Homework 4
CS 210

1. (Heath 6.3) For each of the following functions, what do the first- and second- order optimality conditions say about whether 0 is a minimizer on $\mathbb{R}$?

   (a) $f(x) = x^2$
   (b) $f(x) = x^3$
   (c) $f(x) = x^4$
   (d) $f(x) = -x^4$

2. (Heath 6.4) Determine the critical points of each of the following functions and characterize each as a minimum, maximum, or inflection point. Also determine whether each function has a global minimum or maximum on $\mathbb{R}$.

   (a) $f(x) = x^3 + 6x^2 - 15x + 2$
   (b) $f(x) = 2x^3 - 25x^2 - 12x + 15$
   (c) $f(x) = 3x^3 + 7x^2 - 15x - 3$
   (d) $f(x) = x^2 e^x$

3. (Heath 6.5) Determine the critical points of each of the following functions and characterize each as a minimum, maximum, or saddle point. Also determine whether each function has a global minimum or maximum on $\mathbb{R}^2$.

   (a) $f(x, y) = x^2 - 4xy + y^2$
   (b) $f(x, y) = x^4 - 4xy + y^4$
   (c) $f(x, y) = 2x^3 - 3x^2 - 6xy(x - y - 1)$
   (d) $f(x, y) = (x - y)^4 + x^2 - y^2 - 2x + 2y + 1$

4. (Heath 6.8) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(\mathbf{x}) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2.$$

   (a) At what point does f attain a minimum?
   (b) Perform one iteration of Newton's method for minimizing f using as starting point $\mathbf{x}_0 = (2, 2)^T$.
   (c) In what sense is this a good step?
   (d) In what sense is this a bad step?

5. (Heath 6.9) Let $f : \mathbb{R}^n \to \mathbb{R}$ be given by

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A \mathbf{x} - \mathbf{x}^T \mathbf{b} + c$$

   where A is an $n \times n$ symmetric positive definite matrix, $\mathbf{b}$ is an n-vector, and c is a scalar.

(a) Show that Newton's method for minimizing this function converges in one iteration from any starting point $\mathbf{x}_0$.

(b) If the steepest descent method is used on this problem, what happens if the starting value $\mathbf{x}_0$ is such that $\mathbf{x}_0 - \mathbf{x}^*$ is an eigenvector of A, where $\mathbf{x}^*$ is the solution?