physics-based animation
Physics-Based Animation

Physics

Mathematics

Numerical Methods and Algorithms
Guendelman et al. 2003
Chentanez et al., 2009
• **Newton’s Second Law** \((F = ma)\)

The acceleration \(a\) of a body is parallel and directly proportional to the net force \(F\) acting on the body, is in the direction of the net force, and is inversely proportional to the mass \(m\) of the body.

• **Newton’s Third Law** (Action/Reaction)

When a body exerts a force \(F_1\) on a second body, the second body simultaneously exerts a force \(F_2 = -F_1\) on the first body. This means that \(F_1\) and \(F_2\) are equal in magnitude and opposite in direction.
Math of Natural Phenomena

Ordinary Differential Equations

\[ \dot{x} = f(x, t) \]

- \( x(t) \): a moving point.
- \( f(x,t) \): \( x \)'s velocity.
Math of Natural Phenomena

Partial Differential Equations

$c_t + \vec{u} \cdot \nabla c = f(t)$
Numerical Solution of Diff. Eq.

Euler’s Method

\[ x(t + \Delta t) = x(t) + \Delta t f(x, t) \]
I. Advance velocity $\mathbf{v}^n \rightarrow \hat{\mathbf{v}}^{n+\frac{1}{2}}$

II. Apply collisions $\mathbf{v}^n \rightarrow \hat{\mathbf{v}}^n$, $\hat{\mathbf{v}}^{n+\frac{1}{2}} \rightarrow \hat{\mathbf{v}}^{n+\frac{1}{2}}$

III. Apply contact and constraint forces $\hat{\mathbf{v}}^{n+\frac{1}{2}} \rightarrow \mathbf{v}^{n+\frac{1}{2}}$

IV. Advance positions $\mathbf{x}^n \rightarrow \mathbf{x}^{n+1}$ using $\mathbf{v}^{n+\frac{1}{2}}$, $\hat{\mathbf{v}}^n \rightarrow \hat{\mathbf{v}}^n$

V. Advance velocity $\overline{\mathbf{v}}^n \rightarrow \mathbf{v}^{n+1}$
Particles
Particle: basic dynamic object
Particle: basic dynamic object

mass $m$
Particle: basic dynamic object

mass \quad m

3 dof

\vec{X} = (x, y, z)
Particle: basic dynamic object

3 dof

\[ \vec{X} = (x, y, z) \]

forces: e.g., gravity

\[ \vec{F} = -m\vec{g} \]

mass \( m \)
Particle: basic dynamic object

Equations of motion:
Newton’s 2nd Law

\[ \vec{F} = m\vec{a} \]
Particle: basic dynamic object

Equations of motion: Newton’s 2nd Law

\[ \vec{F} = m\vec{a} \]

\[ \frac{d\vec{x}}{dt} = \vec{v} \]

\[ m \frac{d\vec{v}}{dt} = \vec{F} \]

System of ODEs
Deformable bodies
Connect a bunch of particles into a 1D line segment with springs
A Mass Spring Model for Hair Simulation

Connect a bunch of particles into a 2D mesh
Connect a bunch of particles into a **3D mesh**

![Mesh Image]

**tetrahedron**
Deformable bodies: equations of motion

Equations of motion: Newton’s 2nd Law

\[ \vec{F} = m\vec{a} \]

\[ \frac{d\vec{x}}{dt} = \vec{v} \]

\[ m\frac{d\vec{v}}{dt} = \vec{F} \]

System of PDEs contains spatial derivatives
Rigid bodies
Rigid bodies

6 dofs
forces and torques
elastic collisions
ODEs

\((\vec{X}, \vec{\Omega})\)

\((\vec{F}, \vec{\tau})\)
Rigid body phenomena

stacking

collisions, contact

friction

articulation, control
Articulated rigid bodies
Rigid body simulation

[Weinstein et al 2006]
Rigid and deformable solids coupled together...
Fracture

[Molino et al. 2004]
Contact and collision
Simultaneous resolution of contact, elastic deformation, articulation constraints
our rigid/deformable simulator in Pixar’s WALL-E
Fluid simulation
In fluid simulation, we often use a grid-based representation

\[ \phi(x, y) \]

2d level set function

\[ (x_0(s), y_0(s)) \]

1d interface
An implicit representation has certain advantages over an explicit representation

- naturally handles topological changes
- very easy to extend from 2D to 3D
Fluid equations of motion: Navier-Stokes equations

\[ \vec{F} = m\vec{a} \]

\[ \rho(\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u}) = \mu \Delta \mathbf{u} - \nabla p + \mathbf{f} \]
Two-way Coupled SPH and Particle Level Set Fluid Simulation

Control of virtual character

[Shinar et al. 2008]
rigid/deformable simulator in Pixar’s WALL-E