Affine Image Matching is $\text{TC}^0$-complete

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The Image Matching Problem

Remember CPM08 and CPM09 ...
The Image Matching Problem

Affine Image Matching is $\text{TC}^0$-complete
The Image Matching Problem

Given:

\( F \)

Find:

\( f \in F \)

\( f(A) \)

Goal:

\( \Delta(f(A), B) \rightarrow \min \)

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Affine Image Matching is \( \text{TC}^0 \)-complete
The Image Matching Problem

Given: $F$

Find: $f \in F$

Goal: $\Delta(f(A), B) \rightarrow \min$

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Affine Image Matching is $TC^0$-complete
The Image Matching Problem

Given: any subimage $B$

image $A$

Goal: $\Delta(f(A), B) \rightarrow \min$

Affine Image Matching is $\text{TC}^0$-complete
The Image Matching Problem

Given: \( B \uparrow n \) \[ \begin{array}{c} B \\ \uparrow n \end{array} \]

Find: \( f \in \mathcal{F} \) \[ \begin{array}{c} f \in \mathcal{F} \\ \uparrow n \end{array} \]

\[ \begin{array}{c} f(A) \\ \uparrow n \end{array} \]
The Image Matching Problem

Given: \( B \uparrow n \)

Find: \( f \in \mathcal{F} \quad f(A) \uparrow n \)

Goal: \( \Delta(f(A), B) \rightarrow \min \)

Affine Image Matching is \( \mathrm{TC}^0 \)-complete
$F(x, y) = (a_1a_2a_4a_5)x \cdot (x, y) + (a_3a_6)$

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Affine Image Matching is $\mathsf{TC}^0$-complete
Transformation Class – Affine Transformations

\[ f(x, y) = \begin{pmatrix} a_1 & a_2 \\ a_4 & a_5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a_3 \\ a_6 \end{pmatrix} \]
Consider \( D(A) = \{ f(A) \mid f \in \mathcal{F} \} \)
Discrete Problem

For $D(A)$ contains

These are only 1250 out of hundreds and thousands!
Consider all points

\[ p = (a_1, a_2, a_3, a_4, a_5, a_6)^T \]

of the \( \mathbb{R}^6 \).
Consider all points \( p = (a_1, a_2, a_3, a_4, a_5, a_6)^T \) of the \( \mathbb{R}^6 \).

Every point stands for \( f \in \mathcal{F} \):
Consider all points

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of the \( \mathbb{R}^6 \).

Every point stands for \( f \in \mathcal{F} \):

\[ f(x, y) = \begin{pmatrix} a_1 & a_2 \\ a_4 & a_5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a_3 \\ a_6 \end{pmatrix}. \]
There are hyperplanes

\[ l_{ijk} : ia_1 + ja_2 + a_3 = k - 0.5 \]

\[ J_{ijk} : ia_4 + ja_5 + a_6 = k - 0.5 \]
There are hyperplanes

\[ l_{ijk} : ia_1 + ja_2 + a_3 = k - 0.5 \]
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for \( i, j \in \{-n, \ldots, n\} \) and
for \( k \in \{-n, \ldots, n+1\} \)
There are hyperplanes

\[ l_{ijk} : i a_1 + j a_2 + a_3 = k - 0.5 \]
\[ J_{ijk} : i a_4 + j a_5 + a_6 = k - 0.5 \]

for \( i, j \in \{-n, \ldots, n\} \) and for \( k \in \{-n, \ldots, n + 1\} \)

cutting \( \mathbb{R}^6 \) into a set \( \mathcal{A}_n \)
of convex regions.
Theorem

Let $\varphi \in \mathcal{A}_n$ and let

$$f(A) = f'(A)$$
Theorem

Let $\varphi \in A_n$ and let

$p = (a_1, \ldots, a_6)^T \in \varphi,
\quad p' = (a'_1, \ldots, a'_6)^T \in \varphi$
Theorem

Let $\varphi \in A_n$ and let $p = (a_1, \ldots, a_6)^T \in \varphi$, $p' = (a'_1, \ldots, a'_6)^T \in \varphi$ represent $f$ and $f' \in \mathcal{F}$. 
Theorem

Let \( \varphi \in A_n \) and let

\[
p = (a_1, \ldots, a_6)^T \in \varphi,
p' = (a'_1, \ldots, a'_6)^T \in \varphi
\]

represent \( f \) and \( f' \in \mathcal{F} \).

Then \( f(A) = f'(A) \).
Theorem

Let $\varphi \in \mathcal{A}_n$ and let 

\[ p = (a_1, \ldots, a_6)^T \in \varphi, \]
\[ p' = (a'_1, \ldots, a'_6)^T \in \varphi \]

represent $f$ and $f' \in \mathcal{F}$.

Then $f(A) = f'(A)$.

Theorem

$|\mathcal{A}_n| \in O(n^{18})$. 
A sequential Affine Image Matching Algorithm [MFCS08]

**Input:** Images $A$ and $B$ of size $n$

**Output:** $f \in \mathcal{F}$ with minimum distortion $\Delta = (f(A), B)$

1. 
2. 
3. 
4. 
5. 
6.
A sequential Affine Image Matching Algorithm [MFCS08]

Input: Images $A$ and $B$ of size $n$
Output: $f \in \mathcal{F}$ with minimum distortion $\Delta = (f(A), B)$

1. construct $\mathcal{A}_n$ \hspace{2cm} $O(n^{18})$ time (Edelsbrunner '86)
2.
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2. traverse all faces $\varphi$ of $\mathcal{A}_n$ \hspace{1cm} $O(n^{18})$ time (DFS)
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It was open for a long time, whether Affine Image Matching is \text{TC}^0-complete.
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4. get $f$ from $p$ and compute $f(A)$  \hspace{1cm} $O(1)$ time (amortized)

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**Input:** Images $A$ and $B$ of size $n$
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1. **construct** $\mathcal{A}_n$ \hspace{2cm} $O(n^{18})$ time (Edelsbrunner '86)
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5. **obtain** $\Delta = (f(A), B)$ \hspace{2cm} $O(1)$ time (amortized)
6. **return** $f$ with minimum distortion \hspace{2cm} $O(1)$ time

It was open for a long time, whether Affine Image Matching is $\text{TC}^0$-complete.

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A sequential Affine Image Matching Algorithm [MFCS08]

Input: Images $A$ and $B$ of size $n$

Output: $f \in \mathcal{F}$ with minimum distortion $\Delta = (f(A), B)$

1. construct $A_n$ $\quad O(n^{18})$ time (Edelsbrunner '86)
2. traverse all faces $\varphi$ of $A_n$ $\quad O(n^{18})$ time (DFS)
3. find $p = (a_1, \ldots, a_6)^T \in \varphi$ $\quad O(1)$ time
4. get $f$ from $p$ and compute $f(A)$ $\quad O(1)$ time (amortized)
5. obtain $\Delta = (f(A), B)$ $\quad O(1)$ time (amortized)
6. return $f$ with minimum distortion $\quad O(1)$ time

overall time $\quad O(n^{18})$ time

It was open for a long time, whether Affine Image Matching is polynomial time solveable.
Recently raised Questions

1. What is the “exact” complexity class of Affine Image Matching?
2. To which degree can parallel computation speed-up Affine Image Matching?

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Affine Image Matching is \( \text{TC}^0 \)-complete
Recently raised Questions

1. What is the “exact” complexity class of Affine Image Matching?
2. To which degree can parallel computation speed-up Affine Image Matching?
Theorem

**Affine Image Matching is \( \text{TC}^0 \)-complete.**
New Result

Theorem

**Affine Image Matching is $\text{TC}^0$-complete.**

$$\text{AC}^0 \subset \text{TC}^0 \subset \text{NC}^1 \subset \ldots \subset L \subset P$$
The Complexity Class $\text{TC}^0$

All functions $f : \{0, 1\}^* \rightarrow \{0, 1\}^*$ that can be computed by

1. a family of circuits $C_1, C_2, \ldots, C_n, \ldots$ that contain
2. and-gates, and
3. where every circuit has constant depth and
4. contains a polynomial number of gates.
The Complexity Class $\text{TC}^0$

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2. and-gates, or-gates and
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\[ C_5 \]

\[ C_4 \]

\[ C_3 \]

\[ C_1, C_2 \]

\[ C_6 \]

\[ C_7, C_8 \]
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Affine Image Matching is $\text{TC}^0$-complete
The Complexity Class $\mathsf{TC}^0$

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Affine Image Matching is $\mathsf{TC}^0$-complete
The Complexity Class $\text{TC}^0$

majority function ($\text{TC}^0$-complete)
The Complexity Class $TC^0$

- majority function ($TC^0$-complete)
- integer addition, multiplication ($TC^0$-complete), division ($TC^0$-complete), ...

However, it is unknown how $A_n$ can be computed in $TC^0$. 

Affine Image Matching is $TC^0$-complete
The Complexity Class $\text{TC}^0$

- majority function ($\text{TC}^0$-complete)
- integer addition, multiplication ($\text{TC}^0$-complete), division ($\text{TC}^0$-complete), ...
- repeated integer addition, i.e., $\sum$-operations ($\text{TC}^0$-complete)

However it is unknown how $A_n$ can be computed in $\text{TC}^0$. 

**Affine Image Matching is $\text{TC}^0$-complete**
The Complexity Class $\text{TC}^0$

- majority function ($\text{TC}^0$-complete)

- integer addition, multiplication ($\text{TC}^0$-complete), division ($\text{TC}^0$-complete), ...

- repeated integer addition, i.e., $\sum$-operations ($\text{TC}^0$-complete)

- integer minimum ($\text{TC}^0$-complete), integer sorting ($\text{TC}^0$-complete)
The Complexity Class $\text{TC}^0$

majority function ($\text{TC}^0$-complete)

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integer minimum ($\text{TC}^0$-complete), integer sorting ($\text{TC}^0$-complete)

However it is unknown how $A_n$ can be computed in $\text{TC}^0$. 
A $\text{TC}^0$ Affine Image Matching Approach

To get $D(A)$...
A $\text{TC}^0$ Affine Image Matching Approach

To get $D(A)$:

the old algorithm processes all these points.
A \( \text{TC}^0 \) Affine Image Matching Approach

However, consider:

**Definition**

\( G_n \), the grid of points

\[
\begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
a_6 \\
\end{pmatrix} = 10^{-7} n^{-7} \begin{pmatrix} t_1 + 0.5 \\ t_2 \\ t_3 \\ t_4 \\ t_5 + 0.5 \\ t_6 \end{pmatrix}
\]

for all \( t_1, \ldots, t_6 \) in \( \{-10^{12} n^{13}, \ldots, 10^{12} n^{13}\} \).
Theorem

1. $|G_n| \in O(n^{78})$.
2. Every point $p = (a_1, \ldots, a_6) \in G_n$ fulfills $a_1a_5 \neq a_2a_4$.
3. For every $\varphi \in A_n$ there is a grid point $p$ with $p \in \varphi$. 

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Affine Image Matching is $\mathsf{TC}^0$-complete
A $\text{TC}^0$ Affine Image Matching Approach

**Theorem**

1. $|\mathcal{G}_n| \in O(n^{78})$.
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Theorem

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Affine Image Matching is $\text{TC}^0$-complete
Affine Image Matching is $\text{TC}^0$-complete.
A $\text{TC}^0$ Affine Image Matching Approach

\[ \min f \text{ with minimum } \Delta(f_1(A), B), \Delta(f_2(A), B), \Delta(f_3(A), B), \ldots, \Delta(f_{|G_n|}(A), B) \]

essentially arithmetics, constant depth, $O(n^{78})$ times polynomially many processors

essentially repeated addition, constant depth, $O(n^{78})$ times polynomially many processors

Hence, Affine Image Matching is in $\text{TC}^0$.

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Affine Image Matching is $\text{TC}^0$-complete
A $\textbf{TC}^0$ Affine Image Matching Approach

Affine Image Matching is $\textbf{TC}^0$-complete

essentially arithmetics, constant depth, $O(n^{78})$ times polynomially many processors

essentially repeated addition, constant depth, $O(n^{78})$ times polynomially many processors

minimum computation, constant depth, polynomially many processors

$\Delta(f_1(A), B)$

$\Delta(f_2(A), B)$

$\Delta(f_3(A), B)$

$\Delta(f_{|G_n|}(A), B)$

$\Delta(f_1(A), B)$

$\Delta(f_2(A), B)$

$\Delta(f_3(A), B)$

$\Delta(f_{|G_n|}(A), B)$

$f$ with minimum $\Delta(f(A), B)$
Affine Image Matching Approach

\[ \min f \text{ with minimum } \Delta(f(A), B) \]

\[ \Delta(f_1(A), B), \Delta(f_2(A), B), \Delta(f_3(A), B), \ldots, \Delta(f_{|G_n|}(A), B) \]

\[ f \text{ with minimum } \Delta(f(A), B) \]

Hence, Affine Image Matching is in $\text{TC}^0$.
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

$$\text{Majority} = \left\{ s \in \{0, 1\}^* \mid \sum_{i=1}^{\left| s \right|} s_i \geq \left\lfloor \frac{|s|}{2} \right\rfloor \right\}$$
The $\text{TC}^0$-completeness of Affine Image Matching

string $s$

0 1 0 1 1 1 0
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

Majority $= \{ s \in \{0,1\}^* \mid \sum_{i=1}^{\|s\|} s_i \geq \lfloor \|s\|/2 \rfloor \}$

string $s$

$0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0$

image $A_s$

$\Delta(f_{opt}(A_s), B_s) = \sum_{i=1}^{\|s\|} 1 - s_i$
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

$$\text{Majority} = \{ s \in \{0, 1\}^* | \sum_{i=1}^{\text{|s|}} s_i \geq \lfloor \text{|s|}/2 \rfloor \}$$

for string $s$.

Image $A_s$:

Image $B_s$:

Affine Image Matching is $\text{TC}^0$-complete.
The $\text{TC}^0$-completeness of Affine Image Matching

String $s$ is in Majority iff $\Delta(A_s, B_s) \leq \left\lfloor \frac{|s|}{2} \right\rfloor$. 

\[
\Delta(A_s, B_s) = \sum_{i=1}^{\text{|s|}} 1 - s_i
\]
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem $\text{Majority} = \{ s \in \{0,1\}^* \mid \sum_{i=1}^{\lvert s \rvert} s_i \geq \lfloor \lvert s \rvert^2 \rfloor \}$

$\text{image } A_s$

$\text{image } f(A_s)$

$\text{image } B_s$

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Affine Image Matching is $\text{TC}^0$-complete
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

Majority $= \{ s \in \{0, 1\}^* | \sum_{i=1}^{|s|} s_i \geq \lfloor \frac{|s|}{2} \rfloor \}$

string $s$

0 1 0 1 1 1 0

image $A_s$

image $B_s$
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

$$\text{Majority} = \{s \in \{0, 1\}^* \mid \sum_{i=1}^{\|s\|} s_i \geq \lfloor \|s\|^2 \rfloor\}$$

image $A_s$

image $B_s$

image $f_{opt}(A_s)$

String $s$

$0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0$

$$f_{opt}(x,y) \approx \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$a = \frac{16|s|}{16|s| - 7}$$
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem $\text{Majority} = \{s \in \{0, 1\}^* \mid \sum_{i=1}^{|s|} s_i \geq \lfloor |s|^2 \rfloor\}$ string $s$

image $f_{\text{opt}}(A_s)$

image $B_s$
**The $\text{TC}^0$-completeness of Affine Image Matching**

Consider the $\text{TC}^0$-complete problem $\text{Majority} = \{ s \in \{0,1\}^* \mid |\sum_{i=1}^{|s|} 1-s_i| \leq \left\lfloor \frac{|s|}{2} \right\rfloor \}$.

String $s$ is in Majority iff $\Delta(f_{opt}(A_s), B_s) \leq \left\lfloor \frac{|s|}{2} \right\rfloor$.

<table>
<thead>
<tr>
<th>string $s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 0 1 1 1 1 0</td>
</tr>
</tbody>
</table>

image $f_{opt}(A_s)$

image $B_s$

$$\Delta(f_{opt}(A_s), B_s) = \sum_{i=1}^{|s|} 1-s_i$$
The $\text{TC}^0$-completeness of Affine Image Matching

Consider the $\text{TC}^0$-complete problem

$$\text{Majority} = \{ s \in \{0, 1\}^* | \sum_{i=1}^{|s|} s_i \geq \lfloor |s|/2 \rfloor \}$$

For a string $s = 01011110$,

$$\Delta(f_{\text{opt}}(A_s), B_s) = \sum_{i=1}^{|s|} 1 - s_i$$

Because $A_s$ and $B_s$ can be computed easily, Affine Image Matching is hard in $\text{TC}^0$. 

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Conclusions

- New insights into the structure of affine image transformations.
  - Projective Image Matching is $\text{TC}^0$-complete.
  - $\text{TC}^0$-approach does not apply to weaker transformation classes in a straight-forward manner.
  - $\text{TC}^0$-hardness remains even for the weakest cases like Scaling or Rotation Image Matching.
Conclusions

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End of Talk

Thank you for your attention!