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**Part 1: vectors, dot and cross product**

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1. Solve the given vector equation for  $x$ :

$$3 * [1, 2, -1] + 4 * [2, 0, x] = [11, 6, 17]$$

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2. Solve the given vector equation for the scalar  $x$ . Is there a solution?

$$x * [1, 2, -1] + 4 * [3, 4, 2] = [-1, 0, 4]$$

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3. Calculate the cosine of the angle between the vectors

$[2, 4, 4]$  and  $[4, 3, 0]$ .

4. Given 2 vectors,  $a$  and  $b$ , explain the geometrical relationship between  $a$  and  $b$  for the following cases:

a)  $a \cdot b = 0$  :

b)  $a \cdot b > 0$  :

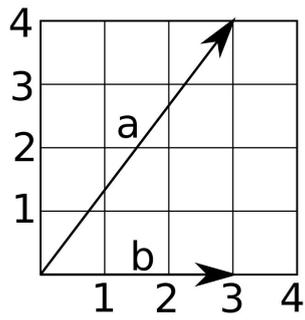
c)  $a \cdot b < 0$  :

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5. Calculate the cross product:  $[1, 2, 3] \times [4, 5, 6]$ .

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6. Given the triangle with vertices  $[0, 2, -1]$ ,  $[2, 0, -1]$  and  $[1, 0, 0]$ , calculate the normal of the plane that contains the triangle.



7. Calculate the vector that bisects the angle between the vectors  $a$  and  $b$  in the figure above.

8. Calculate a vector  $c$  in the same direction of the vector  $a$  and that has the same length as the vector  $b$ .

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Part 2: Matrices

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & 19 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 1 & -3 \\ -1 & 1 \end{bmatrix}$$

*You don't have to do the divisions, just keep the values in division format.*

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1. Calculate:

a)  $A + B^T$

b)  $AB$

c)  $(AB)^{-1}$

2. Solve  $(AB)x = c$ , where  $c$  is the vector  $[1, 2]$  and  $x$  is  $[x_1, x_2]$ . Show the following steps:

a) Isolate  $x$  in the left-hand side of the equation.

b) Compute  $(AB)^{-1}c$  to find the values of  $x$ .

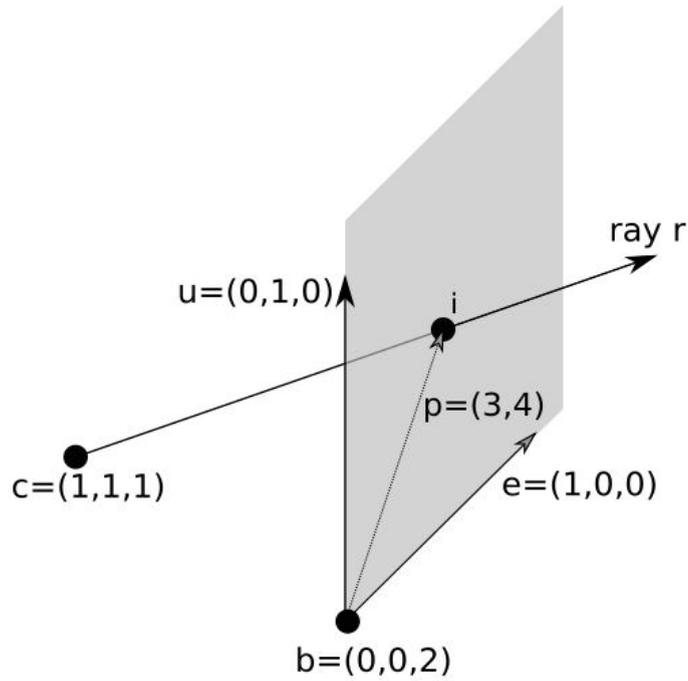
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**Part 3: Ray/plane intersection**

1. Calculate the endpoint and direction of the ray  $r$  in the figure below.

- $u$  and  $e$  are unitary vectors that define the shaded plane.
- $u$  and  $e$  originate at  $b$ .
- The ray passes through the plane at the intersection point  $i$ .
- The 2D vector  $p$  is on the plane.  $p$  goes from  $b$  (origin) to  $i$ .



ray endpoint:

ray direction (don't forget to normalize):

2. Consider a ray with endpoint  $e$  and (unitary) direction  $u$ . Consider a plane with (unitary) normal vector  $n$  and with a point  $x_0$  located anywhere on the plane. The equations for the ray and the plane are:

- ray:  $R(t) = e + ut$
- plane:  $P(x) = (x - x_0) \cdot n = 0$

Any point  $r$  on the ray can be found using the real value  $t \geq 0$ . Any point  $x$  that satisfies the plane equation is on the plane.

e) Write a code in C++ that receives  $e$ ,  $u$ ,  $n$  and  $x_0$ , and returns *true* if the ray intersects the plane. Assume all vectors have the same size.

```
// you can use vec_f as a shortcut for a vector of floats
typedef vector<float> vec_f;
```

```
bool intersects(vec_f &e, vec_f &u, vec_f &n, vec_f &x0){
```

```
}
```

```
// computes dot product between the vectors a and b
```

```
float dot(vec_f &a, vec_f &b) {
    float d = 0;
```

```
        return d;
```

```
}
```

```
// compute the difference between vectors a and b
```

```
vec_t sub(vec_f &a, vec_f &b) {
    vec_t result(a.size());
```

```
        return result;
```

```
}
```

3. Consider a ray with endpoint  $e$  and (unitary) direction  $u$ . Consider a sphere with center  $c$  and radius  $r$ . The equations for the ray and the sphere are:

- ray:  $R(t) = e + ut$
- sphere:  $S(x) = (x - c) \cdot (x - c) = r^2$

Any point  $r$  on the ray can be found using the real value  $t \geq 0$ .

Any point  $x$  that satisfies the sphere equation is on the sphere.

Calculate  $t$  such that the point in the ray intersects the sphere.

Hint: You can use steps and the reasoning that we used for ray-plane intersections.