CS30 Spring 2015
Lab 10

Use the command `diary` to record your answers and submit them. Submit code for the functions and scripts you write. Submit any figures as indicated.

1. (40 points) Consider the following (noisy) height data, sampled from the trajectory of a projectile moving in time.

   \[
   \text{times} = \text{linspace}(0,4.6,10);
   \text{heights} = [101.4476, 97.9845, 92.7696, 89.1982, 81.2081, ... \\
   66.4828, 57.4775, 37.5667, 20.2902, 0.8299];
   \]

   For a set of data \( \{(t_i, f_i) , i = 1 \ldots 10\} \) such as the data above, the Matlab command `polyfit` finds a polynomial \( f(t) \) of given degree that minimizes the sum of squares of residual error

   \[
   r = \sum_{i=1}^{10} (f(t_i) - f_i)^2
   \]  

(a) Use `polyfit` to find a degree 2 polynomial that best fits the given data. What polynomial \( f(t) \) does `polyfit` find?

(b) Plot the data and your polynomial in the same figure and include your plot.

(c) Compute the residual \( r \) using the formula in equation (1).

(d) What is the residual if the polynomial is \( f(t) = -5t^2 - t + 10 \).

(e) Use `polyfit` to fit a degree 9 polynomial to the data, and plot the resulting polynomial in your figure. What is the residual \( r \) using this polynomial? Does this polynomial provide a better or worse fit to the data than the one computed in part (a)?

2. (30 points) Consider the following (noisy) data.

   \[
   \text{x} = \text{linspace}(5,10,10);
   \text{g} = [3372.70, 4689.14, 8925.88, 11371.34, 14775.29, ... \\
   16054.18, 21977.08, 29157.39, 41272.75, 52215.04];
   \]

(a) Find constants \( a \) and \( b \) such that \( g(x) = ax^b \) fits the given data. Do this by transforming the problem into a related linear problem and using `polyfit`.

(b) Plot the data and your function \( g(x) \) in the same figure.

(c) Define a finer sampling of the interval

   \[
   \text{xFiner} = \text{linspace}(5,10,100);
   \]

   and use the Matlab function `interp1` to do a piecewise cubic interpolation of the data on this sample. Plot your result in the same figure. How does it compare with the curve determined in part (a)?

3. (30 points) Consider the polynomial

   \[
   f(x) = 3x^3 + 2x^2 - x + 5;
   \]

(a) Sample the polynomial at 10 sample points in the interval \([0,1]\).

(b) Use `diff` to compute backward difference estimates of the first derivative \( f'(x) \) at points 2-10.

(c) Compute central differences estimates of the first derivatives \( f'(x) \) at points 2-9.

(d) Plot the estimated derivatives as well as the exact derivative \( f'(x) \) in a figure.