Problem 1. (30 points)
You are given a list of $n$ jobs $j_1, j_2, \ldots, j_n$ each with a time $t(j)$ to perform a job and a weight $w(j)$. We want to order the jobs as $j_{\sigma(1)}, j_{\sigma(2)}, \ldots, j_{\sigma(n)}$ in such a way that the quantity
\[
\sum_{i=1}^{n} \left( w(j_{\sigma(i)}) \sum_{k=1}^{i} t(j_{\sigma(k)}) \right)
\]
is minimized. In other words, we want to minimize the weighted sum of the time each job has to wait before being executed. Give an efficient greedy algorithm for this problem, analyze the complexity, and prove its optimality.

Problem 2. (35 points)
Consider the following Reverse-Delete algorithm for the Minimum Spanning Tree (MST) problem. Start with a connected edge-weighted graph $G = (V, E)$. Consider edges in order of decreasing weight (breaking ties arbitrarily). When considering an edge $e$, delete $e$ from $E$ unless doing so would disconnect the current graph. Analyze the complexity of Reverse-Delete and prove that it computes an (optimal) MST.
Problem 3. (35 points)

Let $G = (V, E)$ be a weighted directed graph with weight function $w : E \rightarrow \{0, 1, \ldots, W\}$ for some nonnegative integer $W$. Modify Dijkstra’s algorithm to compute the shortest paths from a given vertex $s$ in $O((n + m) \log W)$-time.