1 True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively. You get -0.25 points for answering the question incorrectly and 0.5 points for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60% confident that your answer is correct).

1. ($\boxed{\text{T}}$/F) OpenGL is a platform independent software interface to graphics hardware.
2. ($\boxed{\text{T}}$/F) $f(\mathbf{p}) = \mathbf{N} \cdot (\mathbf{p} - \mathbf{q}) = 0$, where $\mathbf{N}$ is a normal vector, and $\mathbf{q}$ a point, is an implicit line equation in 2D or implicit plane equation in 3D.
3. ($\boxed{\text{T}}$/F) The Phong reflectance model can be used both within the OpenGL graphics pipeline approach to rendering and within a ray tracing framework.
4. (T/$\boxed{\text{F}}$) Matrix multiplication is commutative.
5. ($\boxed{\text{T}}$/F) Let T be a translation matrix, R be a rotation matrix, and $\mathbf{p}$ be a point. Then the product $TR\mathbf{p}$ first rotates the point and then translate it.
6. ($\boxed{\text{T}}$/F) Scaling and rotating are both examples of linear transformations.
7. ($\boxed{\text{T}}$/F) The camera transformation in the graphics pipeline is an example of a rigid transformation.
8. ($\boxed{\text{T}}$/F) This matrix is a rigid body transformation

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

9. ($\boxed{\text{T}}$/F) This matrix reflects about the yz-plane

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

10. ($\boxed{\text{T}}$/F) The z-buffer stores the depth of each pixel in order to resolve fragment visibility issues.
11. ($\boxed{\text{T}}$/F) The order of operations in the graphics pipeline is as follows: modelview transformation, projection transformation, divide by w, viewport transform.
12. ($\boxed{\text{T}}$/F) Sets of parallel lines remain parallel under orthographic projection in the OpenGL graphics pipeline.
13. ($\boxed{\text{T}}$/F) When rasterizing a triangle under a perspective projection, linear interpolation with screen-space barycentric weights can be used to determine fragment depth values.
14. ($\boxed{\text{T}}$/F) In keyframe character animation, interpolation between keyframe character poses is used to produce the illusion of continuous motion.
15. (T/$\boxed{\text{F}}$) When doing physical simulation, use of a smaller time step for the numerical time integration has the advantage of speeding up the overall computation and improving accuracy.

16. (T/$\boxed{\text{F}}$) A Bezier curve of degree $n+1$ has n distinct control points.

17. ($\boxed{\text{T}}$/F) If all control points of a Bezier curve lie on a line, then the Bezier curve lies on that line.

18. ($\boxed{\text{T}}$/F) Newton's second law states that the mass times acceleration of a particle is equal to the net force on the particle.

2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

21. Which statement about transformations in the OpenGL pipeline is true?

    (a) Changing the order of applied transformation matrices would not affect the final position of a point.

    (b) Vertex positions and normal vectors are both translated the same way when multiplied by a transformation matrix representing a translation.

    (c) The 4-vector $(x, y, z, 0)$ represents the three-dimensional point located at (x, y, z).

    (d) $\boxed{\text{The 4-vectors } (1,2,3,4) \text{ and } (2,4,6,8) \text{ represent the same physical point.}}$

    (e) None of the above.

22. Match the type of transformation in the left column with the example transformation matrix in the right by drawing lines between the matching boxes.

translation	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
rotation	$\begin{pmatrix} .5 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
uniform scale	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
identity	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

23. OpenGL perspective projection

    (a) preserves parallel lines.

    (b) $\boxed{\text{preserves the } z \text{ ordering between the near and far planes, but not necessarily everywhere else.}}$

    (c) preserves the z ordering everywhere.

    (d) is a linear transformations in z.

    (e) is an affine transformations.

24. Perspective correct interpolation

   (a) is needed for implementing the z-buffer in the graphics pipeline.
   (b) is rarely needed.
   (c) requires completely reversing all the geometric transformations in the graphics pipeline.
   (d) $\boxed{\text{is important when using texture maps in the graphics pipeline.}}$
   (e) none of the above.

25. Which statement about curves is true?

   (a) OpenGL has built-in functions for drawing Bezier curves.
   (b) Bezier curves go through all of their control points.
   (c) High order polynomials are often used to interpolate large collections of points representing an object surface.
   (d) Spline curves do not have good smoothness properties.
   (e) $\boxed{\text{Bezier curves are contained in the convex hull of their control points.}}$

26. Which statement about curves is false?

(a) $\boxed{\text{The cubic Bezier curve approximating 4 data points is the unique cubic that interpolates the 4 points.}}$

(b) There is a unique n degree polynomial that interpolates $n+1$ distinct data points.

(c) The monomial basis for curves up to degree 3 is the set $1, u, u^2, u^3$.

(d) The blending functions for a degree 3 Bezier curve are all of degree 3.

(e) When using piecewise polynomial curves to interpolate a set of data points, care must be taken at join points to ensure the desired level of continuity.

27. Which statement about physics-based simulation is true?

(a) The coefficient of restitution is applied to the tangential velocity of a particle to create an effect/illusion of friction.

(b) In particle simulations, applying the forward Euler method to update velocities and positions would naturally handle collisions.

(c) A large time step reduces the effects of errors due to numerical integration in time.

(d) $\boxed{\text{Using a rigid body approximation can significantly reduce the number of degrees of freedom of a very stiff ob}}$

(e) None of the above.

X

28. ~~Which statement about OpenGL is false?~~

not fully covered

~~(a) In OpenGL, the ModelView matrix can represent rotation, scale, and translation of an object.~~

~~(b) A quadrilateral can be represented by two triangles in OpenGL.~~

~~(c)~~ $\boxed{\text{In an OpenGL program, the number of popMatrix operations can be higher or equal to the number of pushM}}$

~~(d) Translating an object by (1,0,0) before rotating by 90 degrees is not the same as rotating the object by 90 degrees after translating by (1,0,0).~~

~~(e) The modern OpenGL graphics pipeline is a programmable pipeline.~~

29. Which of the following statements about animation is true?

(a) $\boxed{\text{A triangle mesh consisting of masses and springs can be used to simulate falling cloth.}}$

(b) Determining the joint angles on a skeleton arm so that its hand is in a desired end position is the problem of Forward Kinematics.

(c) Principles such as squash and stretch apply to hand-drawn animation, not computer-generated animation.

(d) In keyframe animation, a character's motion is determined by the shape of the interpolating curve but not by the particular time parameterization of the curve.

(e) None of the above.

3 Written Response

31. Write down the transformation matrices as stated in each part.

(a) Write down the 4×4 3D matrix to translate a point by (x, y, z).

(b) Write down the 4×4 3D matrix to rotate counter-clockwise by an angle θ about the z-axis.

(c) Write down the 4×4 3D matrix to scale an object by 50% in all directions.

(d) Write down the 2D rotation matrix that rotates by 90 degrees clockwise.

(a) $$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

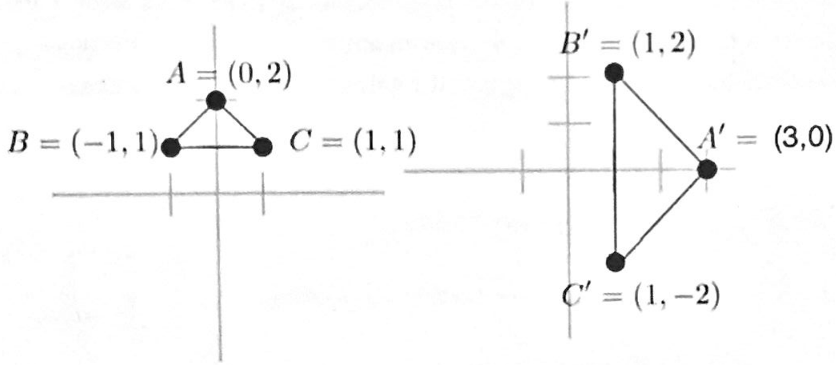
(b) $$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) $$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(d) $$\begin{pmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & 0 \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

or $$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

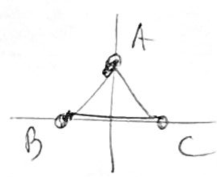
32. Find a sequence of transformation matrices that map the triangle ABC to the triangle $A'B'C'$. Sketch the triangle after each transformation. How are these transformation matrices combined into a single tranformation matrix to map ABC to $A'B'C'$?



Matrix

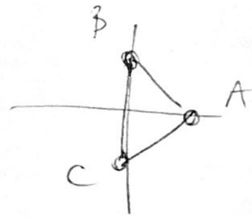
no a

① translate $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$



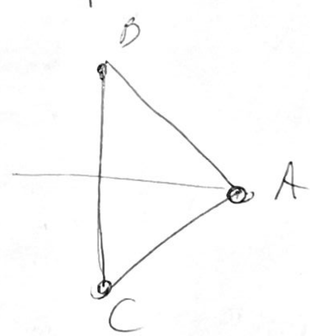
$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \hline 0 & 0 & 1 \end{array}\right)$$

② rotate $\frac{\pi}{2}$ CW



$$\left(\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

③ scale uniformly by 2



$$\left(\begin{array}{cc|c} 2 & 0 & 0 \\ 0 & 2 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

④ translate $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

Final transformation:

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c} 2 & 0 & 0 \\ 0 & 2 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \hline 0 & 0 & 1 \end{array}\right)$$

33. Find the inverse of the rigid body transformation

$$\begin{pmatrix} R & \mathbf{t} \\ 0\,0\,0 & 1 \end{pmatrix},$$

where R is a 3×3 rotation matrix and $\mathbf{t}$ is a 3-vector.

First, note that

$$\begin{pmatrix} R & t \\ 0\,0\,0 & 1 \end{pmatrix} = \left(\begin{array}{c|c} I & t \\ \hline 0\,0\,0 & 1 \end{array}\right)\left(\begin{array}{c|c} R & \begin{smallmatrix}0\\0\\0\end{smallmatrix} \\ \hline 0\,0\,0 & 1 \end{array}\right)$$

$$\begin{pmatrix} R & t \\ 0\,0\,0 & 1 \end{pmatrix}^{-1} = \left(\begin{array}{c|c} R & \begin{smallmatrix}0\\0\\0\end{smallmatrix} \\ \hline 0\,0\,0 & 1 \end{array}\right)^{-1}\begin{pmatrix} I & t \\ 0\,0\,0 & 1 \end{pmatrix}^{-1}$$

$$= \left(\begin{array}{c|c} R^T & 0 \\ \hline 0 & 1 \end{array}\right)\left(\begin{array}{c|c} I & -t \\ \hline 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{c|c} R^T & -R^T t \\ \hline 0 & 1 \end{array}\right)$$

34. Consider the cubic Bezier curve $f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3$ with control points p_0, p_1, p_2, p_3, which is given by the conditions

$$f(0) = p_0$$
$$f'(0) = 3(p_1 - p_0)$$
$$f'(1) = 3(p_3 - p_2)$$
$$f(1) = p_3$$

(a) Write down the 4×4 matrices C and P such that

$$C \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = P \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

(b) Given that $B = C^{-1}P$ is given by the following matrix

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{pmatrix},$$

what are the cubic Bezier blending functions $b_0(u), b_1(u), b_2(u), b_3(u)$?

(b)

$$u^T B = (1 \; u \; u^2 \; u^3) \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{pmatrix}$$

$$= (\underbrace{1 - 3u + 3u^2 - u^3}_{b_0(u)}, \underbrace{3u - 6u^2 + 3u^3}_{b_1(u)}, \underbrace{3u^2 - 3u^3}_{b_2(u)}, \underbrace{u^3}_{b_3(u)})$$

(a)

$$f'(u) = a_1 + 2a_2 u + 3a_3 u^2$$
$$f(0) = a_0 = p_0$$
$$f'(0) = a_1 = 3(p_1 - p_0)$$
$$f'(1) = a_1 + 2a_2 + 3a_3 = 3(p_3 - p_2)$$
$$f(1) = a_0 + a_1 + a_2 + a_3 = p_3$$

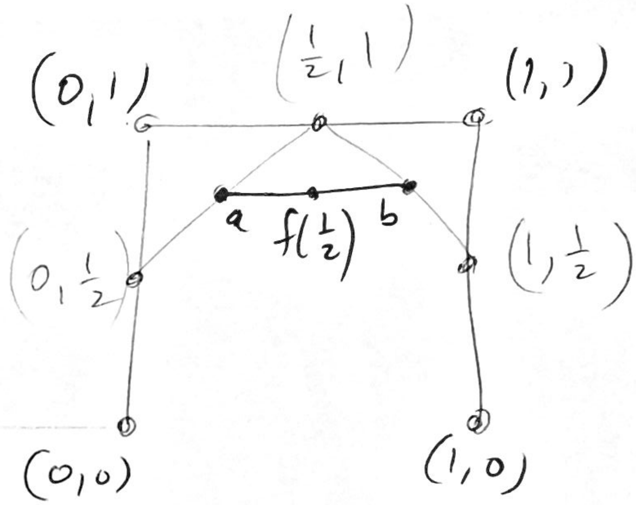
$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

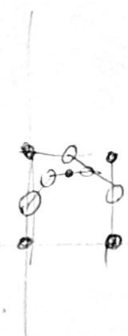
$$\| $$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

35. Use the de Casteljau algorithm to evaluate the position of the cubic Bezier curve with its control points at $(0,0), (0,1), (1,1), (1,0)$ for parameter values $u = 0.5$ and $u = 0.75$. Drawing a sketch will help you do this.

$\boxed{u = 0.5}$



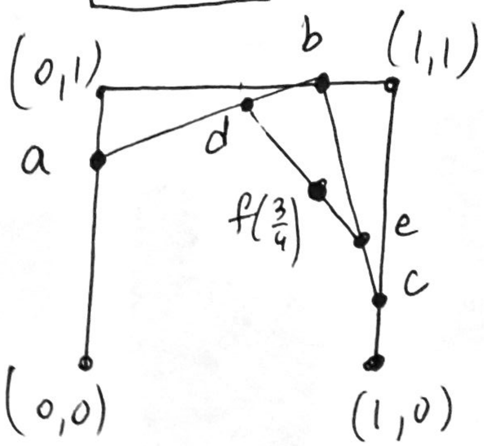


$$a = \frac{1}{2}\left(0, \frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}, 1\right) = \left(0, \frac{1}{4}\right) + \left(\frac{1}{4}, \frac{1}{2}\right)$$
$$= \left(\frac{1}{4}, \frac{3}{4}\right)$$

$$b = \frac{1}{2}\left(\frac{1}{2}, 1\right) + \frac{1}{2}\left(1, \frac{1}{2}\right) = \left(\frac{1}{4}, \frac{1}{2}\right) + \left(\frac{1}{2}, \frac{1}{4}\right) = \left(\frac{3}{4}, \frac{3}{4}\right)$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}a + \frac{1}{2}b = \frac{1}{2}\left(\frac{1}{4}, \frac{3}{4}\right) + \frac{1}{2}\left(\frac{3}{4}, \frac{3}{4}\right) = \left(\frac{1}{8}, \frac{3}{8}\right) + \left(\frac{3}{8}, \frac{3}{8}\right)$$
$$= \left(\frac{4}{8}, \frac{6}{8}\right) = \left(\frac{1}{2}, \frac{3}{4}\right)$$

$\boxed{u = \frac{3}{4}}$



$$a = \frac{1}{4}(0,0) + \frac{3}{4}(0,1) = \left(0, \frac{3}{4}\right)$$
$$b = \frac{1}{4}(0,1) + \frac{3}{4}(1,1) = \left(0, \frac{1}{4}\right) + \left(\frac{3}{4}, \frac{3}{4}\right) = \left(\frac{3}{4}, 1\right)$$
$$c = \frac{1}{4}(1,1) + \frac{3}{4}(1,0) = \left(\frac{1}{4}, \frac{1}{4}\right) + \left(\frac{3}{4}, 0\right) = \left(1, \frac{1}{4}\right)$$
$$d = \frac{1}{4}a + \frac{3}{4}b = \left(0, \frac{3}{16}\right) + \left(\frac{9}{16}, \frac{3}{4}\right) = \left(\frac{9}{16}, \frac{15}{16}\right)$$
$$e = \frac{1}{4}b + \frac{3}{4}c = \left(\frac{3}{16}, \frac{1}{4}\right) + \left(\frac{3}{4}, \frac{3}{16}\right) = \left(\frac{15}{16}, \frac{7}{16}\right)$$

$$f\left(\frac{3}{4}\right) = \frac{1}{4}d + \frac{3}{4}e = \left(\frac{9}{64} + \frac{45}{64}, \frac{15}{64} + \frac{21}{64}\right) = \left(\frac{54}{64}, \frac{36}{64}\right)$$