Name:

Student ID #:

• You are expected to work on this assignment on your own

• Use pseudocode, Python-like or English to describe your algorithms. Absolutely no C++/C/Java

• When designing an algorithm, you are allowed to use any algorithm or data structure we explained in class, without giving its details, unless the question specifically requires that you give such details

• Always remember to analyze the time complexity of your algorithms

• Homework has to be submitted electronically on Gradescope by the deadline. No late assignments will be accepted
Problem 1. (40 points)

Let $I_1, I_2, \ldots, I_n$ be a set of closed intervals on the real line, with $I_i = [a_i, b_i]$. Design an efficient greedy algorithm to compute the smallest set $S$ of points such that each interval contains at least one point. Analyze the time complexity of your algorithm and prove that it always produces the optimal solution.

Answer:
Problem 2. (60 points)

In the United States, coins are minted with denominations of 1, 5, 10, 25, and 50 cents. Now consider a country whose coins are minted with denominations of \(\{d_1, \ldots, d_k\}\) units. They seek an algorithm that will enable them to make change of \(n\) units using the minimum number of coins.

1. (20 pts) The greedy algorithm for making change repeatedly uses the biggest coin smaller than the amount to be changed until it is zero. Provide a greedy algorithm for making change of \(n\) units using US denominations. Prove its correctness and analyze its time complexity.

2. (20 pts) Show that the greedy algorithm does not always give the minimum number of coins in a country whose denominations are \(\{1, 6, 10\}\).

3. (20 pts) Give dynamic programming algorithm that correctly determines the minimum number of coins needed to make change of \(n\) units using denominations \(\{d_1, \ldots, d_k\}\). Analyze its running time.

Answer: