Problem 1. (30 points) Give a tight bound (using the big-theta notation) on the time complexity of following method as a function of \( n \). For simplicity, you can assume \( n \) to be a power of two.

Algorithm WeirdLoop \((n : \text{integer})\)

\[
i \leftarrow n \\
\text{while } i \geq 1 \text{ do} \\
\quad \text{for } j \leftarrow 1 \text{ to } i \text{ do} \\
\quad \quad k \leftarrow 1 \\
\quad \quad \text{while } k \leq n \text{ do} \\
\quad \quad \quad k \leftarrow 2k \\
\quad \quad i \leftarrow i/2
\]
Problem 2. (40 points)
You are given an array $A$ of size $n$ which contains integers in the range $[0, n - 1]$. Give a $O(n)$-time algorithm that prints all integers that appear more than once in $A$, using only $O(1)$ additional memory space. Note that since you can use only constant additional space, the use of external data structures or hash tables is not allowed.

Problem 3. (30 points)
Given the following recurrence relation

$$T(n) = \begin{cases} 1 & n = 1 \\ T\left(\frac{n}{4}\right) + \sqrt{n} & n > 1 \end{cases}$$

1. Solve it exactly (i.e., without using any asymptotic notation) by iterative substitutions
2. Prove by induction that your solution is correct