Optimal(almost) edit distance "1" dictionary

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Edit distance

- The edit distance between two strings x and y is the minimal number of edit operations needed to get string y from string x (which is the same as number of edit operations needed to get string x from string y).
- Usually considered edit operations are: insertion, deletions and substitution.
  - Insertion: insert a character c at some position in the string.
  - Deletion: delete some character from the string.
  - Substitution: substitute some character of the string with another character.

Problem definitions (Approximate dictionary with edit distance "1")

- We work in the word RAM model with word length w. All standard operations including multiplication, division and shift take constant time.
- We have a set S of n strings.
- Total number of characters in all strings is m.
- Each character is encoded using b bits.
- The size of alphabet is noted by α where $\alpha = 2^b$.
- We have to build a data structure on the set S so that we can answer to an approximate queries that asks for all strings of the set S that are at edit distance at most "1" from a query string q.

Applications

- Spell checking in word processors.
- Data-cleaning in databases. The same name is spelled differently in different databases.
- Optical character recognition. Correct the mis-recognized characters or complete the unrecognized characters.
- Information retrieval(Search engines): correct for user's typing errors.
- Bio-informatics.

Problem definitions(Approximate full-text indexing with edit distance "1")

- We work in the word RAM model.
- We have a text T of length n characters .
- We have to build a data structure on the set S so that we can answer to an approximate query that asks for all sub-strings of the text Tthat are at edit distance at most 1 from a query string q.

Results

- A dictionary that occupies $O(mb)$ bits space.
- Space is optimal up to a constant factor.
- Query time of the dictionary is $O(k)$, where $k = |q|$ is the length of query string.
- Query time is almost optimal.
- Application: a full text index that occupies space $O(n(\lg(n)\lg\lg(n))^2 b)$ bits with query time $O(|q|)$.

Related work(approximate dictionary for edit distance"1")

General case:

- Recall that α is alphabet size and $\alpha = 2^b$(each character is represented using b bits).
- Length of query string q is $k = |q|$

Method	Query time	Space usage	Construction time
BG96(1)	$O(k)$	$O(\alpha m)$ words	N.A
BG96(2)	$O(bk + \lg(n))$	$O(m)$ words	$O(m)$
BG96(3)	$O(\alpha k)$	$O(m)$ words	$O(m)$
CGL04	$O(k + \lg(n) \lg \lg(m))$	$O(m + n \lg(n))$ words	$O(m \lg(m))$
New result	$O(k)$	$O(mb)$ bits(optimal)	$O(m)$

BG96 : Brodal, Gasieniec. Approximate Dictionary Queries. CPM 1996.
CGL04 : Cole, Gottlieb, Lewenstein. Dictionary matching and indexing with errors and don't cares. STOC 2004.

Related work(approximate dictionary for edit distance"1")

Constant sized alphabets case:

- We have $b = O(1)$ and hence $\alpha = 2^b = O(1)$.
- Recall that a word occupies w bits.

Method	Qeury time	Space usage	Construction time
BG96(1)	$O(k)$	$O(mw)$ bits	N.A
BG96(2)	$O(k + \lg(n))$	$O(mw)$ bits	$O(m)$
BG96(3)	$O(k)$	$O(mw)$ bits	$O(m)$
CGL04	$O(k + \lg(n) \lg\lg(m))$	$O((m + n\lg(n))w)$ bits	$O(m\lg(m))$
New result	$O(k)$	$O(m)$ bits(optimal)	$O(m)$

Related work(approximate full-text indexing for edit distance"1")

- We have to index a text T of length n characters. Each character is chosen from a set of $\alpha = 2^b$ possible characters.
- Queries: report all sub-strings of the text T that are at edit distance "1" from query string q.
- We have length of q is $k = |q|$.

Method	Query time	Space usage
CGL04	$O(k + \lg(n) \lg\lg(n))$	$O(n \lg^2(n))$ bits
New result	$O(k)$	$O(n(\lg^2(n) \lg\lg^2(n))b)$ bits

Naive solution (using exact dictionary)

- Using a standard hash based dictionary occupying space $O(mb)$ bits.
- Query time for exact queries is $O(|s|)$ for a string s.
- For approximate queries on a string q, we can simply generate all strings at edit distance "1" from q and query exact dictionary for each string.
- We have k,$k2^b$ and $(k-1)2^b$ candidate strings that can be obtained from q using deletion,substitution and insertion respectively.
- Total query time becomes $O(k2^b \cdot k) = O(k^2 2^b)$.

Overview of the new solution (Reducing number of candidate strings)

- number of candidate strings for deletions is k.
- Reduce the number of candidate strings for substitutions and insertions to k and $k+1$ respectively instead of $k2^b$ and $(k+1)2^b$. For each possible position for insertion or substitution we have a list of candidate characters. Hence we have to explore k and $k+1$ lists in total for substitutions and insertions.
- Sufficient to check just for first character of the each list.

Overview of the new solution (Reduce query time for each candidate string to $O(1)$)

- Look-up for a candidate string involves two step: compute a hash function on candidate string and compare the candidate string with the string from the dictionary pointed by the hash function.
- Use a preprocessing step that takes $O(k)$ time.
- Each subsequent hash function evaluation takes $O(1)$ time.
- Each subsequent string comparison takes takes $O(1)$ time also.

Components

We make use of the following components:

- Two Succinctly encoded tries.
- Injective hash functions from sets of strings into sets of integers.
- Minimal perfect hash functions.
- Succinctly encoded sequences of integers.
- Dictionary based on Minimal perfect hashing.
- Dictionary that stores Succinctly encoded lists, where each list is associated with a key.

Components: succinctly encoded tries

- Succinctly encoded trie. A trie can be encoded using space $O(mb)$ bits.
- Time to construct the trie is $O(m)$.
- Time to traverse the trie is $O(k)$ for a string of length k.
- At each step of the traversal, we get a unique identifier in interval $[0, m-1]$.

Components: minimal perfect hash function

Minimal perfect hash function for integers

- A minimal perfect hash functions (mphf) maps a set of n' integer keys to interval $[0, n-1]$.
- Time to construct the perfect hash function if $O(n')$.
- Query time is $O(1)$.

Minimal perfect hash function for strings

- For a set of n' strings having a total of m' characters, we first apply a simple hash function that maps the set of strings to integers of $O(\lg(n))$ bits.
- We can now build the mphf on the set of integers.

Components: hash function for strings

- Goal: reduce each of the variable length strings to integers occupying $O(w)$ bits each, so that each string is mapped to a distinct integer.
- The hash functions is a polynomial over prime field modulo a prime P, where $P \geq 2^b$ and $P >= mn^2$.
- For a string s, the hash function is evaluated as

$$h_t(s) = \sum_{i=1}^{|s|} s[i] \otimes t^i$$

  where t is a number from interval $[1, P]$ that characterizes the hash function.
- All additions and multiplications are done modulo P.
- With a randomly chosen t, the hash function h_t will be injective over interval $[0, P-1]$ with probability at most $1/2$.
- Keep choosing a new value for t until all strings are mapped to a distinct integer.

Components:Succinctly encoded Compressed sequences of integers

- We have a sequence of n integers.
- The sum of the integers in the sequence is m.
- We can encode the integers to use space $n(2+\lg(m/n))$ bits.
- The encoding permits to retrieve the sum of integers $sum_k = x_0 + x_1 + \cdots + x_k$ for any k in constant time.
- Retrieving the integer x_k can be deduced in $O(1)$ time using formulae $x_k = sum_k - sum_{k-1}$.

Components: mphf based dictionary for variable length strings

- We use mphf to map n strings to integers in the range $[0, n-1]$.
- Store the lengths of strings using a succinctly encoded sequence of integers.
- Store the strings in array in the order given by the *mphf*.
- Constant access to the start position of every string.

Components: retrieval only dictionary of lists

- Very similar to the dictionary for variable length strings.
- Instead of storing a set of strings, we store a set of lists where each list is associated with a key(we do not store the key itself).
- returns size of a list associated with a key in constant time.
- Constant time access any element of a list.
- The data structure is retrieval only: the data structure returns the correct list for an existing key, but returns an arbitrary list for a non-existing key.

Putting elements together

Our data structure will contain the following elements:

- A trie Tr that stores the strings of S.
- A reverse trie $\overline{Tr}$ that stores the strings of $\overline{S}$ where the set $\overline{S}$ is the set of strings of S written in reverse order.
- a variant of mphf based dictionary.
- dictionary of lists.

Putting elements together (trie and reverse trie)

Our data structure will contain the following elements:

- Construction of Tr and $\overline{Tr}$ takes time $O(m)$.
- The trie Tr and reverse trie $\overline{Tr}$ are succinctly encoded.
- Space usage of Tr and $\overline{Tr}$ is $O(mb)$ bits.
- traversal of the trie and reverse trie returns a unique identifier at each step.

Putting elements together (mphf based dictionary)

- A dictionary for variable length strings augmented with signatures for long strings.
- Short strings are stored unmodified.
- A string is considered as a long one if its length exceeds w bits.
- Each long string will be stored in triple its original size.
- We use a parameter $u = \lg(m)/b$.
- For a long string s we store signatures of prefixes of s of lengths $u, 2u, 3u, \ldots$.
- We also store signatures of suffixes of s of lengths $u, 2u, 3u, \ldots$.

Putting elements together (mphf based dictionary)

- Each signature occupies $\lg(m)$ bits.
- Signatures of prefixes of s are obtained by traversing trie Tr for the string s.
- Signatures of suffixes of s are obtained by traversing trie $\overline{Tr}$ for the string $\overline{s}$ (string s written in reverse order).
- The signature of a prefix of length iu is the unique identifier returned by the trie Tr at step iu of traversal for the string s.
- The signature of a suffix of length iu is the unique identifier returned by the trie $\overline{Tr}$ at step iu of the traversal for the string $\overline{q}$.
- Total space used by signatures does not exceed $2|s|b$ bits.

Putting elements together (lists dictionary)

For each string s from S of length $k = |s|$, we do the following pre-processing:

- We traverse the trie Tr and store in an array $L[0..k]$, the labels encountered at each step of the traversal.
- Likewise we traverse the trie $\overline{Tr}$ for the string $\overline{s}$ and store the labels encountered at each step of traversal in array $R[0..k]$.
- In total we store exactly k characters in the lists for th string s.
- Total space used by all lists is the same as the space occupied by the strings themselves.

Queries(Preprocessing for signatures)

- We have a string q of length k.
- Traverse the trie Tr for the string q and store in a an array $L[1..k]$, the labels (integers) returned at each step of the traversal.
- Traverse the the trie $\overline{Tr}$ for the string $\overline{q}$ (q written in reverse order), and store the returned labels in an array $R[1..k]$.
- Time for traversal of Tr and $\overline{Tr}$ is $O(k)$.

Queries(Preprocessing for hashing)

- Compute an array $A_t[0, k+1]$ of powers of t (recall that t is the number that characterizes the hash function). This takes time $O(k)$: first set $A_t[0] = 1$, then set $A_t[i+1] = A_t[i] \cdot t$ for every $i \in [1, k]$).
- Compute an array $F[0..k]$ of the hash values of all prefixes of q: first set $F[0] = 0$, then set $F[i] = F[i-1] + (q[i] \cdot A_t[i])$ for each $i \in [1, k]$. Total time is $O(k)$.
- Compute the array $G[1..k]$ of hash values of all suffixes of q: first set $F[0] = 0$, then set $F[i] = F[i-1] + (q[i] \cdot A_t[i])$ for each $i \in [1, k]$. Total time $O(k)$.

Queries(Substitution)

For $i \in [1, k]$ such that $L[i-1] \neq \perp$ and $R[k-i] \neq \perp$ we do the following:

- Query the retrieval list for every pair $(L[i-1], R[k-i])$.
- $L[i]$ is the identifier of of $q[1..i-1]$ (prefix of q of length $i-1$)
- $R[k-i]$ is the identifier of $q[i+1..k]$ (suffix of q of length $k-i$).
- First element of list associated with a pair $(L[i-1], R[k-i])$ is a character c that could be substituted at position i in string q.
- We now have to look for the string $q' = q[0..i-1] \cdot cq[i+1..k]$ in the mphf based dictionary. If we have a match we continue to report all remaining elements (characters) of the list.

Queries(Lookup in dictionary)

- Lookup in the dictionary should not take more than $O(1)$ time.
- We can compute $h(q')$ in constant time using formulae

$$h(q') = F[i-1] \oplus (c \oplus G[i+1]) \otimes A_t[i]$$

.

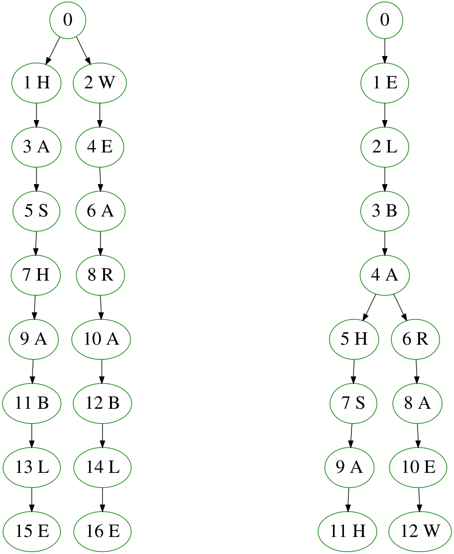
- We can now use the hash value $h(q')$ to compute mphf which will point to a string s in the dictionary. If s is a short string of length $\leq w$, we can compare it with q' in constant time. Otherwise we compare s with q' using the array s_l of left signatures and the array s_r of right signatures.
  - Length of s is $3k$.
  - $l_{q'} = 0 \vee s_l[l_{q'}] = L[l_{q'}]$.
  - $q[u \cdot l_{q'} + 1..i-1] = s_1[u \cdot l_{q'} + 1..i-1]$.
  - $s_1[i] = c$.
  - $q[i+1..k - u \cdot r_{q'}] = s_1[i+1..k - u \cdot r_{q'}]$.
  - $r_{q'} = 0 \vee s_r[r_{q'}] = R[r_{q'}]$.

Example(construction)

- A set of two strings "*wearable*" and "*hashable*".
- Suppose $w = 32$ bits. We use ASCII alphabet: $b = 8$ bits and $\alpha = 256$ (we have 256 distinct characters).
- The two strings are considered as long strings as their length exceeds $w = 32$ bits.
- We have $u = (32/8) = 4$.

Example(construction)

We build the trie *Tr* and reverse trie $\overline{Tr}$.

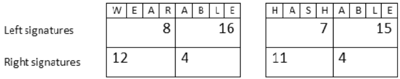


Example(construction)

- Traverse the trie Tr for the strings "*wearable*" and "*hashable*" resulting in sequences $[0, 2, 4, 6, 8, 10, 12, 14, 16]$ and $[0, 1, 3, 5, 7, 9, 11, 13, 15]$.
- Traverse the reverse trie $\overline{Tr}$ for the strings $=$ "*elbaraew*" and "*elvahsah*", resulting in sequences $[0, 1, 2, 3, 4, 6, 8, 10, 12]$ and $[0, 1, 2, 3, 4, 5, 7, 9, 11]$.
- For string "*wearable*", we store the characters w,e,a,r,a,b,l,e in the lists associated with pairs $(0, 12)$,$(2, 10)$,$(4, 8)$,$(6, 6)$,$(8, 4)$,$(10, 3)$,$(12, 2)$,$(14, 1)$,$(16, 0)$ respectively.
- For string "*hashable*", we store the characters h,a,s,h,a,b,l,e in lists associated with pairs $(0, 11)$,$(1, 9)$,$(3, 7)$,$(5, 5)$,$(7, 4)$,$(9, 3)$,$(11, 2)$,$(13, 1)$,$(15, 0)$ respectively.

Example(construction)

Build the mphf based dictionary.



Application to full-text indexing

- We have to index a text n characters each encoded using b bits.
- We first, index the text using the data structure of *CGL*04.
- Space usage of that data structure is $O(n \lg(n) \lg \lg(n))$ words.
- Query time of data structure is $O(k + \lg(n))$ for a pattern of length k which is optimal when $k \geq \lg(n)$.

Application to full-text indexing

- To obtain optimal query time for $k < \lg(n)$, we build our dictionary on all substring of the text of lenghts below $\lg(n)$.
- For each sub-string stored in dictionary, we store a pointer to its location in the text.
- Total number of sub-srings is about $n \lg(n) \lg\lg(n)$, and each sub-string is of length at most $\lg(n) \lg\lg(n)$.
- Total space is thus $O(n(\lg^2(n) \lg\lg^2(n))b)$ bits.

Conclusion

- We have presented a dictionary for approximate edit distance "1" queries that uses optimal space up to constant factor.
- Query time for a string of length k characters is $O(k)$, which is optimal for very large alphabets(characters that occupy w bits), but not for smaller alphabets, for which query time is a factor w away from optimal.
- Straightforward application of our dictionary permits to build a full-text index that uses space $O(n(\lg(n)\lg\lg(n))^2)$ with query time $O(k)$ for a pattern of length k.

Follow-ups

- We plan to investigate practical performance of the dictionary.
- For constant-sized alphabets we can improve query time of our dictionary from $O(k)$ to optimal $O(k/w)$ (factor w speedup) at the expense of using space that is a factor $\lg(w)$ from optimal (we use space $O(m \lg(w))$ bits instead of $O(m)$ bits).
- For approximate full-text indexing, we have an improved solution with space usage reduced to $O(n \lg(n) \lg \lg(n))$ bits for constant sized alphabets and to $O(n \lg(n) \lg \lg(n))$ words for arbitrary alphabets.

Open problems

- Improving space: reduce constant factors, entropy compression.
- Is there any lower bound on space/time trade-off for the dictionary.
- What about external memory. A straightforward adaptation of our dictionary in external memory would use $O(k)$ I/Os. An optimal solution would use $O(k/B)$ I/Os.
- Dynamic version of our dictionary.