This quiz is **closed book, closed notes** and 50 minutes long

- Read the questions carefully
- No electronic equipment allowed (smart phones, tablets, computers, ...)
- Write legibly. What can’t be read will not be graded
- When designing an algorithm, you are allowed to use any algorithm or data structure we explained in class, without giving its details, unless the question specifically requires that you give such details
- Always remember to analyze the time complexity of your solution
- If you have a question about the meaning of a question, raise your hand
Problem 1. [Greedy/Union-Find]

Use Dijkstra’s algorithm to compute the cost of the shortest (i.e., minimum weight) path from vertex a to the other vertices. Indicate the \( D \) value and the vertices in the cloud \( C \) after each iteration of the main loop in the table below.

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**Problem 2.** [Greedy/Union-Find]

Complete the proof for the following statement.

**Fact:** Let $G = (V, E)$ be a weighted undirected graph. If all the edge weights in $G$ are distinct, the minimum spanning tree is unique.

**Proof:** Suppose for contradiction that there are two distinct spanning trees $T$ and $T'$ for $G$, which means that $T$ and $T'$ differ by at least one edge. Among those edges that are in only one of the two trees, let $e$ be one of minimum cost. Assume without loss of generality that $e \in T$ (the other case, $e \in T'$, is symmetric). Adding $e$ to $T'$ creates a simple cycle $C$ . . .
Complete the Theorem we covered in class about the optimal substructure for the Longest Common Subsequence problem. No need to prove the theorem.

**Theorem.** Let \( X =< x_1, \ldots, x_m > \) and \( Y =< y_1, \ldots, y_n > \) be two sequences (strings), and let \( Z =< z_1, \ldots, z_k > \) be any LCS of \( X \) and \( Y \).

1. If \( x_m = y_n \) then ... 

2. If \( x_m \neq y_n \) then \( z_k \neq x_m \) implies ... 

3. If \( x_m \neq y_n \) then ...
Problem 4. [Dynamic Programming]

Given an array \( A = \{a_1, a_2, \ldots, a_n\} \) of integers, we say that a subsequence \( \{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\} \) is (monotonically) increasing if for every \( i_s < i_t \), we have \( a_{i_s} < a_{i_t} \). Given an array \( A \) of size \( n \), we want to compute the length of the longest increasing subsequence (LIS) in \( A \). For instance, if \( A = \{9, 5, 2, 8, 7, 3, 1, 6, 4\} \) the length of the LIS is 3, because \((2, 3, 4) \) (or \((2, 3, 6)\)) are LIS of \( A \). Analyze the time- and space-complexity of your solution.

**Hint:** Define \( L(i) \) be the length of the LIS for a prefix \( \{a_1, \ldots, a_i\} \) of \( A \) such that \( a_i \) is the last element in LIS; then \( L(i) \) can be recursively written as:

\[
L(i) = \begin{cases} 
\text{if } i = 1 \\
\text{otherwise}
\end{cases}
\]

The space-complexity is

The time-complexity is