(Lectures 12-19)

Lecture 12

- overdetermined system
- least squares
- normal equations
  - uniqueness of solution to normal equations
  - geometric interpretation
- Least squares solution by pseudo-inverse
  - minimum norm solution

Lecture 13

- least squares solution by QR (for A with linearly independent columns)
  - case 1: A has linearly independent columns
  - case 2: A has linearly dependent columns
- least squares and Tikhonov regularization
  - formulation
  - makes the problem full rank
  - makes the problem better conditioned
- weighted least squares formulation
- condition number of $f$
- condition number of a matrix
  - definition
  - properties
  - well-conditioned vs. ill-conditioned
  - condition number in 2-norm
  - condition number in 2-norm of symmetric matrix
  - and geometric interpretation
- normal equations square the condition number
Lectures 14-15

- conditioning and stability
- backward error
- condition number
- stability and accuracy
- Floating Point
  - finite precision
  - general floating point system: base, precision, exponent range
  - example system
  - normalization

Lecture 16

- Floating Point Numbers
  - underflow level
  - overflow level
  - picture of representable floating point numbers
  - subnormals
  - exceptional values: Inf and Nan

- Floating Point Math
  - Rounding: chop, nearest, even
  - Machine epsilon
  - addition/subtraction
  - multiplication/division
  - Rounding Error analysis
  - Floating Point Issues
    - Finite Precision
    - Overflow
    - Cancellation error
  - Examples of floating point issues
Lecture 17

• iterative methods
• matrix splitting
  – Jacobi
  – Gauss-Seidel
• convergence rate
• eigenvalue problems
  – power method
  – normalized power iteration
  – power method and shifting
  – inverse iteration
  – Rayleigh quotient iteration
  – QR algorithm for eigenvalues and eigenvectors

Lecture 18

• QR algorithm
  – basic algorithm
  – generates similar matrix at each iteration
  – converges to Schur form (revealing eigenvalues)
• optimizations
  – first reduce matrix to upper Hessenberg/Tridiagonal via Householder, cuts cost of QR decomposition
  – shifted QR algorithm to accelerate convergence
• Krylov subspaces
  – good for large, sparse A
• Arnoldi iteration
  – generates projections of A onto Krylov subspaces
  – generates upper Hessenberg matrix
  – then do QR on upper Hessenberg matrix to find approximate eigenvalues of A
• Arnoldi reduces to Lanczos for symmetric matrices
• upper Hessenberg reduces to tridiagonal for symmetric matrices
• residual
  – relation to error
  – and stopping criteria for iterative methods
• solvers based on Krylov subspaces
• $Ax = b$ by GMRES
  – minimizes 2-norm of residual over each Krylov subspace

Lecture 19
• line search method
• step size for exact line search on quadratic function
• steepest descent method
• spd $A$, $Ax = b$ by Conjugate Gradients (CG)
  – recast as minimization of quadratic function
  – minimizes $A$-norm of error over each Krylov subspace
  – $A$-orthogonal search directions
  – compared with steepest descent directions
  – termination in $n$ steps (theoretical)
  – Gram-Schmidt $A$-orthogonalization