This quiz is **closed book, closed notes** and 35 minutes long

- Read the questions carefully
- No electronic equipment allowed (cell phones, tablets, computers, ...)
- Write legibly. What can’t be read will not be graded
- Use pseudocode (or English) to describe your algorithms
- Always remember to analyze the time complexity of your solution
- If you have a question about the meaning of a question, raise your hand

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Problem 1. (40 points: 4 points if correct, 2 if unanswered, 0 if wrong)
Mark by true or false each of the following (no need to prove).

\[ \sqrt{n \log_2 n^3} + n \log_2 n \in O(n \log_2 n) \]
\[ 9^{\log_2 n} \in \Omega(n^2 \log n) \]
\[ \log_2 2^{n^2} - n \in \Theta(n \log_3 3^n) \]
An array sorted in increasing order is always a min-heap
A min-heap is always a sorted array (in increasing order)

The following questions are on graphs; assume that \( n = |V| \) is the number of vertices, and \( m = |E| \) is the number of edges; DFS is “depth first search”; BFS is “breadth first search”; in DFS/BFS the the set of edges visited during the execution of these algorithms are called tree or discovery edges; non-tree edges are the others (also called back edges in DFS, cross edges in BFS)

A directed complete graph with \( n \) nodes has exactly \( n(n - 1)/2 \) edges
Given the spanning tree \( T \) formed by the discovery (tree) edges of a BFS traversal of a connected undirected graph \( G \) started from node \( s \), for each vertex \( v \), the path on tree \( T \) is the shortest path between \( s \) and \( v \)
An edge \( e \) whose removal disconnects the graph is called a bridge; if BFS is run on a connected undirected graph \( G \), it is a possible for a bridge in \( G \) to be a cross (non-tree) edge
For a connected undirected graph \( G \), the absence of back (non-tree) edges with respect to a DFS tree implies that \( G \) is acyclic
If one runs a DFS on a connected undirected graph, the number of back (non-tree) edges is exactly \( m - n + 1 \)
Problem 2. (24 points: 6 points each)
For each of the concepts listed below write a precise (possibly formal) definition. Do not explain or comment about the corresponding algorithm, if any.

1. topological ordering of a directed acyclic graph $G = (V, E)$

2. directed cycle in a directed graph $G = (V, E)$

3. spanning tree of an undirected graph $G = (V, E)$

4. transitive closure of a directed graph $G = (V, E)$
Problem 3. (36 points)

Suppose you are given an array $A = \{a_1, a_2, \ldots, a_n\}$ of $n$ distinct integers. You are told that the sequence of values $a_1, a_2, \ldots, a_n$ is unimodal, that is for some index $p \in [1, n]$, the values in the array increase up to position $p$ in $A$, and then decrease the remainder of the way until position $n$. Give an algorithm to find the position $p$ in $O(\log n)$ time. You can assume $n$ to be a power of 2.