Alphabet-Dependent String Searching with Wexponential Search Trees

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July 1, 2015

We consider a fundamental data structure question: how to represent a tree?

(Compacted) Trie

A trie is simply a tree with edges labeled by single characters. A compacted trie is created by replacing maximal chains of unary vertices with single edges labeled by (possibly long) words.

Navigation queries

Given a pattern p, we want to traverse the edges of a compacted trie to find the node corresponding to p. If there is no such node, we would like to compute its longest prefix for which the corresponding node does exist.

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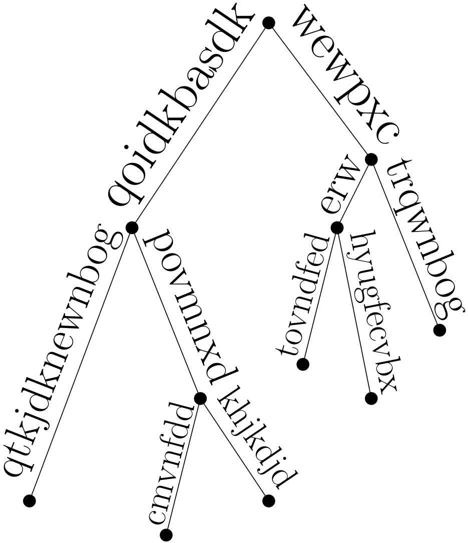

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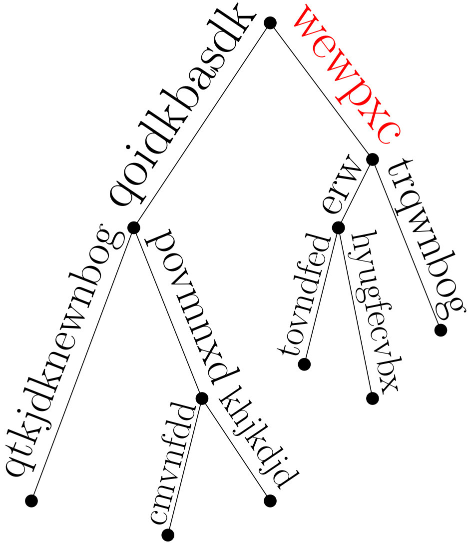

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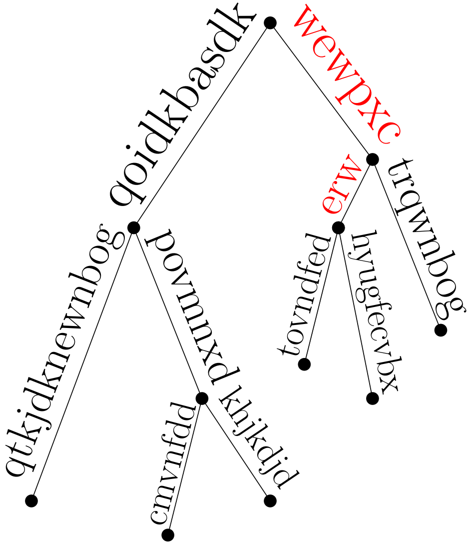

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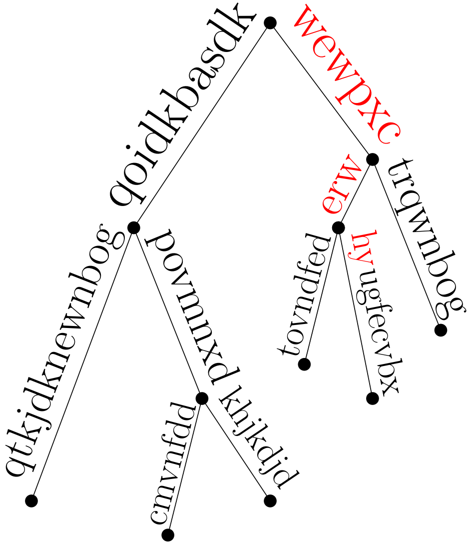

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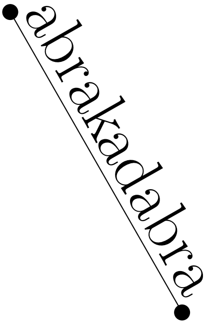
Given an edge, we want to split it into two parts by (possibly) creating a node, and adding a new edge outgoing from this middle node.



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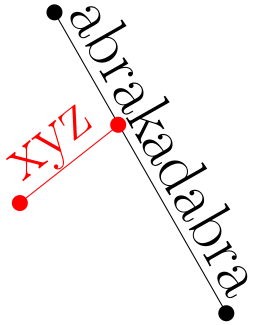

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Static case

Given a compacted trie, can we **quickly** construct a **small** structure which allows us to execute navigation queries **efficiently**?

Dynamic case

Can we maintain a compacted trie so that:

1. the resulting structure is **small**,
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Parameters: the number of nodes in the compacted trie n, the size of the alphabet σ, and the length of the pattern m.

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Hashing

For each node store a hash table mapping characters to the corresponding outgoing edges.

Randomized!

Table

Or, for each node store a table of size σ mapping characters to the corresponding outgoing edges.

Space usage is $n\sigma$!

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Rules of the game:

1. the solution must be deterministic,
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3. bound on the update time must be worst-case.

Then it seems that navigation queries must necessarily take $\mathcal{O}(mf(\sigma))$ time, for some function of σ, for instance $f(\sigma) = \log \sigma$, or something better if we use a more sophisticated predecessor structure. Surprisingly, this is not true.

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There exists a deterministic linear-size structure supporting navigation in $\mathcal{O}(m + \sqrt{\frac{\log n}{\log \log n}})$ time and splitting edges in $\mathcal{O}(\sqrt{\frac{\log n}{\log \log n}})$.

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A static linear-size constant-access dictionary on a set of k keys can be deterministically constructed in time $\mathcal{O}(k \log^2 \log k)$.

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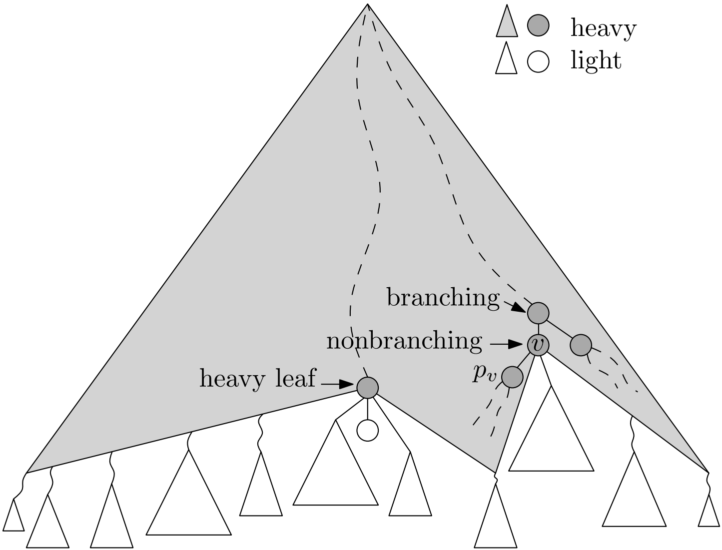
We store the edges outgoing from v in a few different ways depending on the size of the subtree rooted at v.

Heavy nodes

A node is heavy if its subtree contains at least $s = \Theta(\log^2 \log \sigma)$ leaves, and otherwise light. Furthermore, a heavy node is branching if it has more than one heavy child.


heavy
light
branching
nonbranching
v
p_v
heavy leaf

We classify edges into three types, and deal with each type separately:

1. from (any) branching node to a light node,
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At most one such edge per node, can be stored separately.

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The total number of such edges is just $\frac{n}{s}$, hence we can afford the super-linear construction time. More precisely, we compute the perfect hashing function for each such node separately in

$$\mathcal{O}(k \log^2 \log k) = \mathcal{O}(k \log^2 \log \sigma) = \mathcal{O}(ks)$$

time, which takes $\mathcal{O}(\frac{n}{s}s) = \mathcal{O}(n)$ time in total.

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We store all such edges in a predecessor structure. By combining perfect hashing result and Willard's x-fast trees, there exists a linear-size predecessor structure with $\mathcal{O}(\log\log\sigma)$ query time, which can be constructed in linear time.

Observe that any navigation query traverses an edge of type (1) at most once, hence we pay $\mathcal{O}(\log\log\sigma)$ just once (so far). But what happens when we reach a light node?

Each light node contains at most s leaves. We can execute a binary search over those leaves using the suffix array trick, namely in each step we achieve at least one of the following:

1. halve the current interval,
2. consume one character from the pattern.

Hence in $\mathcal{O}(m + \log s)$ time we can locate the predecessor of the pattern among all leaves, and the search actually computes the longest prefix of the pattern which is a prefix of a string corresponding to some leaf.

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The total time complexity for a query is

$$\mathcal{O}(m + \log\log\sigma + \log s) = \mathcal{O}(m + \log\log\sigma)$$

and the total construction time is linear.

Now let us consider the dynamic case.

Reduction

The general case can be reduced to maintaining a collection of trees of size $\mathcal{O}(\sigma)$ each and linear total size, so that any update/query can be efficiently translated into an update/query into at most one smaller tree.

From now on we assume that $n = \mathcal{O}(\sigma)$. Instead of the simple two-level scheme we need to partition the nodes into more groups.

Levels of nodes

Let $f(\ell) = 2^{(\frac{3}{2})^{\ell}}$. We say that a node v is of level ℓ when the number of leaves in its subtree belongs to $[f(\ell), 2f(\ell+1)]$. We will maintain an invariant that a level of v doesn't exceed the level of its parent.

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Those edges are stored in a static dictionary with constant access time. We already know that such dictionary can be construct in close-to-linear time, which is enough because of the way we defined the levels. More precisely, it cannot happen too often that a level of a node increases.

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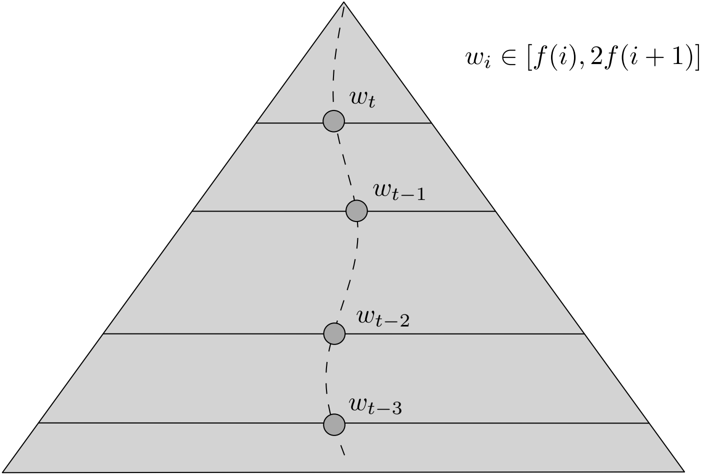
Those edges are stored in a dynamic dictionary structure. For this we develop a weighted variant of the exponential search trees of Andersson and Thorup.

Even without the modification, the query complexity is $\mathcal{O}(m + \frac{\log^3 \log \sigma}{\log \log \log \sigma})$. This is because there are at most $t = \Theta(\log \log \sigma)$ edges of type (2) on any path descending from the root.



Faster!

The subsequent accesses to the dynamic dictionary structures are not completely independent.

Wexponential search trees

There exists a linear-size dynamic structure storing a collection of n weighted elements from $[1, U]$ with the following bounds:

1. predecessor search takes $\mathcal{O}(\log \frac{\log W}{\log w} \frac{\log\log U}{\log\log\log U})$, where W is the current total weight, and w is the weight of the predecessor,
2. inserting a new element of weight 1 takes $\mathcal{O}(\log\log W)$,
3. increasing a weight of an element of weight w by 1 takes $\mathcal{O}(\log \frac{\log W}{\log w})$.

Telescoping

Now if we use this structure instead of the standard exponential search trees, the total complexity of all queries at nodes where we decrease the current level becomes:

$$\sum_{i=t-1}^{0} \log \frac{\log w_{i+1}}{\log w_i} \frac{\log\log U}{\log\log\log U} = \frac{\log\log U}{\log\log\log U} \log\log w_t$$

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$$= \frac{\log\log U}{\log\log\log U} \log\log U = \frac{\log^2 \log U}{\log\log\log U}$$

(ignoring the details necessary to show how to update the structures...)

Wexponential search trees

Imagine that each element of weight w is a fragment of such length, and draw all of them on a $[1, W]$ segment.

Then choose a set of roughly $\sqrt{W}$ evenly spaced splitters. Store them in a static predecessor structure, and recursively build a smaller wexponential search tree for each of the resulting roughly $\sqrt{W}$ subsets.

Beame and Fich STOC'90

A static predecessor search structure with $\mathcal{O}(\frac{\log\log\sigma}{\log\log\log\sigma})$ query time can be constructed in $\mathcal{O}(k^{1+\epsilon})$ time and space, where k is the number of elements.

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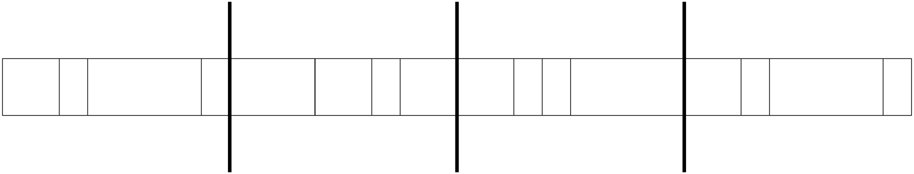

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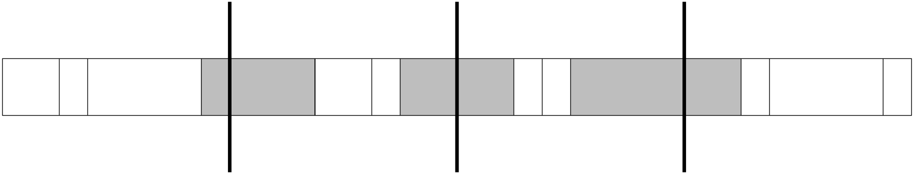

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Intuition:

1. the larger the weight, the sooner the element is stored in a static predecessor structure,
2. rebuilding a static predecessor structure is very costly, but happens only if there have been multiple insertions/increases.

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Very complicated in Andresson&Thorup paper. We follow the simpler idea of Bender, Cole and Raman.

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Questions?