Alphabet-Dependent String Searching with Wexponential Search Trees

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We consider a fundamental data structure question: how to represent a tree?

(Compacted) Trie

A trie is simply a tree with edges labeled by single characters. A compacted trie is created by replacing maximal chains of unary vertices with single edges labeled by (possibly long) words.

Navigation queries

Given a pattern $p$, we want to traverse the edges of a compacted trie to find the node corresponding to $p$. If there is no such node, we would like to compute its longest prefix for which the corresponding node does exist.
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Static case

Given a compacted trie, can we quickly construct a small structure which allows us to execute navigation queries efficiently?

Dynamic case

Can we maintain a compacted trie so that:

1. the resulting structure is small,
2. we can execute navigation queries efficiently,
3. we can split any edge efficiently?

Parameters: the number of nodes in the compacted trie $n$, the size of the alphabet $\sigma$, and the length of the pattern $m$. 
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Given a compacted trie, can we **quickly** construct a **small** structure which allows us to execute navigation queries **efficiently**?

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Parameters: the number of nodes in the compacted trie $n$, the size of the alphabet $\sigma$, and the length of the pattern $m$. 
Hashing

For each node store a hash table mapping characters to the corresponding outgoing edges.

Randomized!

Table

Or, for each node store a table of size $\sigma$ mapping characters to the corresponding outgoing edges.

Space usage is $n\sigma$!

BST

Or, for each node store a binary search tree mapping characters to the corresponding outgoing edges.

Navigation query takes $O(m \log \sigma)$ time!
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Navigation query takes $O(m \log \sigma)$ time!
Rules of the game:

1. the solution must be deterministic,
2. the space usage must be linear in $n$, irrespectively of $\sigma$,
3. bound on the update time must be worst-case.

Then it seems that navigation queries must necessarily take $O(mf(\sigma))$ time, for some function of $\sigma$, for instance $f(\sigma) = \log \sigma$, or something better if we use a more sophisticated predecessor structure. Surprisingly, this is not true.

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There exists a deterministic linear-size structure supporting navigation in $O(m + \log \sigma)$ time, which can be construct in linear time.
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Application to text indexing

Consider a suffix tree of a text. After prepending a letter, one edge should be split. It is easy to locate it in amortized $O(1)$ time, but getting a sublinear worst-case bound is not trivial!
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Yes if $\sigma$ is unbounded in terms of $n$, and navigation queries actually give us the predecessor of the string.
But what if $\sigma$ is non-constant, yet (significantly) smaller than $n$?

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There exists a static deterministic linear-size structure supporting navigation in $O(m + \log \log \sigma)$ time, which can be constructed in linear time.

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To construct a static deterministic linear-size structure, we could simply try to find a perfect hashing function storing pairs \((node, character)\).

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A static linear-size constant-access dictionary on a set of \(k\) keys can be deterministically constructed in time \(O(k \log^2 \log k)\).

Hence we immediately get a static deterministic structure which can be construct in close-to-linear time. Can we do better?
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We store the edges outgoing from $v$ in a few different ways depending on the size of the subtree rooted at $v$.

### Heavy nodes

A node is heavy if its subtree contains at least $s = \Theta(\log^2 \log \sigma)$ leaves, and otherwise light. Furthermore, a heavy node is branching if it has more than one heavy child.
We classify edges into three types, and deal with each type separately:

1. from (any) branching node to a light node,
2. from a nonbranching heavy node to (any) heavy node,
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At most one such edge per node, can be stored separately.
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The total number of such edges is just \( \frac{n}{s} \), hence we can afford the super-linear construction time. More precisely, we compute the perfect hashing function for each such node separately in

\[
O(k \log^2 \log k) = O(k \log^2 \log \sigma) = O(ks)
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time, which takes \( O\left(\frac{n}{s}s\right) = O(n) \) time in total.
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We store all such edges in a predecessor structure. By combining perfect hashing result and Willard’s $x$-fast trees, there exists a linear-size predecessor structure with $O(\log \log \sigma)$ query time, which can be constructed in linear time.
Observe that any navigation query traverses an edge of type (1) at most once, hence we pay $O(\log \log \sigma)$ just once (so far). But what happens when we reach a light node?

Each light node contains at most $s$ leaves. We can execute a binary search over those leaves using the suffix array trick, namely in each step we achieve at least one of the following:

1. halve the current interval,
2. consume one character from the pattern.

Hence in $O(m + \log s)$ time we can locate the predecessor of the pattern among all leaves, and the search actually computes the longest prefix of the pattern which is a prefix of a string corresponding to some leaf.
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The total time complexity for a query is

$$\mathcal{O}(m + \log \log \sigma + \log s) = \mathcal{O}(m + \log \log \sigma)$$

and the total construction time is linear.
Now let us consider the dynamic case.

Reduction

The general case can be reduced to maintaining a collection of trees of size $O(\sigma)$ each and linear total size, so that any update/query can be efficiently translated into an update/query into at most one smaller tree.

From now on we assume that $n = O(\sigma)$. Instead of the simple two-level scheme we need to partition the nodes into more groups.

Levels of nodes

Let $f(\ell) = 2^{(\frac{3}{2})^\ell}$. We say that a node $v$ is of level $\ell$ when the number of leaves in its subtree belongs to $[f(\ell), 2f(\ell + 1)]$. We will maintain an invariant that a level of $v$ doesn’t exceed the level of its parent.
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Those edges are stored in a static dictionary with constant access time. We already know that such dictionary can be construct in close-to-linear time, which is enough because of the way we defined the levels. More precisely, it cannot happen too often that a level of a node increases.
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Those edges are stored in a dynamic dictionary structure. For this we develop a weighted variant of the exponential search trees of Andersson and Thorup.
Even without the modification, the query complexity is $O(m + \frac{\log^3 \log \sigma}{\log \log \log \sigma})$. This is because there are at most $t = \Theta(\log \log \sigma)$ edges of type (2) on any path descending from the root.
Faster!

The subsequent accesses to the dynamic dictionary structures are not completely independent.

Wexponential search trees

There exists a linear-size dynamic structure storing a collection of $n$ weighted elements from $[1, U]$ with the following bounds:

1. predecessor search takes $O\left(\log \frac{\log W}{\log w} \frac{\log \log U}{\log \log \log U}\right)$, where $W$ is the current total weight, and $w$ is the weight of the predecessor,

2. inserting a new element of weight 1 takes $O(\log \log W)$,

3. increasing a weight of an element of weight $w$ by 1 takes $O\left(\log \frac{\log W}{\log w}\right)$. 
Telescoping

Now if we use this structure instead of the standard exponential search trees, the total complexity of all queries at nodes where we decrease the current level becomes:

$$\sum_{i=t-1}^{0} \log \frac{\log w_{i+1}}{\log w_i} \frac{\log \log U}{\log \log \log U} = \frac{\log \log \log U}{\log \log \log \log U} \log \log w_t$$

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(ignoring the details necessary to show how to update the structures...)

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Wexponential search trees

Imagine that each element of weight $w$ is a fragment of such length, and draw all of them on a $[1, W]$ segment.

Then choose a set of roughly $\sqrt{W}$ evenly spaced splitters. Store them in a static predecessor structure, and recursively build a smaller wexponential search tree for each of the resulting roughly $\sqrt{W}$ subsets.

Beame and Fich STOC’90

A static predecessor search structure with $O\left( \frac{\log \log \sigma}{\log \log \log \sigma} \right)$ query time can be constructed in $O(k^{1+\epsilon})$ time and space, where $k$ is the number of elements.
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Intuition:

1. the larger the weight, the sooner the element is stored in a static predecessor structure,
2. rebuilding a static predecessor structure is very costly, but happens only if there have been multiple insertions/increases.

Worst-case bounds

Very complicated in Andresson&Thorup paper. We follow the simpler idea of Bender, Cole and Raman.
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2. rebuilding a static predecessor structure is very costly, but happens only if there have been multiple insertions/increases.

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Very complicated in Andresson&Thorup paper. We follow the simpler idea of Bender, Cole and Raman.
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Questions?