1 True/False

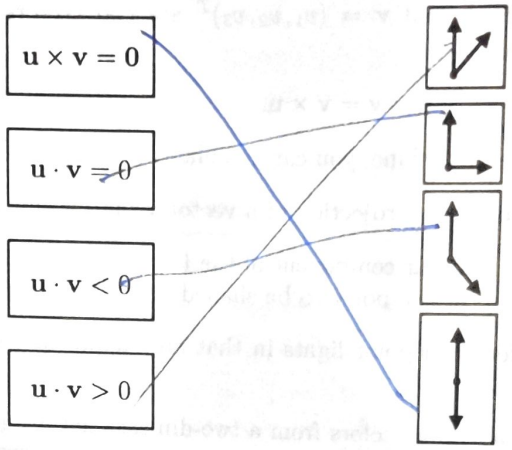
For each question, indicate whether the statement is true or false by circling T or F, respectively. You get 2 points for answering a question correctly, -0.5 points for answering the question incorrectly, and 1 point for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60% percent confident that your answer is correct).

1. (T/F) If $\mathbf{u} = (u_1, u_2, u_3)^T$ and $\mathbf{v} = (v_1, v_2, v_3)^T$ are two vectors in $\mathbb{R}^3$, then their dot product is $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$.
2. (T/F) For vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.
3. (T/F) Given two points on a plane, you can calculate the normal of the plane using the cross product.
4. (T/F) The vector $(\mathbf{v} \cdot \mathbf{n})\mathbf{n}$ is a projection of a vector $\mathbf{v}$ onto a unit vector $\mathbf{n}$.
5. (T/F) To compute the specular component of the Phong reflectance model, we reflect the view vector about the surface normal at the point to be shaded.
6. (T/F) Area lights differ from point lights in that area lights cast soft shadows while point lights do not.
7. (T/F) The process of assigning colors from a two-dimensional image to the points on the surface of a three-dimensional object is called texture mapping.
8. (T/F) Barycentric coordinates can be used to interpolate quantities from the vertices of triangle into the interior.
9. (T/F) When ray tracing with antialiasing, only one view ray is cast per pixel.
10. (T/F) Solving for the intersection of a ray with a sphere leads to a quadratic equation in the ray parameter t.

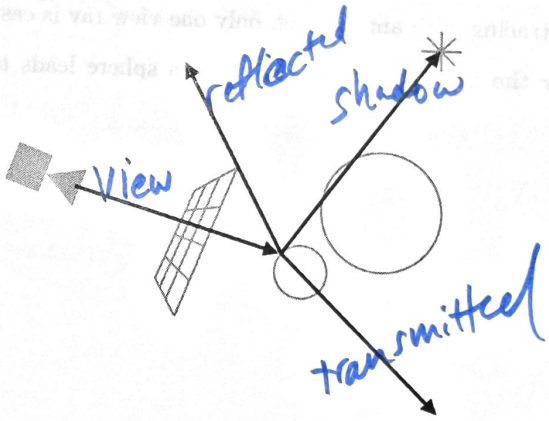
2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

11. For two vectors, $\mathbf{u}$, $\mathbf{v}$, match the expression in the left column with the illustration in the right column by drawing lines between the matching boxes.



12. The image below depicts four rays involved in a ray tracing computation. Indicate the type of each ray by labeling it with the words "view", "shadow", "reflected", or "transmitted".



13. Which of the following statements about the Phong reflectance model are true?

I. The model captures reflections of other objects in the scene.

II. The model has three components: ambient, diffuse, and specular.

III. The model accounts for surface orientation.

(a) II only

(b) I and II only

(c) I and III only

(d) II and III only

(e) I, II, and III

14. Which of the following statements regarding the debugging tools valgrind and gdb are true?

I. Valgrind is a debugging tool that can be used to detect improper memory usage in your program.

II. To run gdb on your program, it must be compiled with debug information.

III. gdb and valgrind are interchangeable, and can each detect the same problems with your code.

(a) I only

(b) II only

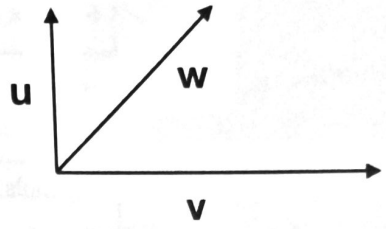
(c) I and II only

(d) II and III only

(e) I, II, and III

3 Written Response

15. Given the vectors $\mathbf{u} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \mathbf{v} = \begin{pmatrix}2\\0\\0\end{pmatrix}, \mathbf{w} = \begin{pmatrix}1\\1\\0\end{pmatrix}$ shown below, calculate the quantities below.



(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $\mathbf{u} \times \mathbf{v}$

(c) Projection of $\mathbf{w}$ on the direction of $\mathbf{v}$.

(c) $P_{\vec{v}}\vec{w} = \dfrac{\vec{v}\vec{v}^T}{\|\vec{v}\|\|\vec{v}\|}\vec{w}$

$\|\vec{v}\| = (v \cdot v)^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$

$\vec{v}/\|\vec{v}\| = \begin{pmatrix}1\\0\\0\end{pmatrix} \triangleq \vec{n}$

$P_{\vec{v}}\vec{w} = nn^Tw = \begin{pmatrix}1\\0\\0\end{pmatrix}(1\ 0\ 0)\begin{pmatrix}1\\1\\0\end{pmatrix} =$

$= \begin{pmatrix}1\\0\\0\end{pmatrix}$ ✓

(a) $u \cdot v = 0 \cdot 2 + 1 \cdot 0 + 0 \cdot 0 = 0$

(b) $u \times v = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\0 & 1 & 0\\2 & 0 & 0\end{vmatrix} = (0, 0, -2)^T = \begin{pmatrix}0\\0\\-2\end{pmatrix}$

16. Consider a ray with ray equation $\mathbf{r}(t) = \mathbf{e} + t\mathbf{u}, t \geq 0$ where $\|\mathbf{u}\| = 1$, and a plane with implicit plane equation $\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0)$, where $\mathbf{n}$ is the unit normal the plane and $\mathbf{p}_0$ is a point in the plane.

(a) Explain how to determine whether the ray and plane intersect and identify the point of intersection if they do intersect.

(b) Now consider three points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ that lie in the above plane and are not colinear. Explain how you would determine if the ray intersects with the triangle $\mathbf{ABC}$.

(a) Plug r(t) into plane eq:

$n \cdot (r(t) - p_0) = 0$

$\Rightarrow n \cdot (e + tu - p_0) = 0$

$\Rightarrow n \cdot [e - p_0 + tu] = 0$

$\Rightarrow n \cdot u\, t = n \cdot (p_0 - e)$

$n \cdot u \neq 0$

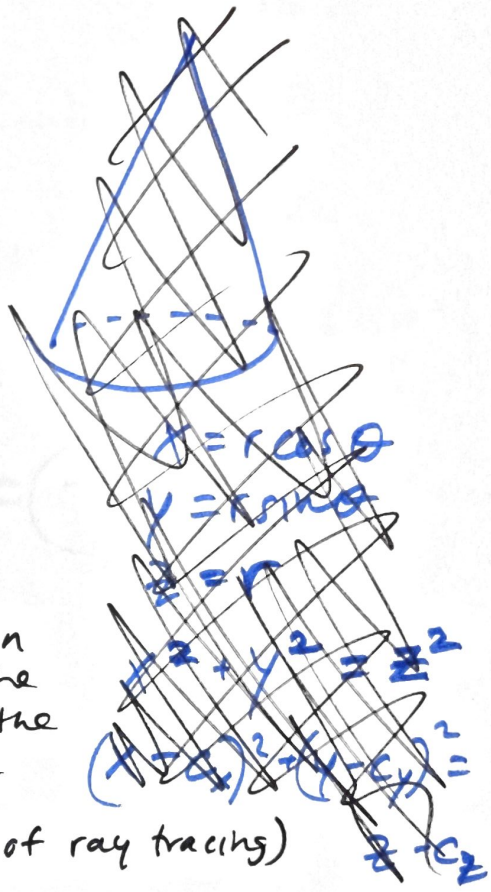
$t = \dfrac{n \cdot (p_0 - e)}{n \cdot u}$

if $t \geq 0$

$\Rightarrow$ intersection

$n \cdot u = 0$

either e is in the plane & then whole ray is in the plane, or e not in the plane. Either way, we will treat this as no intersection (for purpose of ray tracing)



(b) if no intersection in (a) $\Rightarrow$ no tri. intersection

else find the barycentric coords of $p(t)$, α, β, γ s.t.

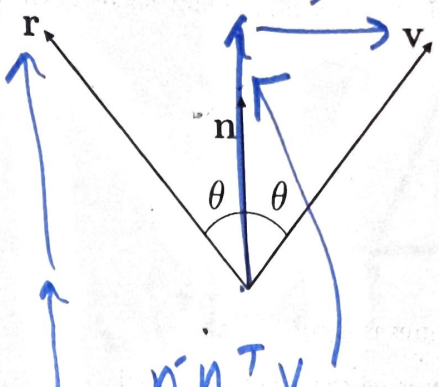
$p(t) = \alpha A + \beta B + \gamma C$

If $0 \leq \alpha, \beta, \gamma \leq 1$, then we have tri. intersection

else no tri. intersection.

Written Response

21. In the figure below, the vector **r** is the reflection of the vector **v** about the unit vector **n**. Write an expression for **r** in terms of **v** and **n**.



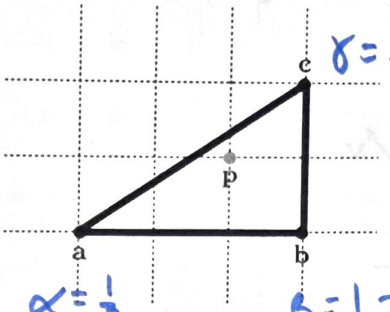
$v - nn^T v$

$nn^T v$

$\vec{r} = nn^T v - (v - nn^T v)$

$= -v + 2nn^T v = (-I + 2nn^T)v$

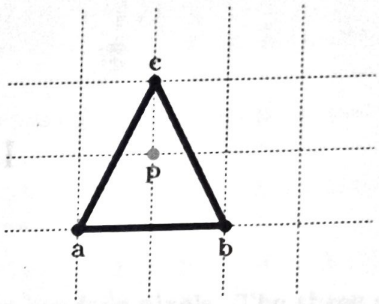
22. Next to each triangle, write the values of the barycentric coordinates α, β, γ for the point $\mathbf{p}$ with respect to the triangle with vertices $\mathbf{a}, \mathbf{b}, \mathbf{c}$. pictured.



$\gamma = \frac{1}{2}$

$\alpha = \frac{1}{3}$

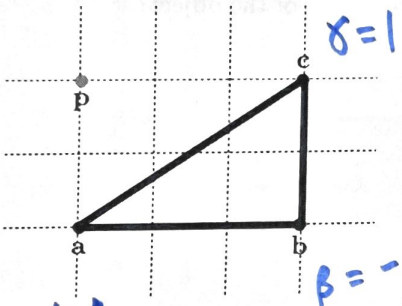
$\beta = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$



$\alpha = x = \frac{1}{4}$

$\beta = x = \frac{1}{4}$

$\gamma = \frac{1}{2}$

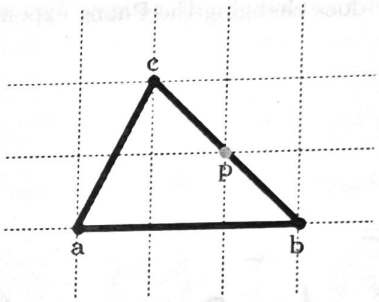


$\gamma = 1$

$\beta = -1$

$\alpha =$ [redacted]

$\alpha + \beta + \gamma = 1$ ✓



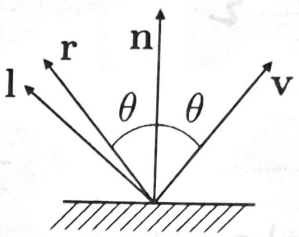
$\alpha = 0$

$\beta = \frac{1}{2}$

$\gamma = \frac{1}{2}$

You can use the grid lines, or use triangle areas. Both strategies are useful at different times.

23. Consider the figure below, depicting a point to be shaded using the Phong Reflectance Model, where **l** is the light vector, **v** is the view vector, **r** is the reflected vector, and **n** is the normal vector.



(a) Write down the ambient, diffuse, and specular components of the Phong Reflectance Model.

(b) How does changing the Phong exponent change the appearance of the object?

(a)

amb = $L_a R_a$

dif = $L_d R_d \max(n \cdot l, 0)$

spec = $L_s R_s \max(r \cdot l, 0)^e$

(b) large exp → small spec highlight → very "shiny" material

small exp → large spec highlight → less "shiny" material

Below is a simple 2D raytracing setup. The 1D image has four pixels. The three objects (red, green, and blue) are made of wood. The yellow circle is a point light. There are no ambient lights. (1) Draw all of the rays that would be cast while raytracing this scene. (2) Label each pixel with R, G, B, or K to indicate that the pixel would appear Red, Green, Blue, or blacK.