 1,000 random matrices of size 256x256 over an alphabet of size *a*
- In each, embed a bicluster of size 64x64 (random content, random positions)
- Run the algorithm for *t* iterations (*t*=5,10,…,100) and compute how many successes out of 1,000

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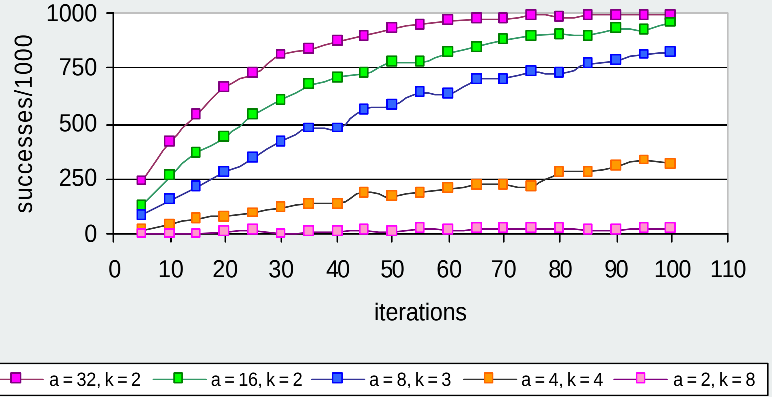
Simulation result (column sampling)



Performance of the randomized algorithm for different choices of the alphabet size a ($k_{min} = 1$, $k = log_a m$)

Findings

- Simple and fast randomized algorithm to find large biclusters in text matrices
- Probabilistic analysis of performance
- Simulations

- Next: approximate biclusters?

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