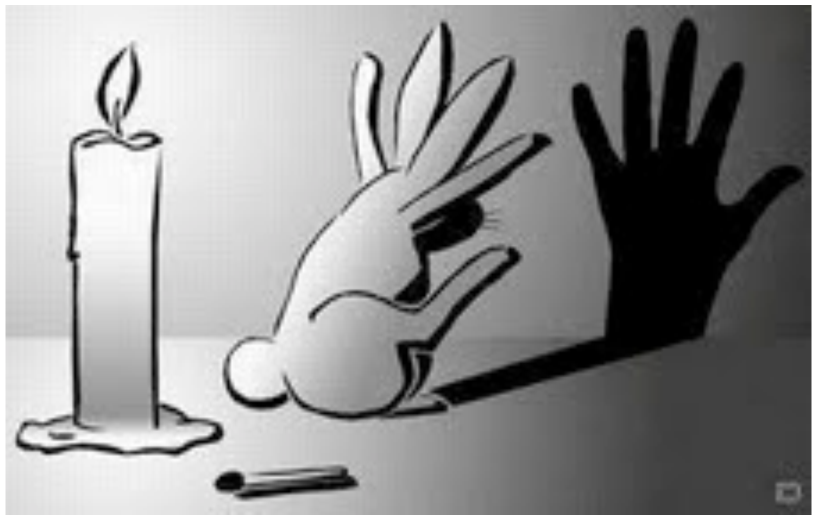
CS130 : Computer Graphics

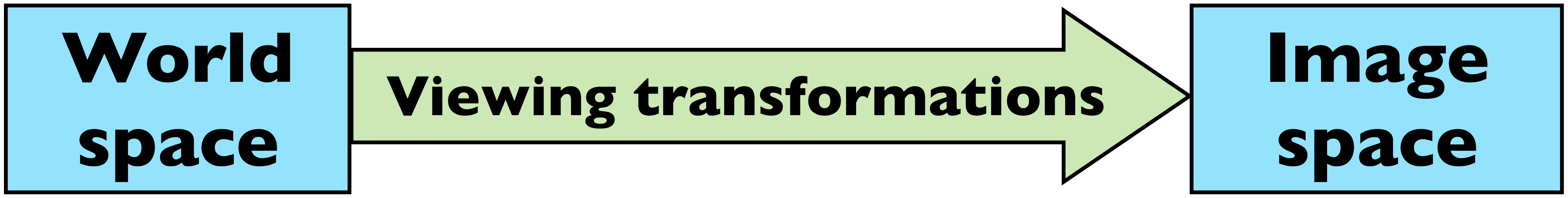
Lecture 6: Viewing Transformations

Tamar Shinar
Computer Science & Engineering
UC Riverside

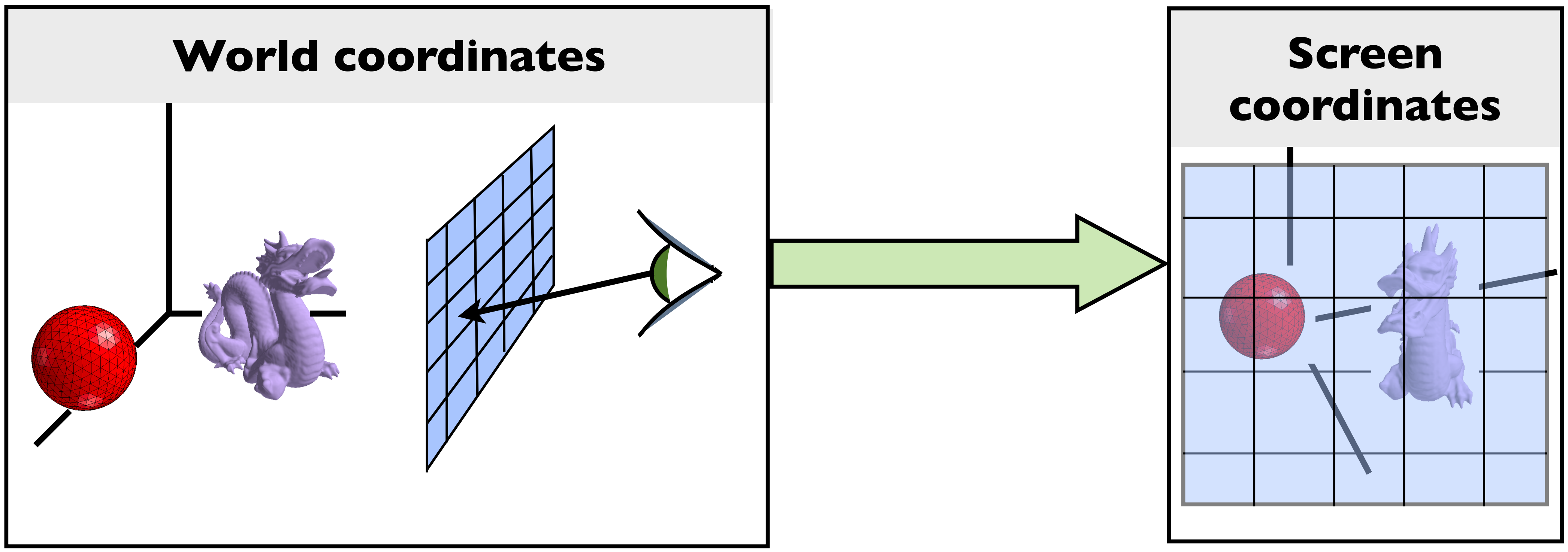
Viewing Transformations



Viewing transformations

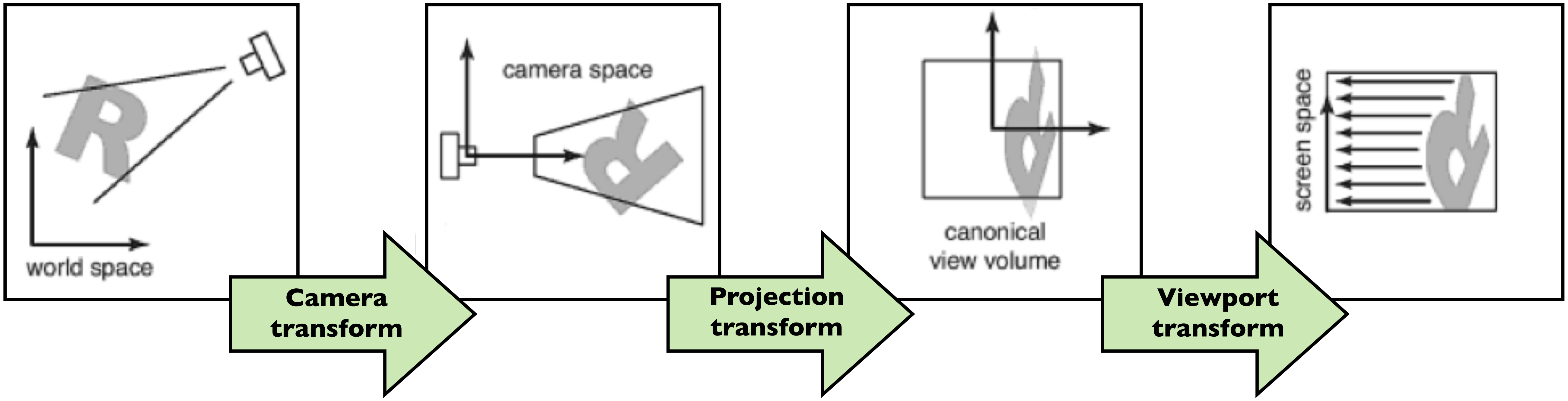


- Move objects from their 3D locations to their positions in a 2D view



The viewing transformation also project any pixels viewing ray back to the pixel's position in **image space**

Decomposition of viewing transforms



- rigid body transformation
- place camera at origin

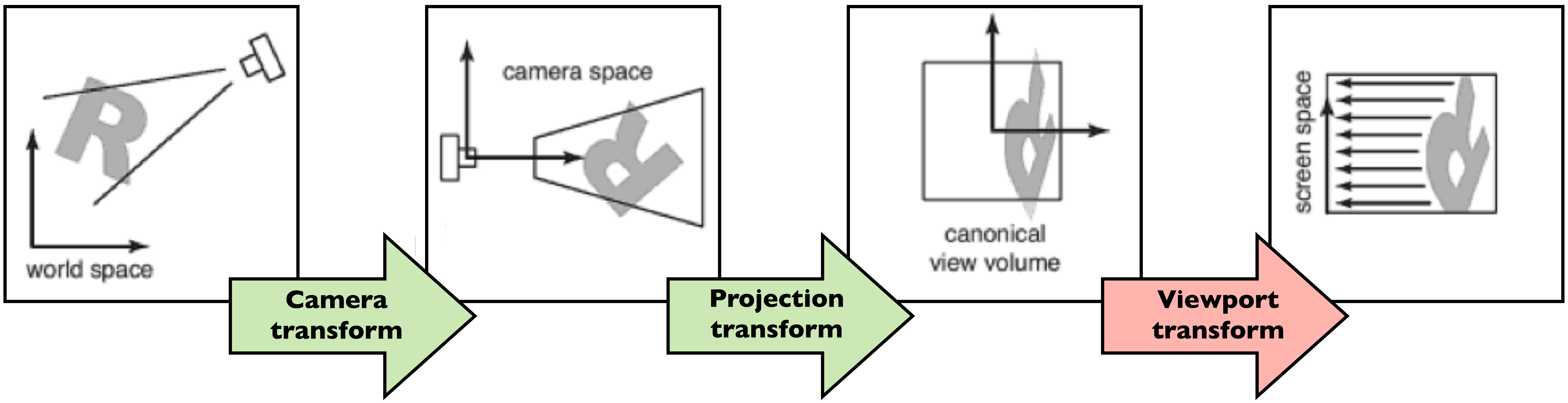
- x, y, z in [-1,1]
- depends on type of projection

- map to pixel coordinates

Viewing transforms depend on: camera position and orientation, type of projection, field of view, image resolution

there are several names for these spaces: “camera space” = “eye space”, “canonical view volume” = “clip space”= “normalized device coordinates”, “screen space=pixel coordinates” and for the transforms: “camera transformation” = “viewing transformation”

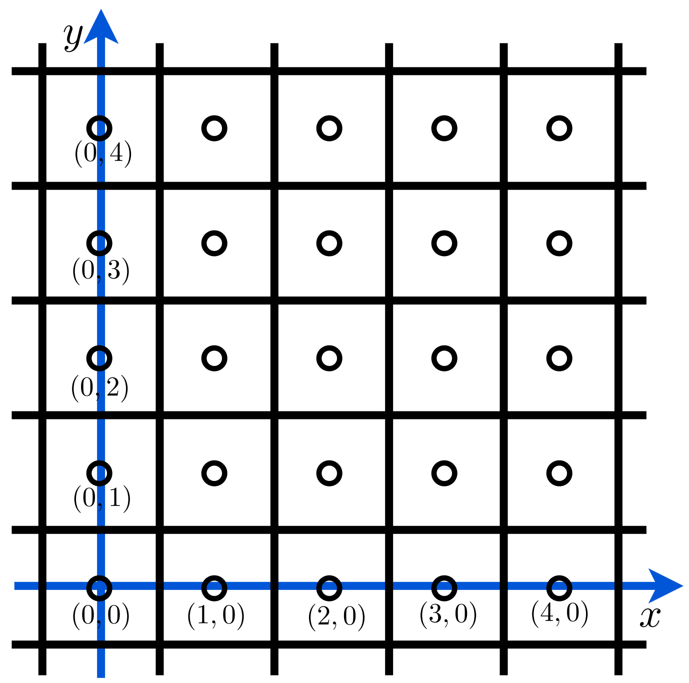
Viewport transform



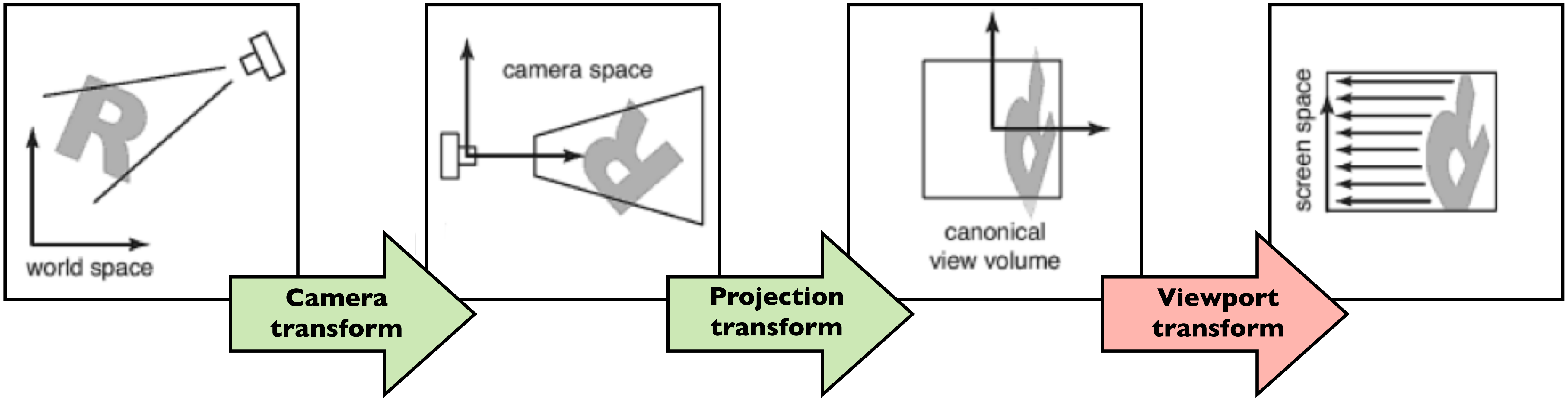
$$(x, y, z) \rightarrow (x', y', z')$$

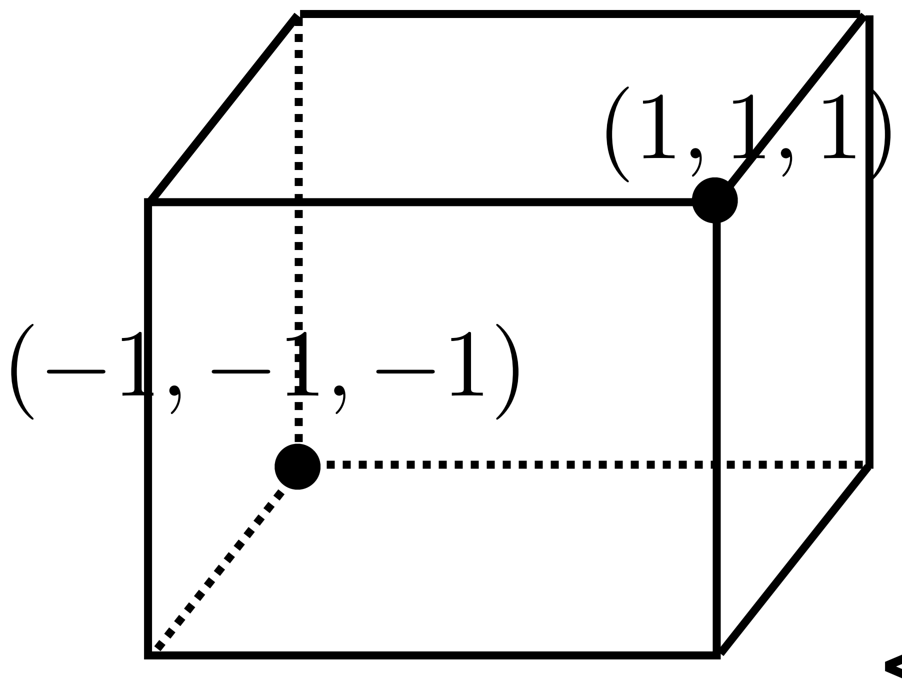
$$(x, y, z) \in [-1, 1]^3$$

$$x' \in [-.5, n_x - .5]$$
$$y' \in [-.5, n_y - .5]$$



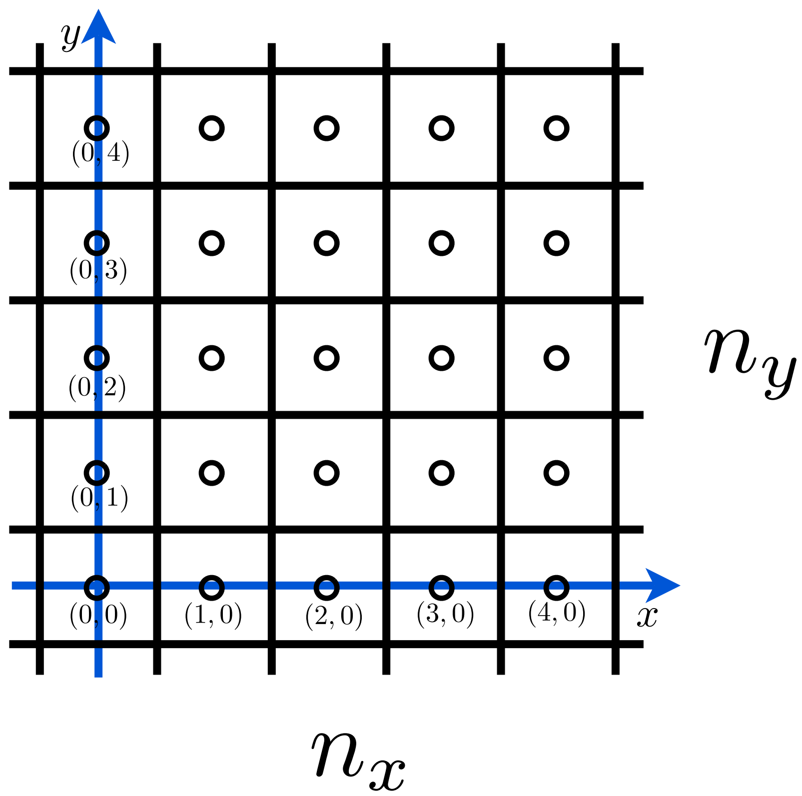
Viewport transform



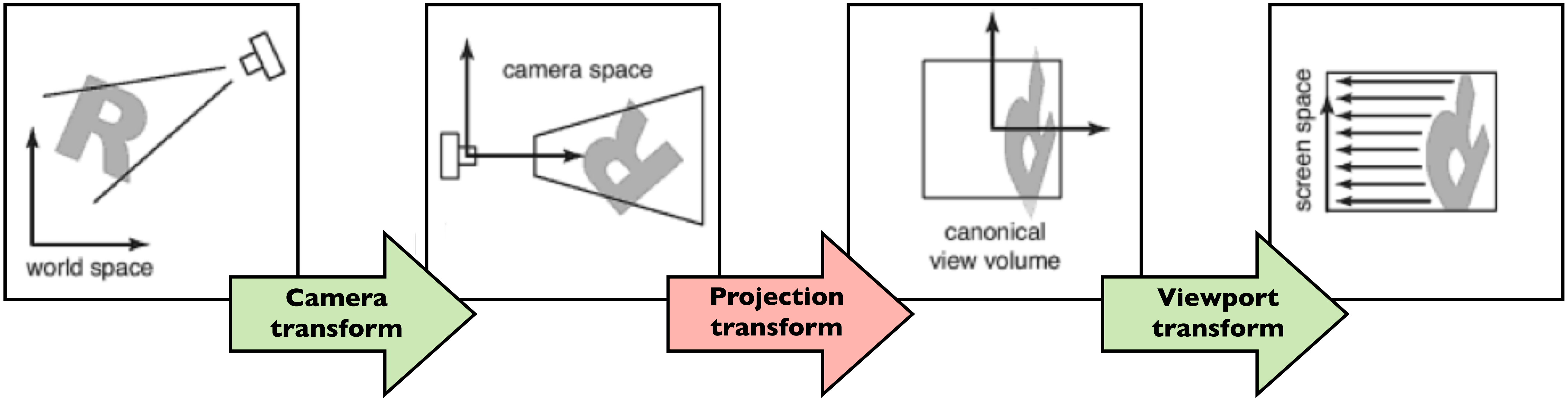


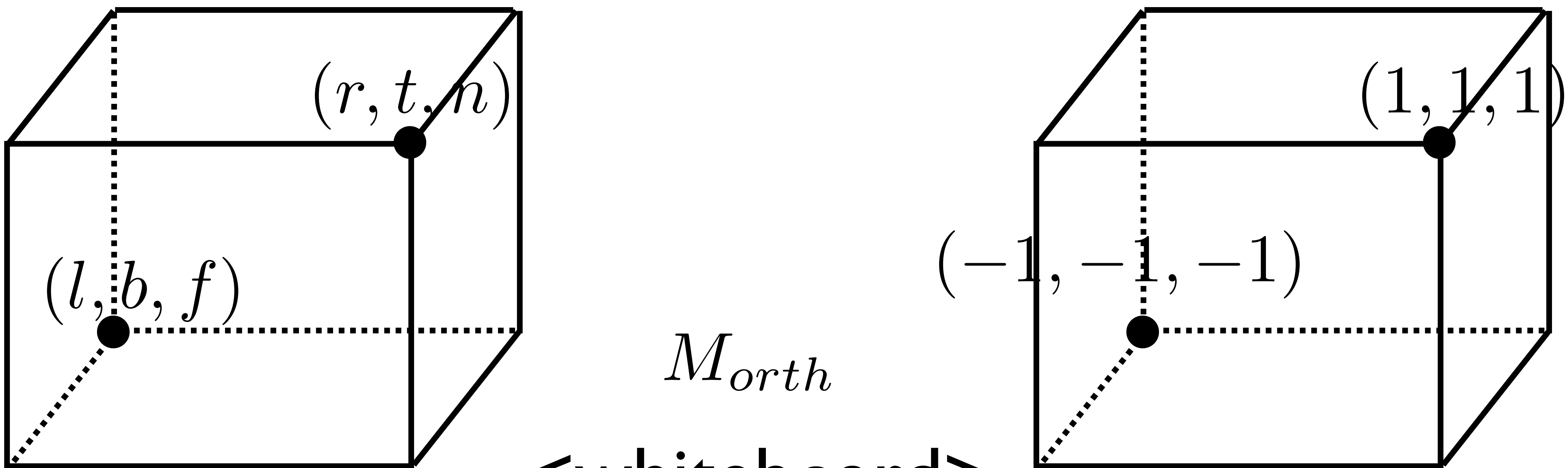
M_{vp}

<whiteboard>



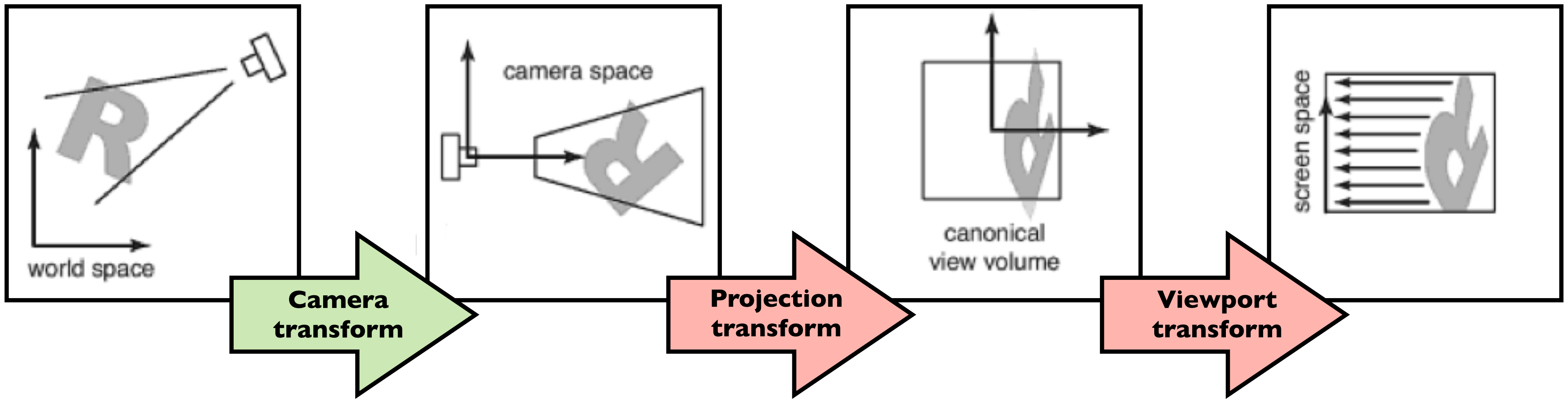
Orthographic Projection Transform





<whiteboard>

Line drawing algorithm



construct M_{vp}

construct M_{orth}

$M = M_{vp} M_{orth}$

for each line segment (a_i, b_i) do

$\quad \mathbf{p} = M\mathbf{a}_i$

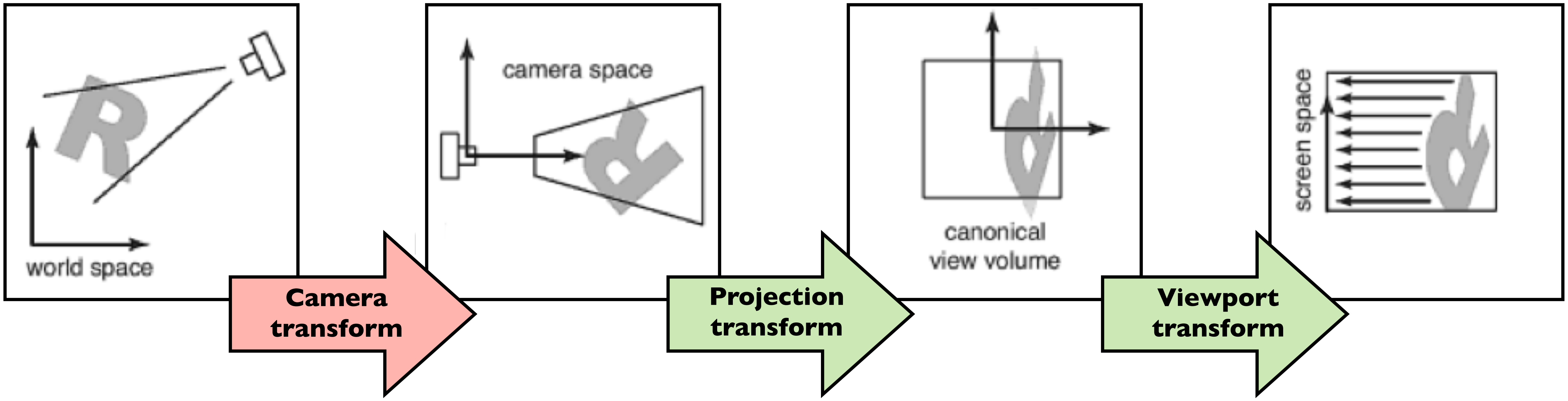
$\quad \mathbf{q} = M\mathbf{b}_i$

drawline (x_p, y_p, x_q, y_q)

Shirley, Marschner 7.1

draw lines specified in camera space

Camera Transform



Camera Transform

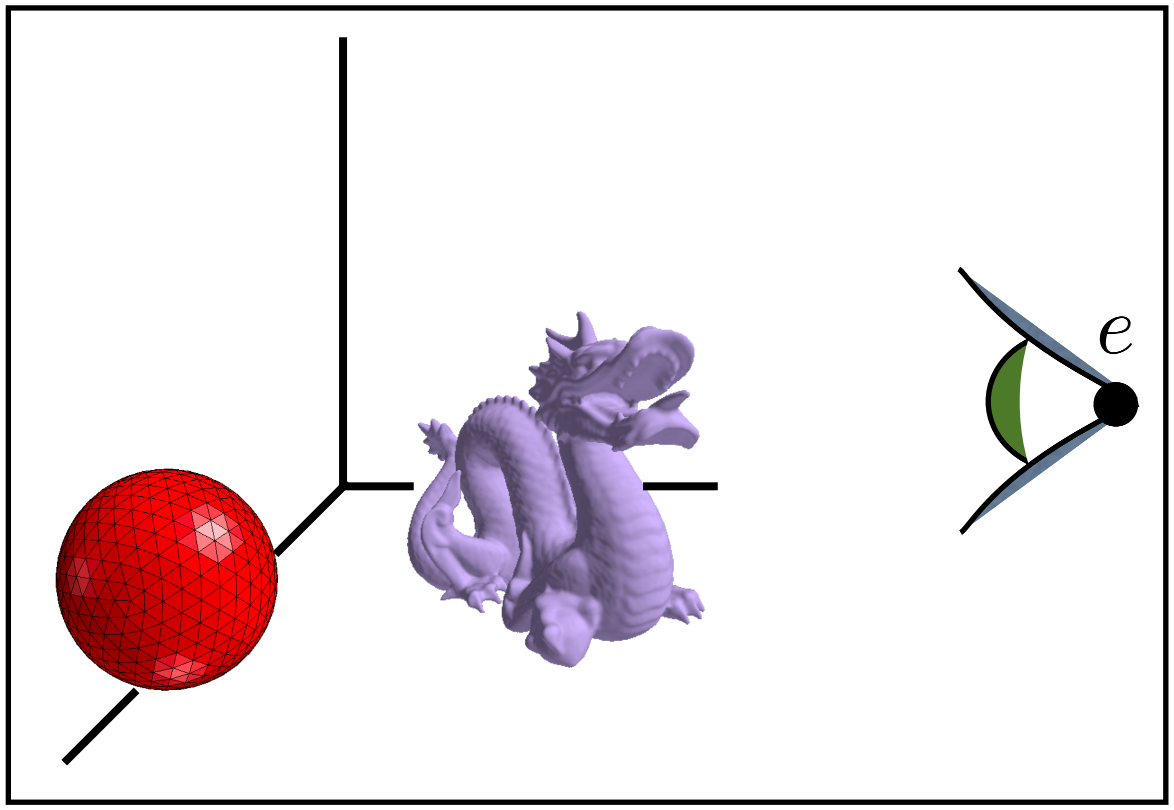
How do we specify the camera configuration?

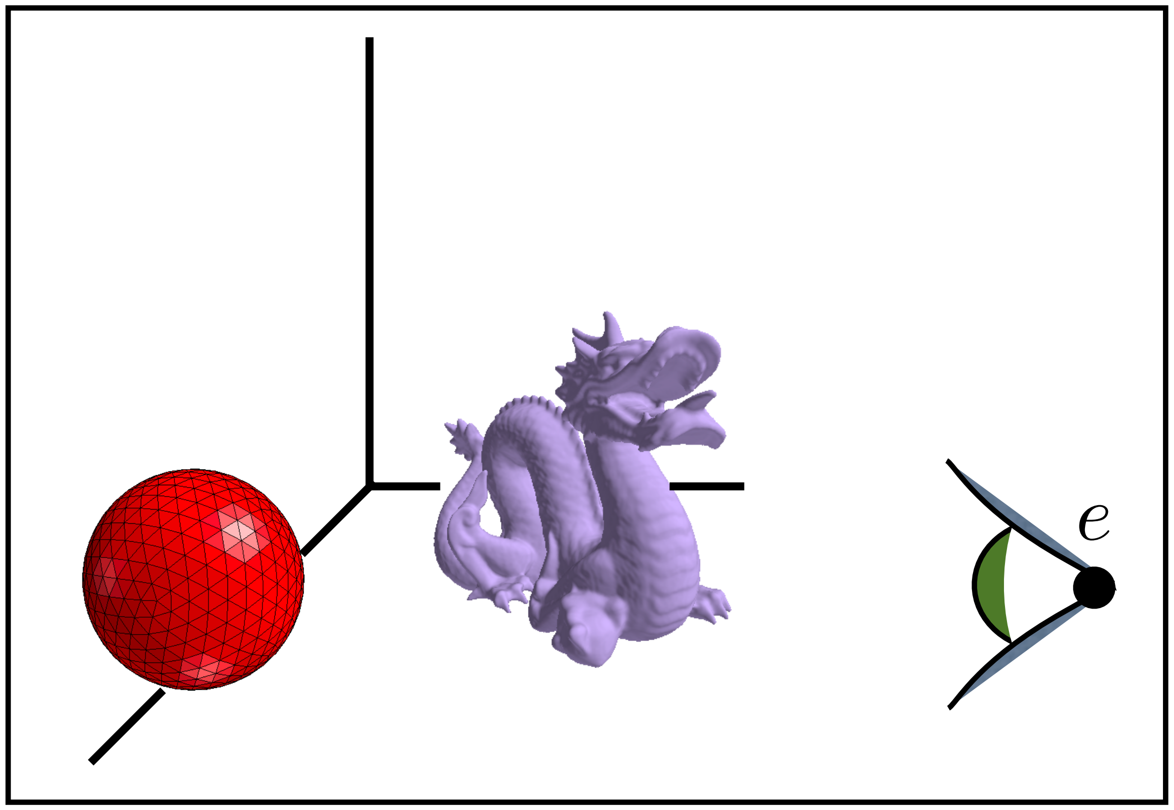
(orthogonal case)

Camera Transform

How do we specify the camera configuration?

eye position

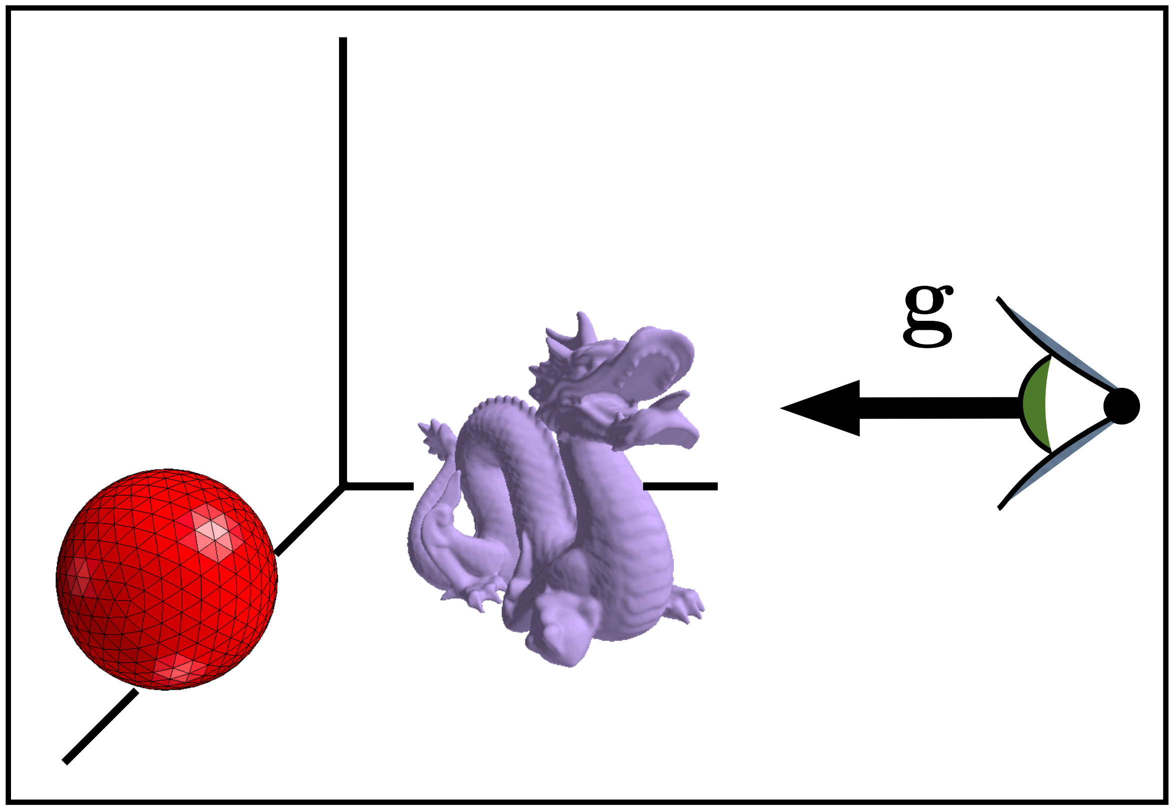


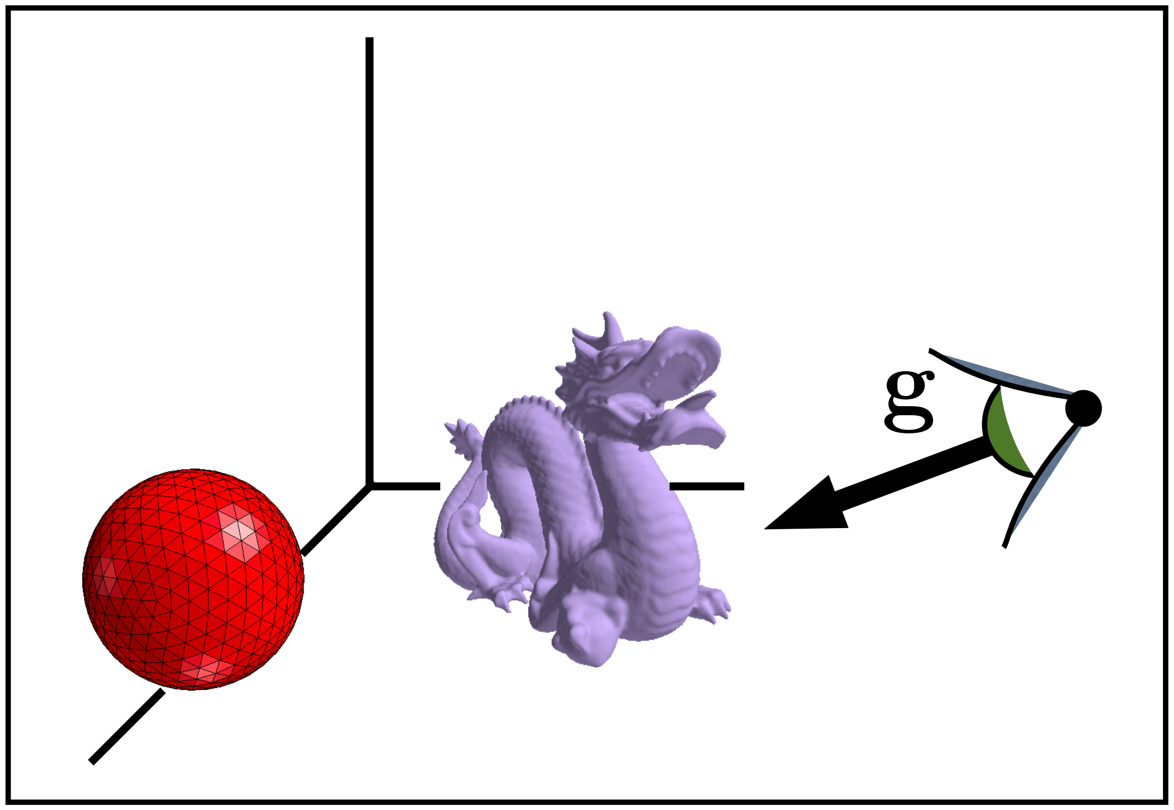


Camera Transform

How do we specify the camera configuration?

gaze direction

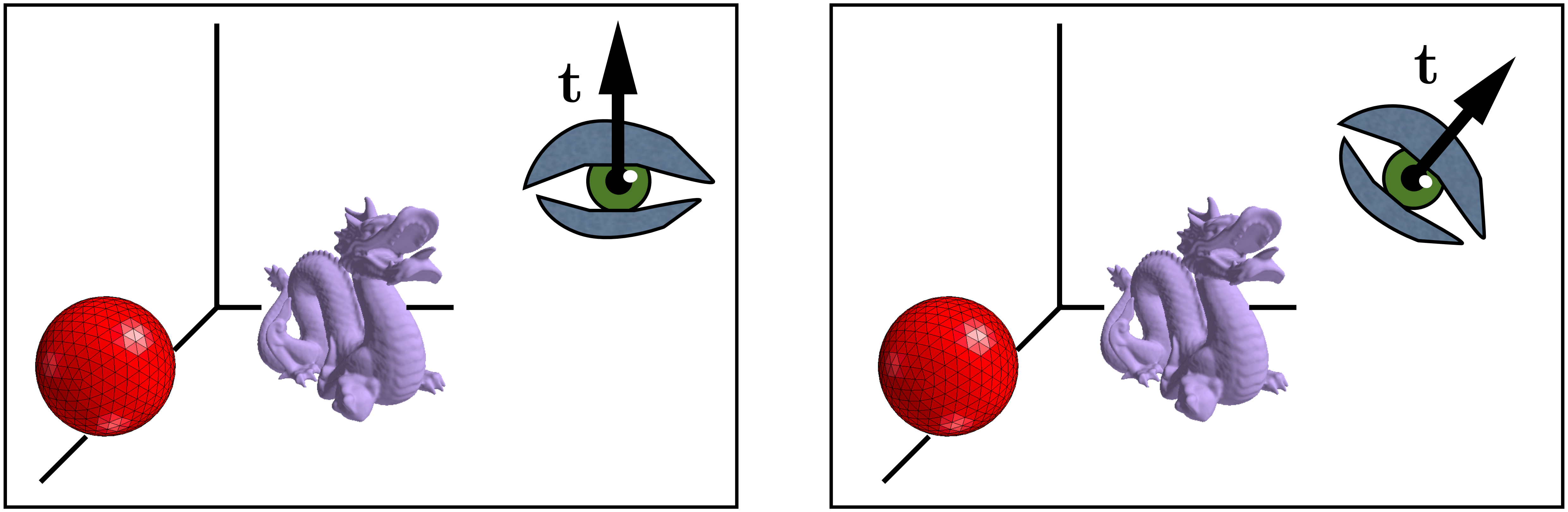




Camera Transform

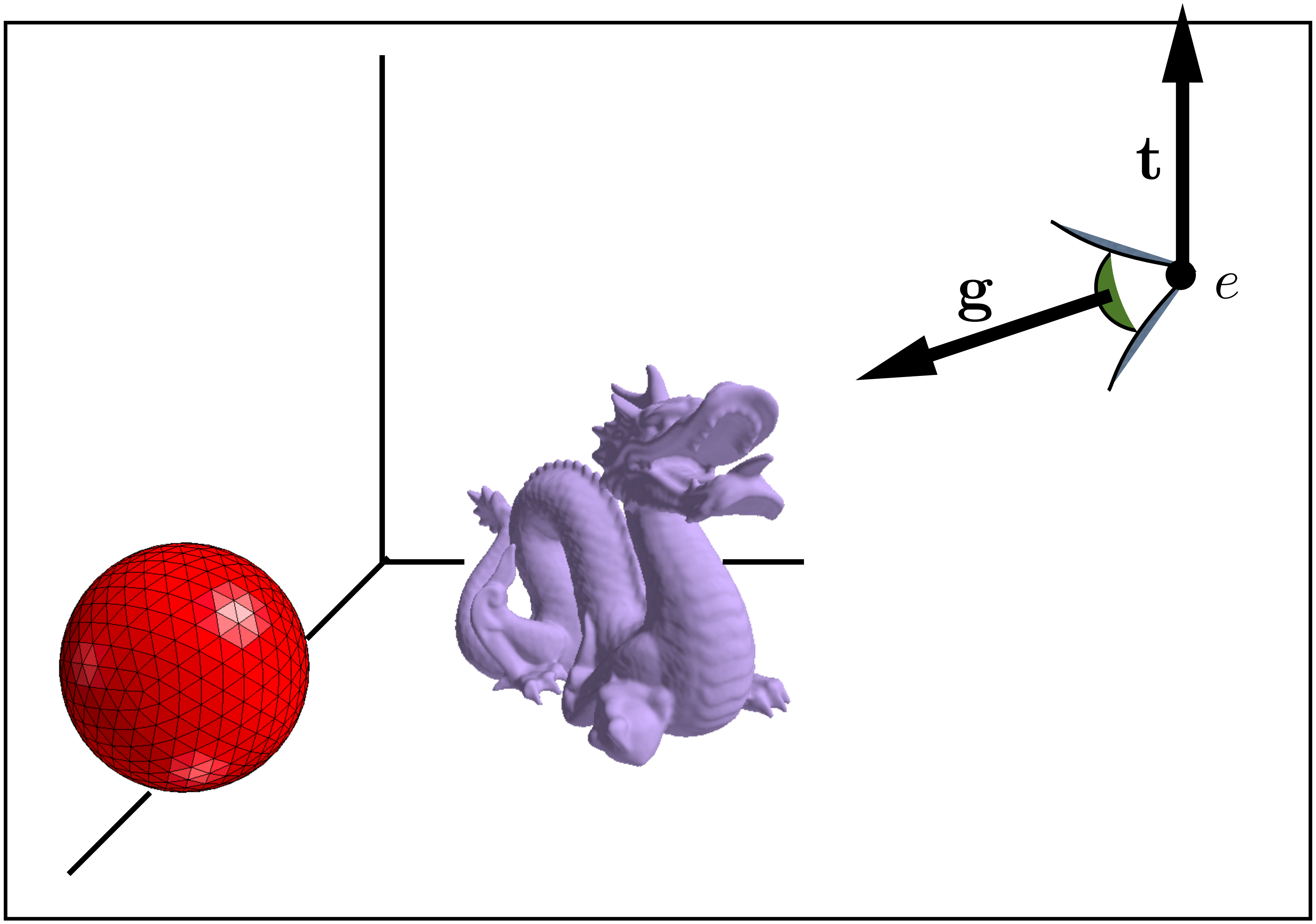
How do we specify the camera configuration?

up vector

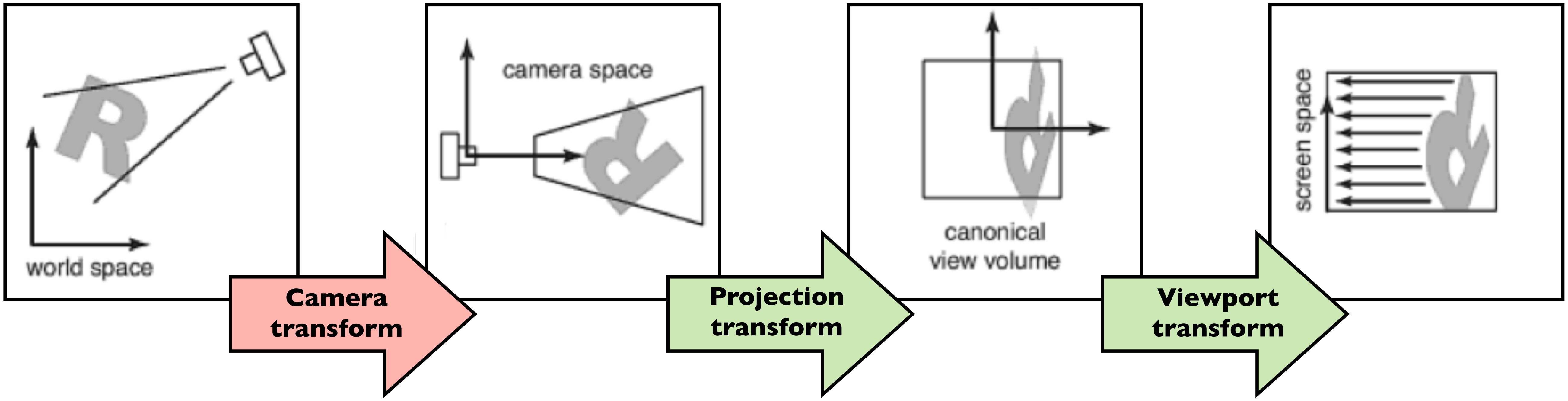


Camera Transform

How do we specify the camera configuration?



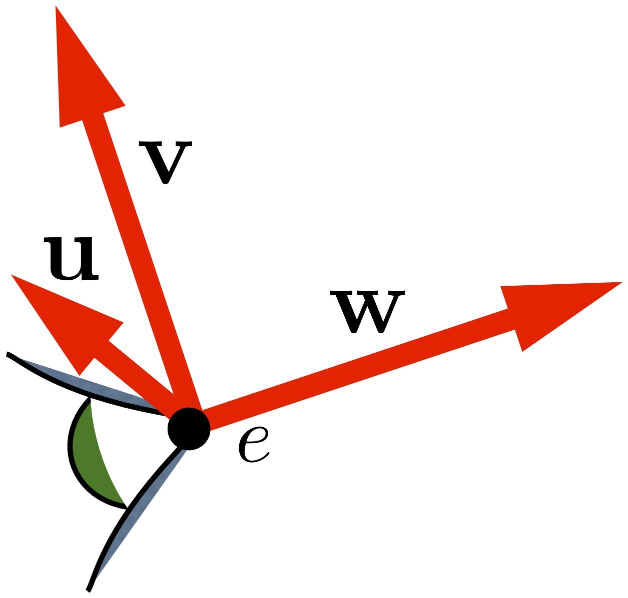
Camera Transform

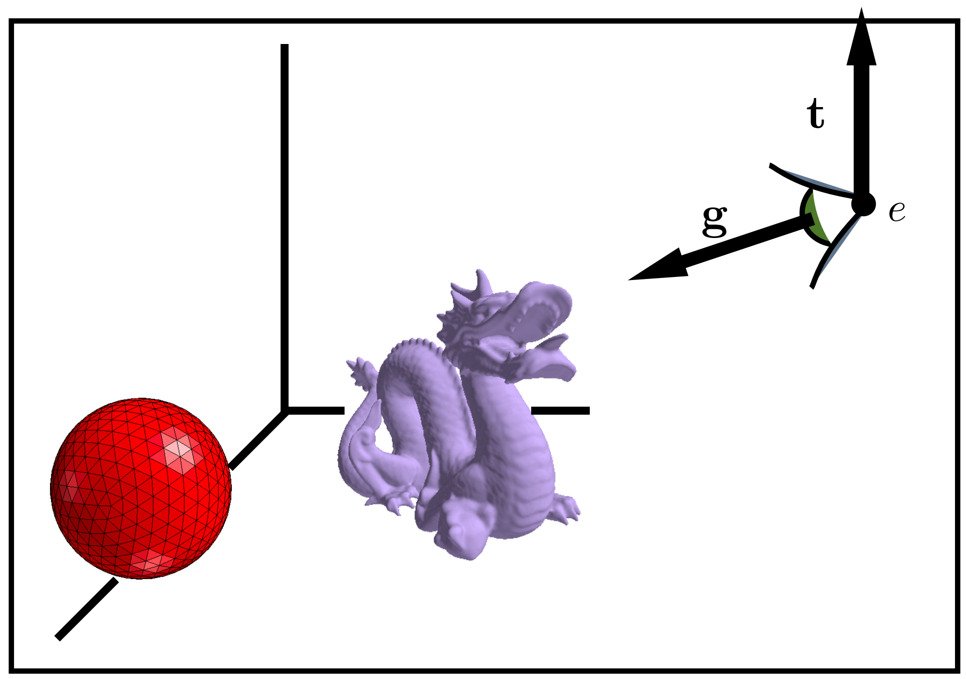


$$\mathbf{w} = -\frac{\mathbf{g}}{||\mathbf{g}||}$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{||\mathbf{t} \times \mathbf{w}||}$$

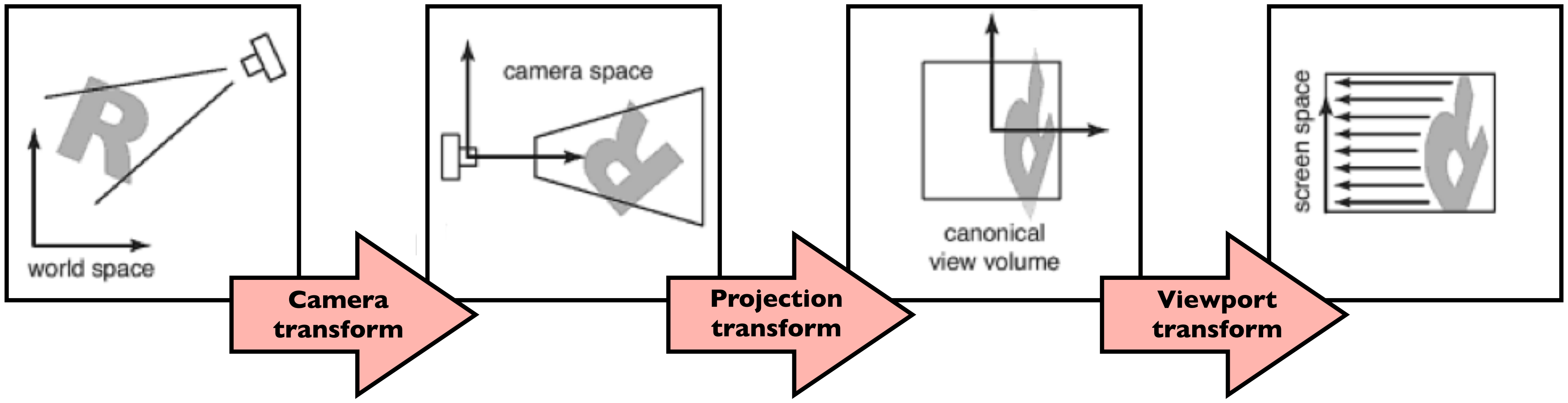
$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$





M_{cam} <whiteboard>

Line drawing algorithm

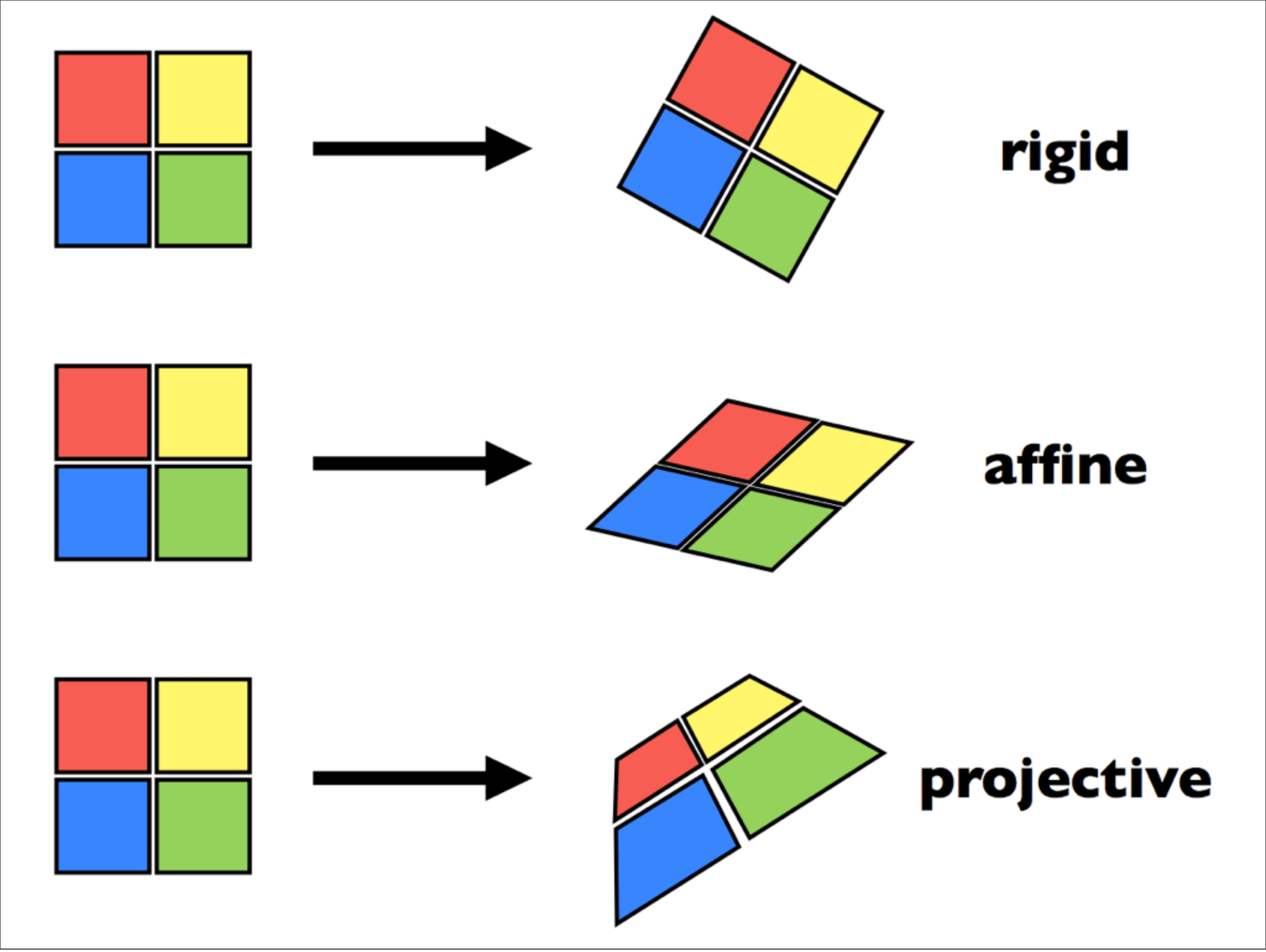


construct M_{vp} M_{cam}
construct M_{orth}
$M = M_{vp} M_{orth} M_{cam}$
for each line segment (a_i, b_i) do
$\mathbf{p} = M\mathbf{a}_i$
$\mathbf{q} = M\mathbf{b}_i$
drawline (x_p, y_p, x_q, y_q)

Shirley, Marschner 7.1

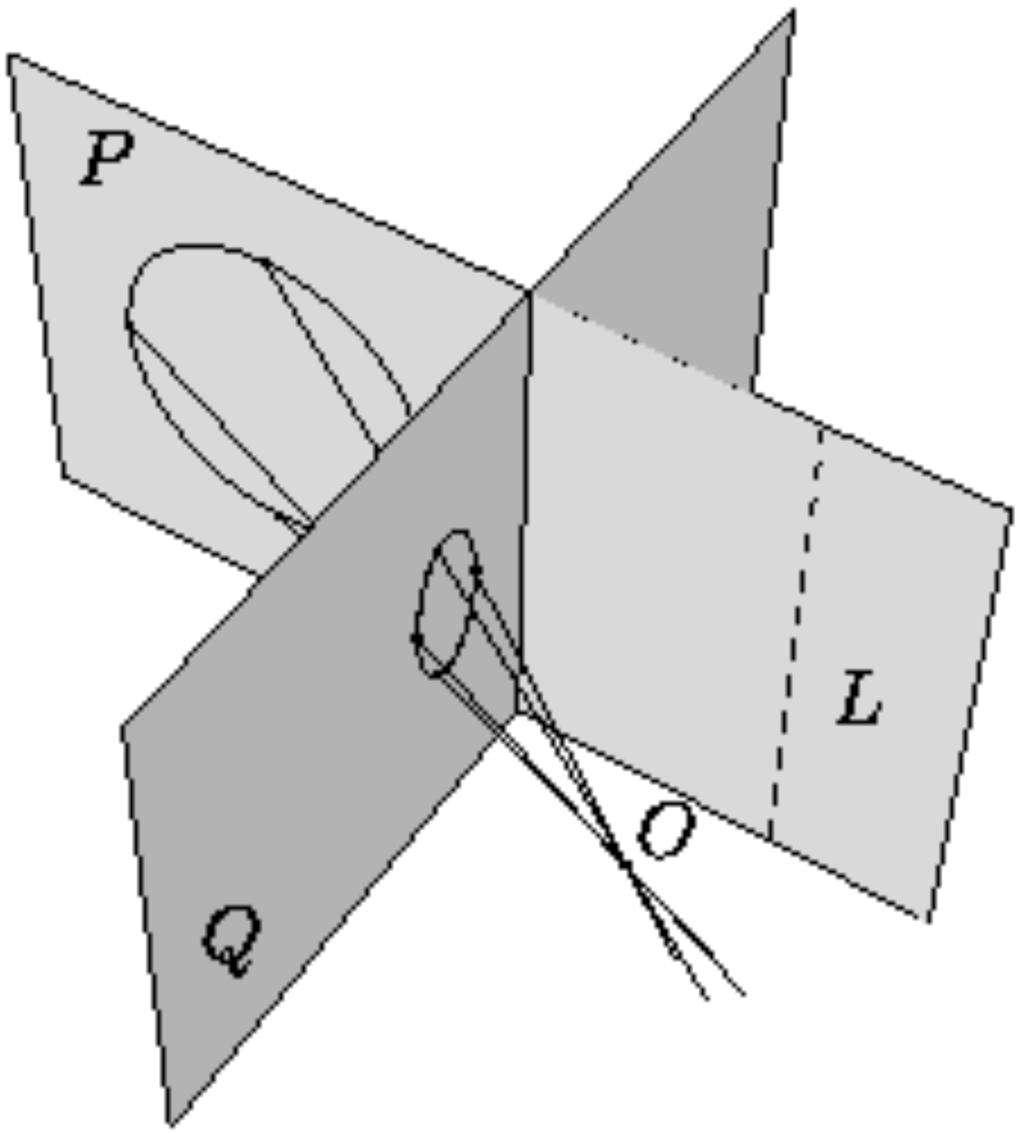
draw lines specified in world space

Perspective Viewing



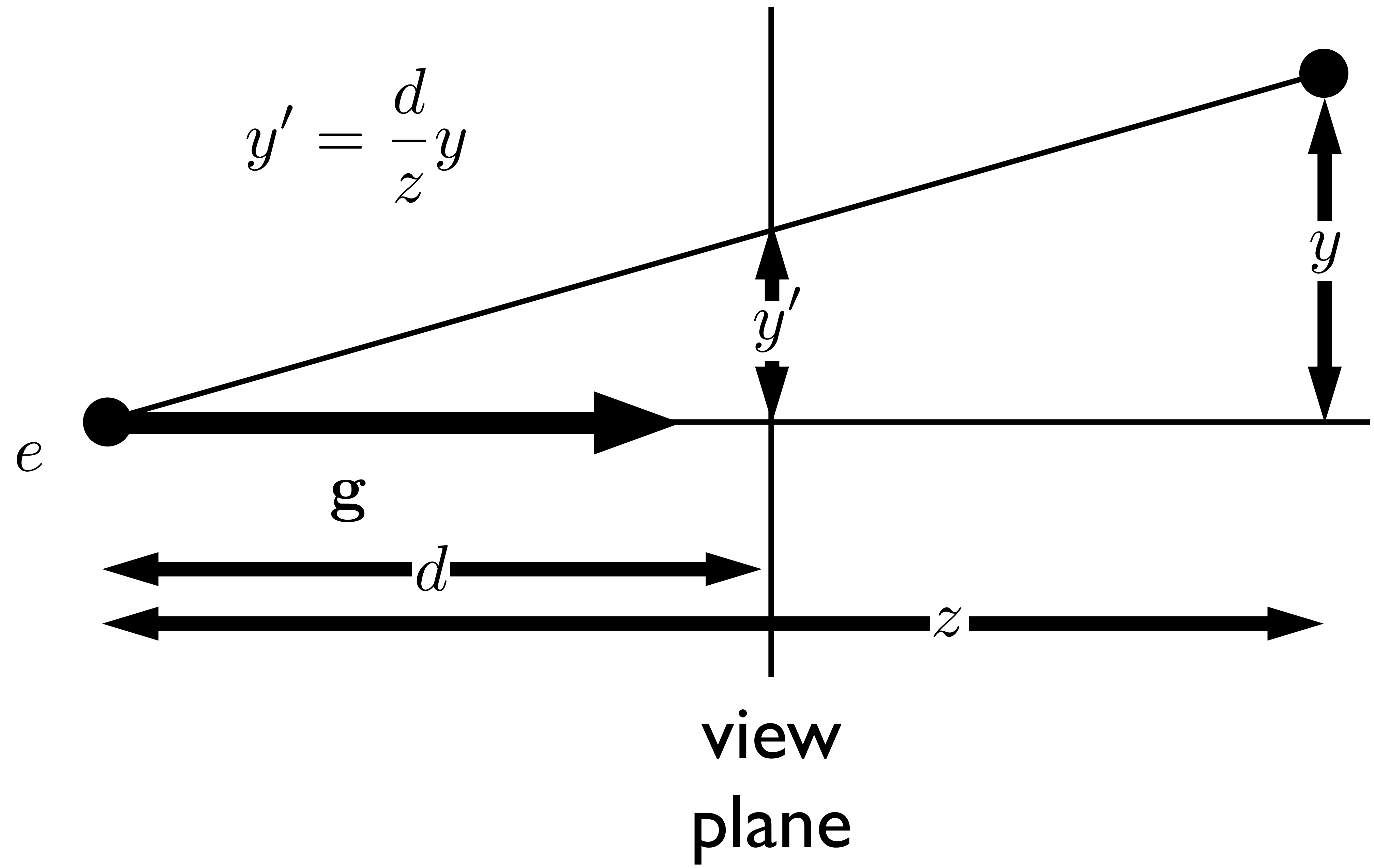
rigid – translation and rotation only – parallel lines and angles are preserved
affine – scaling, shear, translation, rotation – parallel lines preserved, angles **not** preserved projective – parallel lines and angles **not** preserved

Projective Transformations



[geom.uiuc.edu]

Projective Transformations



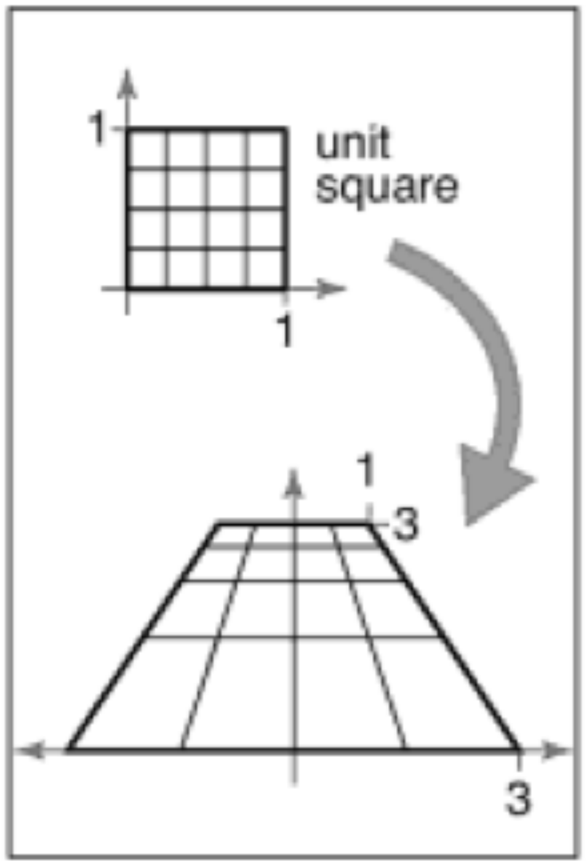
note that the height, **y’,** in **camera space** is proportional to y and inversely proportion to z. We want to be able to specify such a transformation with our **4x4 matrix machinery**

Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \quad x = \frac{\tilde{x}}{w} \quad y = \frac{\tilde{y}}{w} \quad z = \frac{\tilde{z}}{w}$$

<u>Example:</u>

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



[Shirley, Marschner]

Note: this makes our homogeneous representation for points unique only **up to a constant**

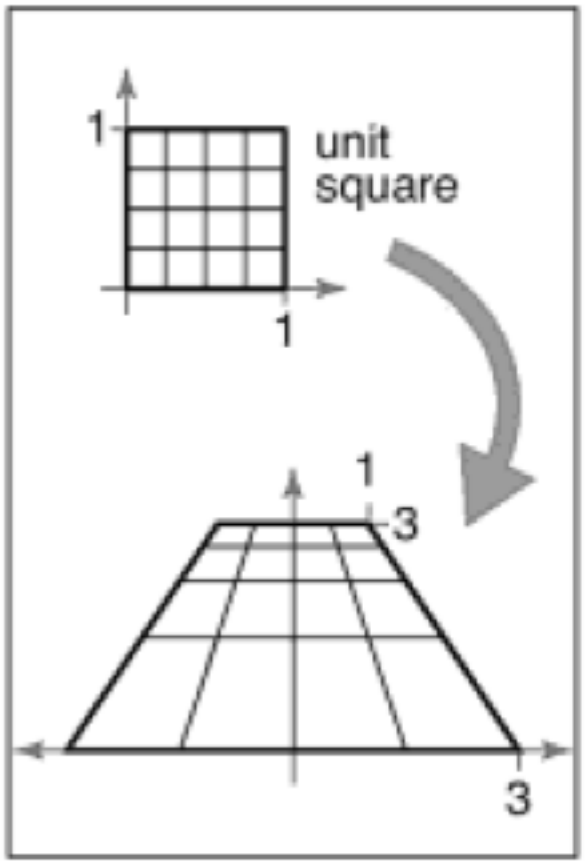
Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \begin{matrix} x = \frac{\tilde{x}}{w} \\ y = \frac{\tilde{y}}{w} \\ z = \frac{\tilde{z}}{w} \end{matrix}$$

We can now implement perspective projection!

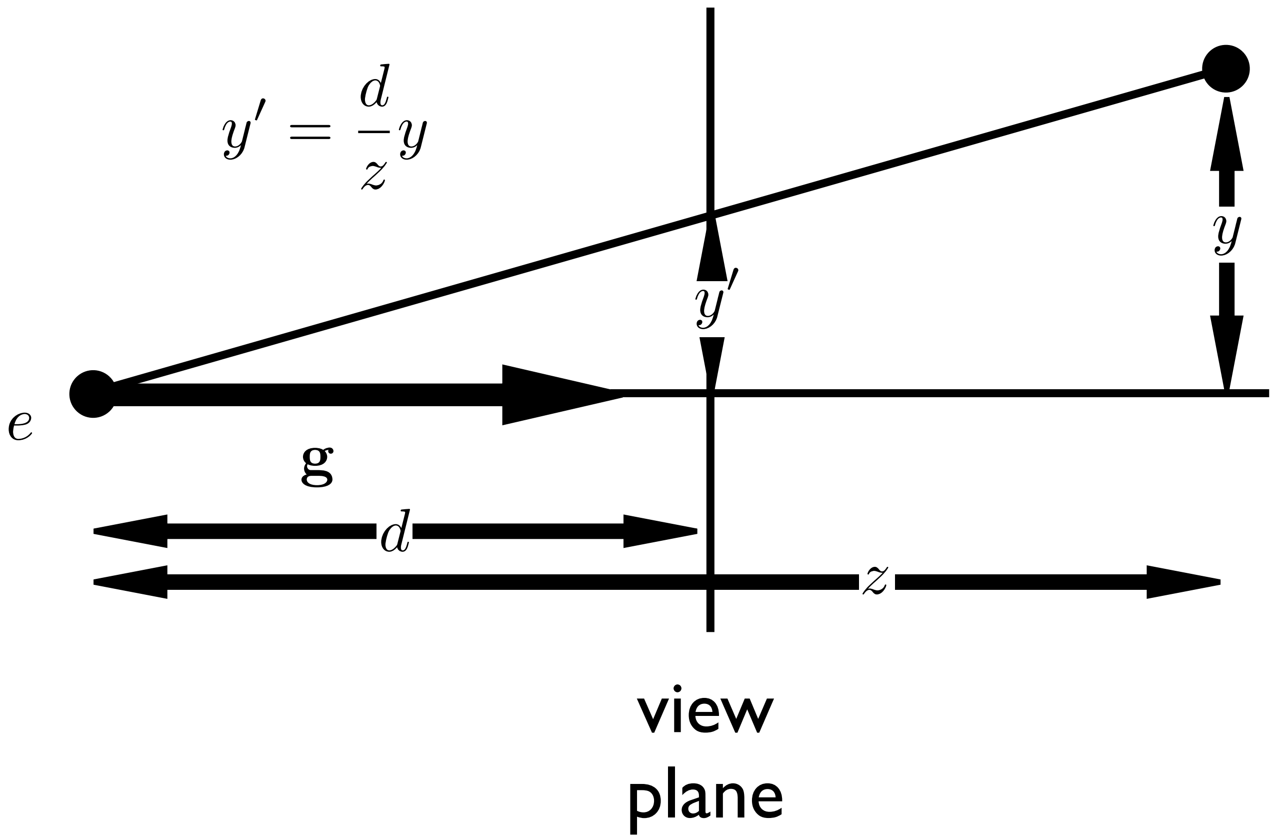
<u>Example:</u>

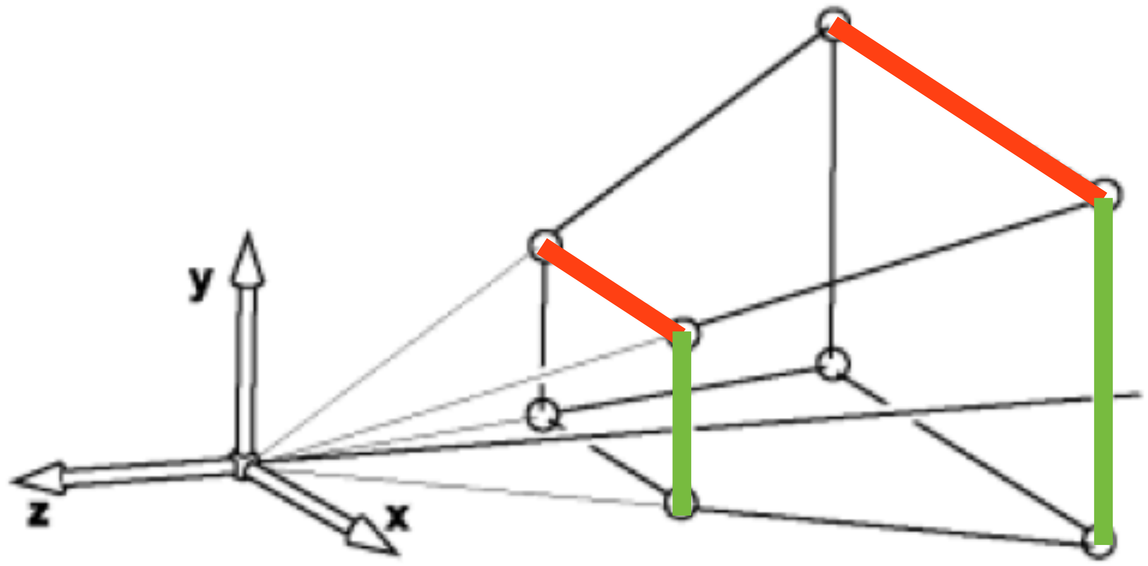
$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



[Shirley, Marschner]

Perspective Projection





both x and y get multiplied by d/z

[Shirley, Marschner]

note that both x and y will be transformed

Simple perspective projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \Rightarrow \begin{cases} x' = \frac{d}{z}x \\ y' = \frac{d}{z}y \\ z' = \frac{d}{z}z = d \end{cases}$$

This achieves a simple perspective projection onto the view plane z = d

but we've lost all information about z!

<whiteboard>

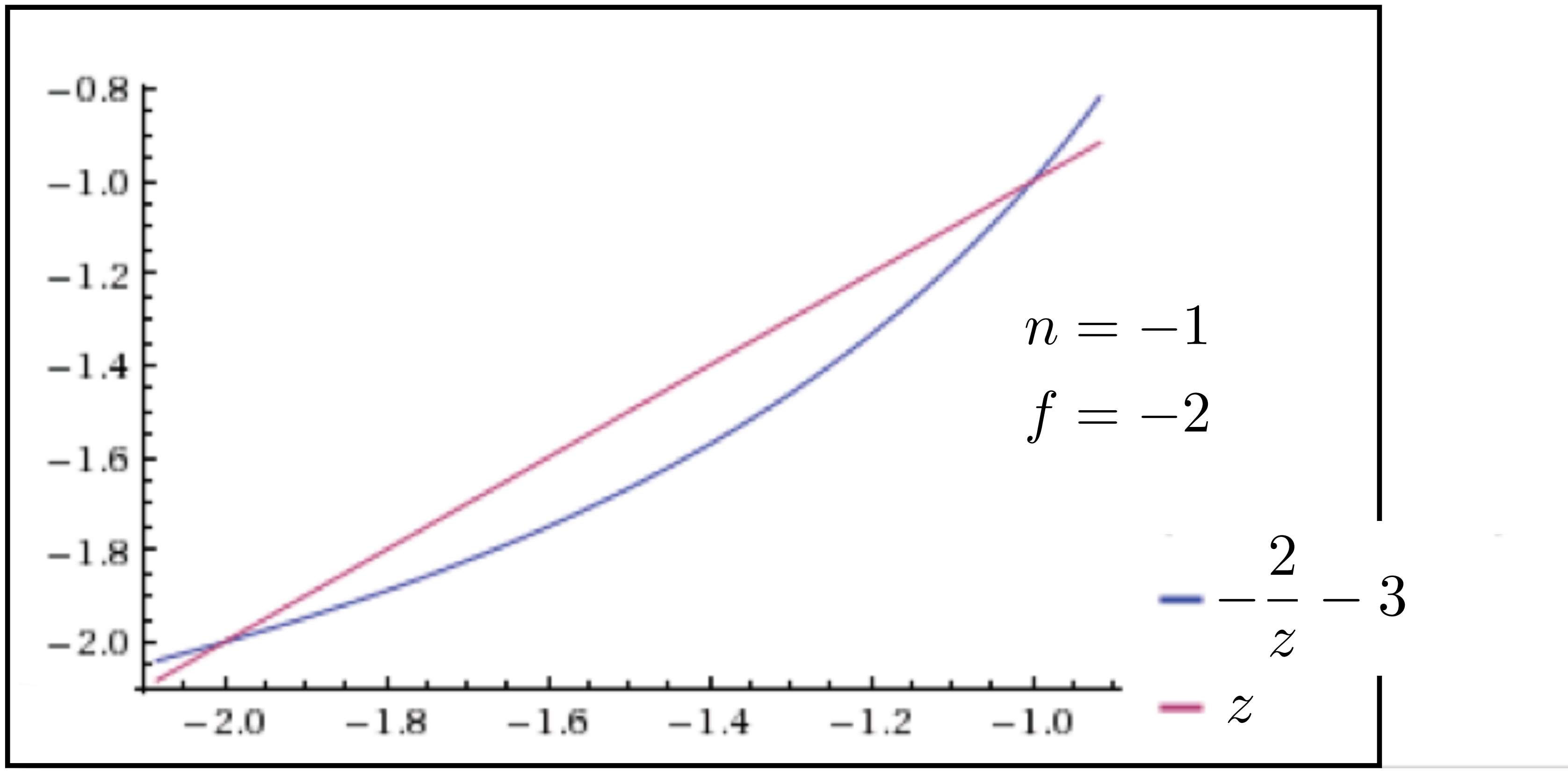
This simple projection matrix won't suffice. We need to preserve z information for later hidden surface removal.

whiteboard: derive P

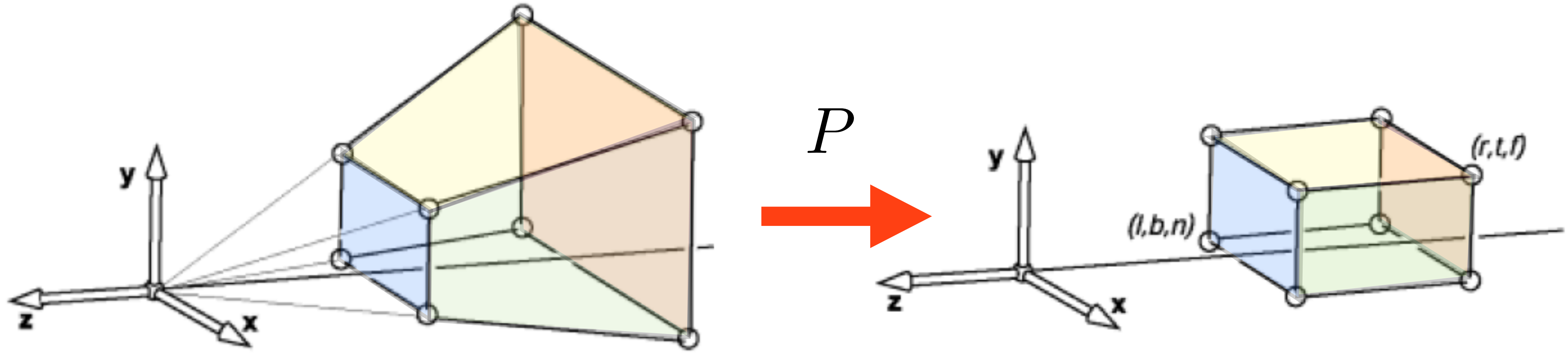
Perspective Projection

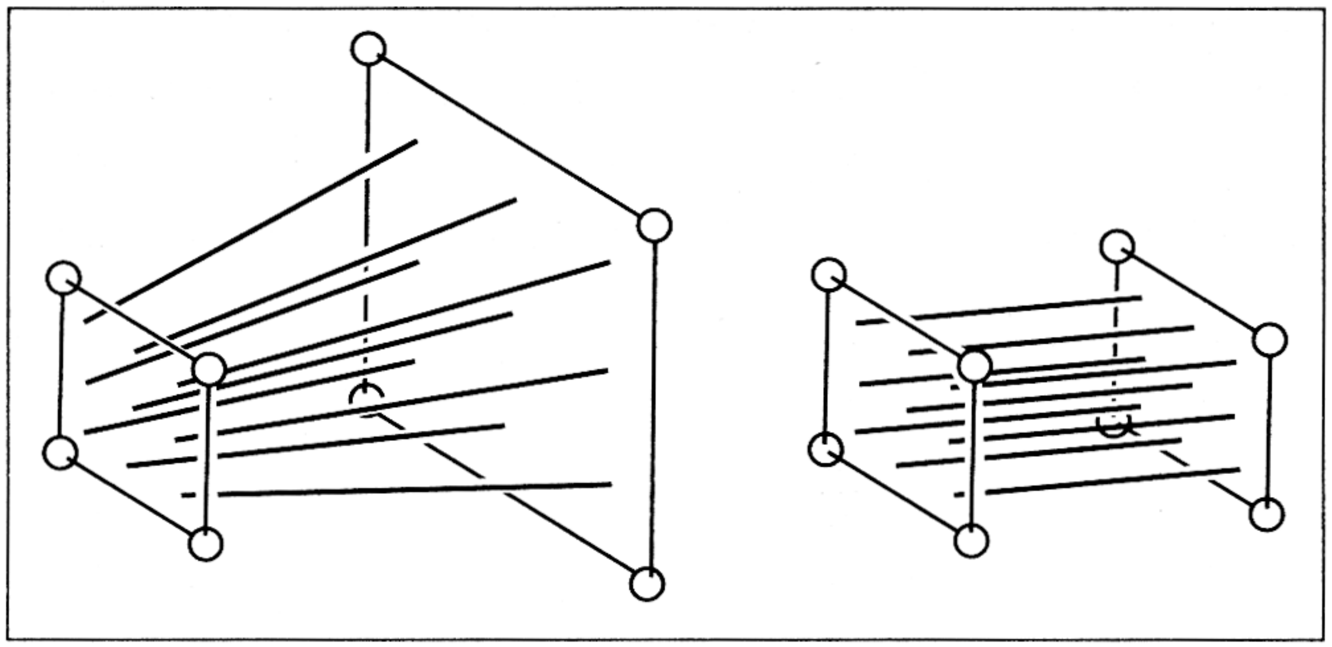
$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad z' = (n+f) - \frac{nf}{z}$$

Example:



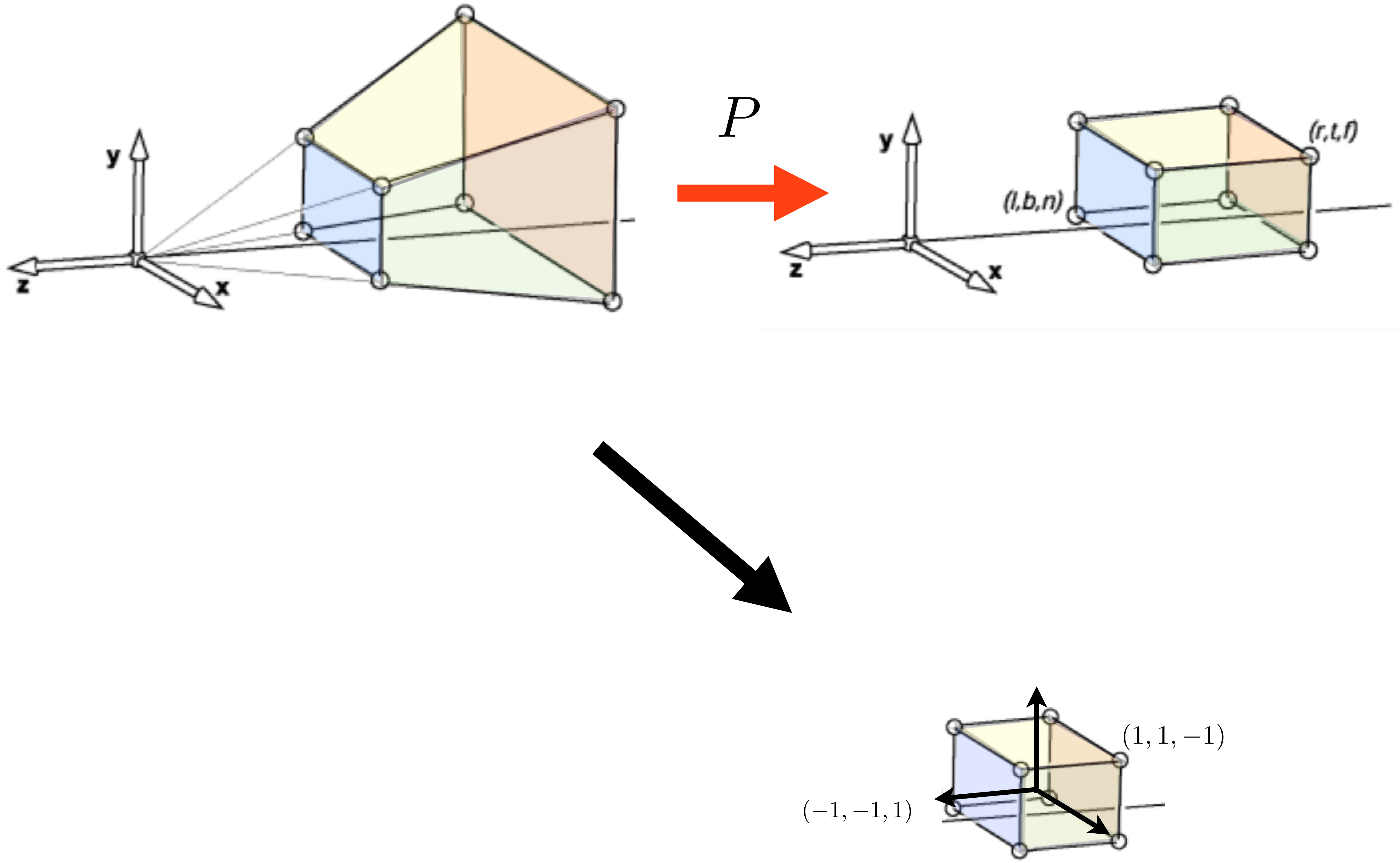
The perspective transformation does not preserve **z** completely, but it preserves **z = n, f** and is **monotone** (preserves ordering) with respect to z



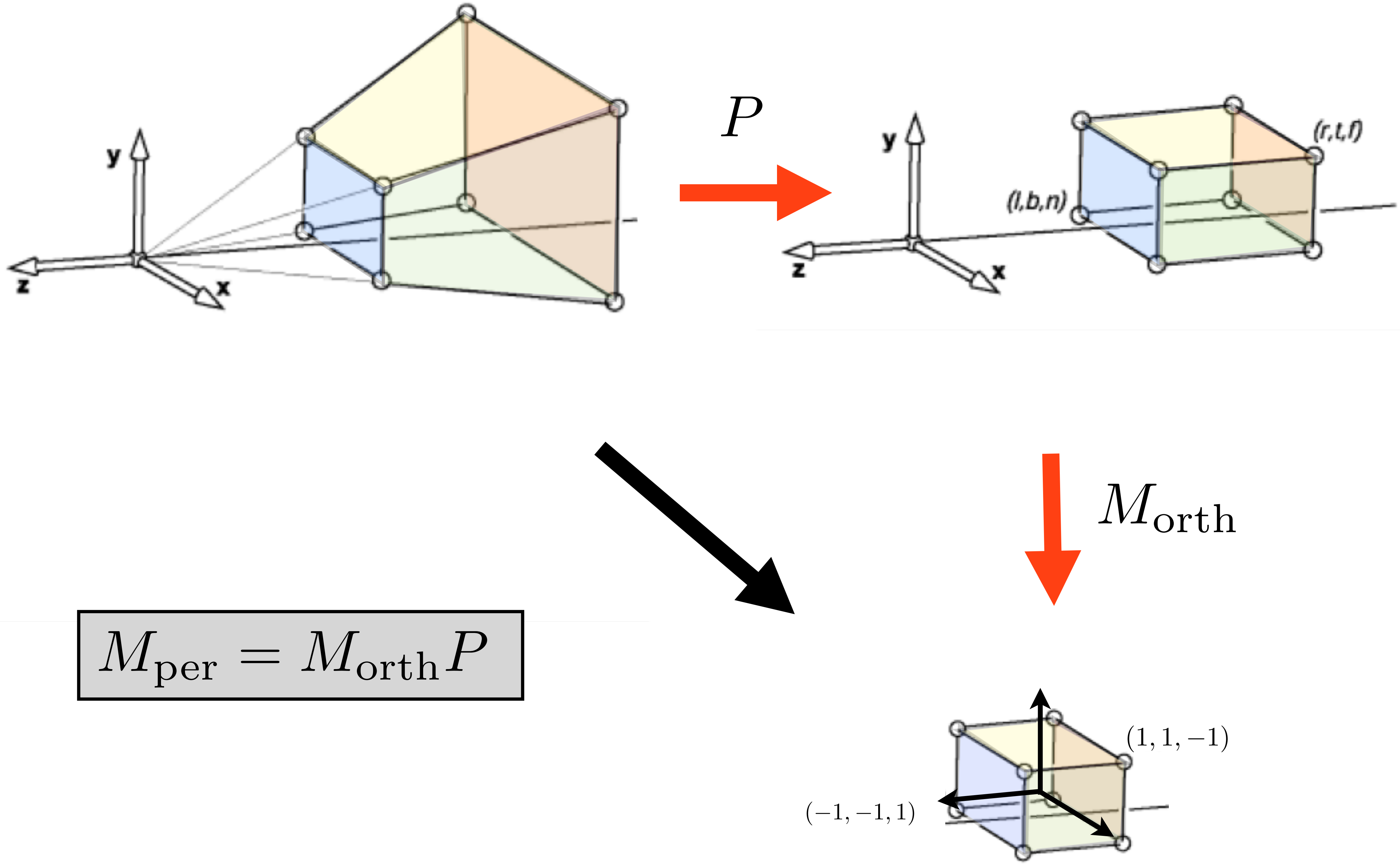


[Shirley, Marschner]

So far we've mapped the view frustrum to a rectangular box. This rectangular box has the same near face as the view frustrum. The far face has been mapped down to the far face of the box. This mapping is given by P. The bottom figure shows how lines in the view frustrum get mapped to the rect. box.

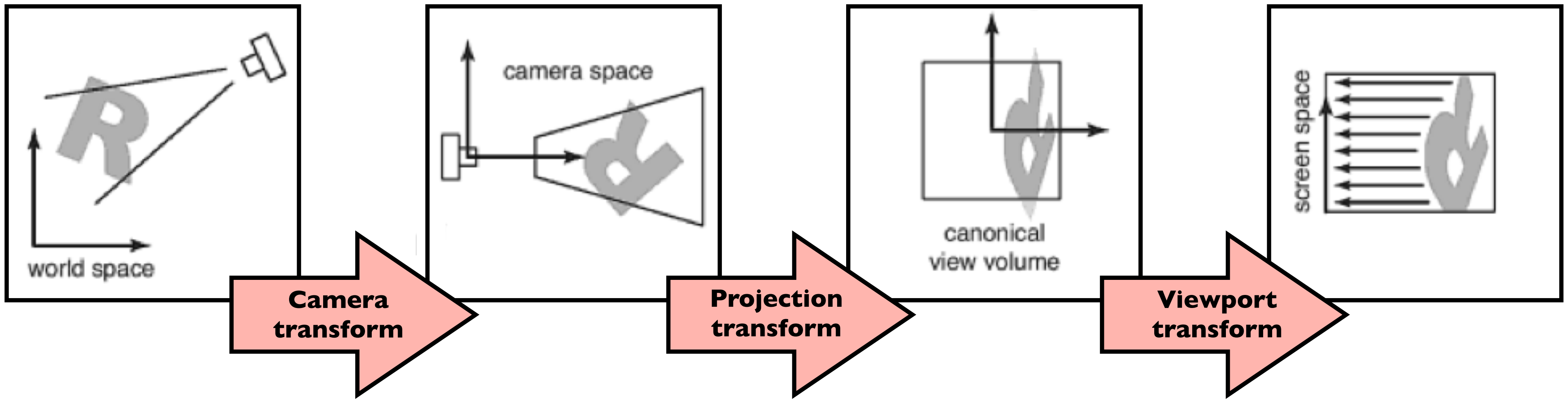


We’re not quite done yet thought, because the projection transform should map the view frustrum to the canonical view volume.



We need a second mapping to get our points into the canonical view volume. This second mapping is a mapping from one box to another. So it's given by an orthographic mapping, M_orth. The final perspective transformation is the composition of P and M_orth.

Line drawing algorithm



construct M_{vp} M_{cam}

construct M_{per}

$M = M_{vp} M_{per} M_{cam}$

for each line segment (a_i, b_i) do

$\mathbf{p} = M\mathbf{a}_i$

$\mathbf{q} = M\mathbf{b}_i$

drawline $(x_p/w_p, y_p/w_p, x_q/w_p, y_q/w_p)$

draw lines specified in <u>world</u> space

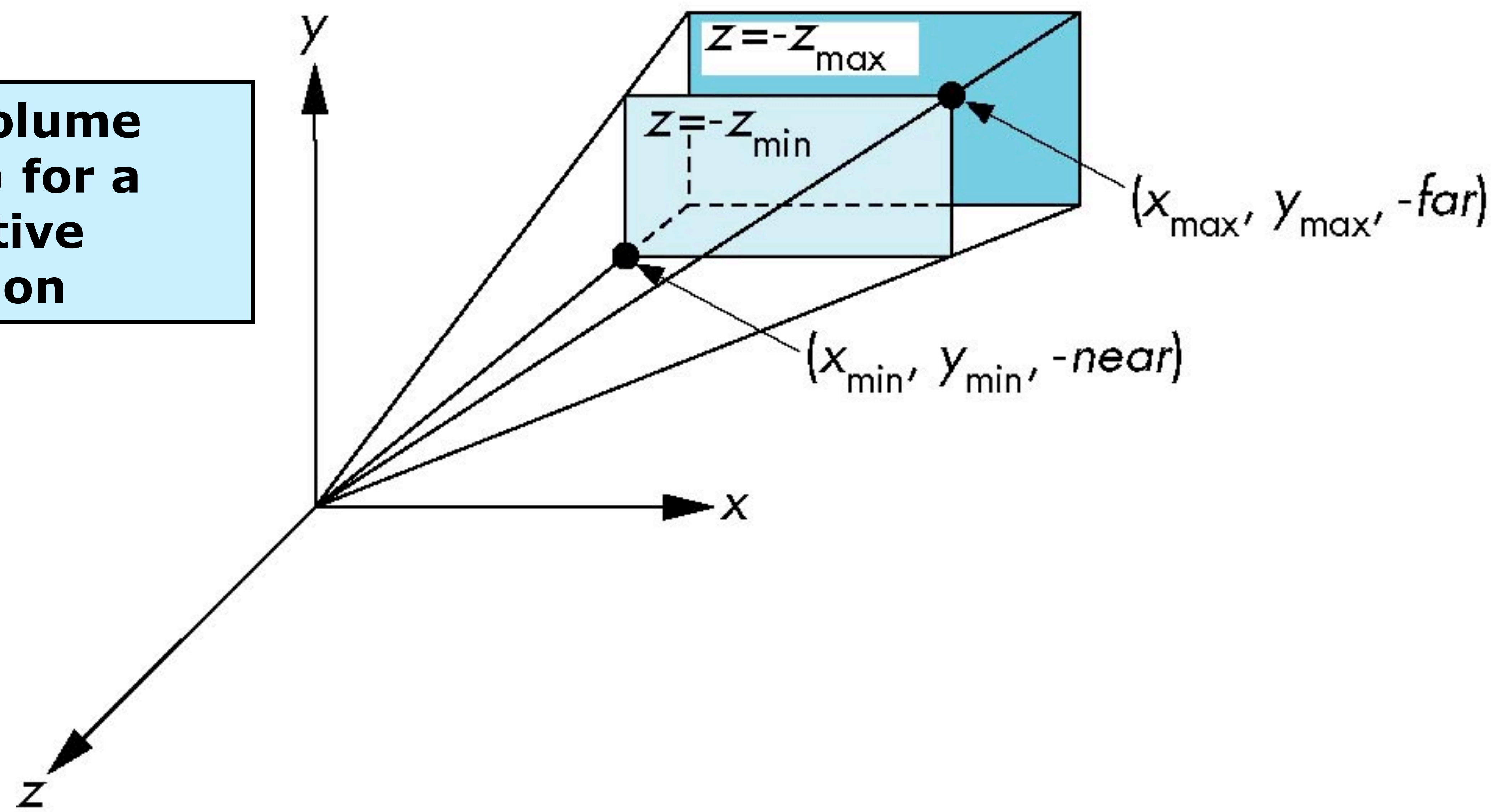
[Shirley, Marschner]

Note the two lines that have changed: 1. we put our perspective transformation into the overall transformation matrix M. 2. When we call the drawline function, we have to divide the x and y coordinates by the w coordinate.

OpenGL Perspective Viewing

`glFrustum(xmin,xmax,ymin,ymax,near,far)`

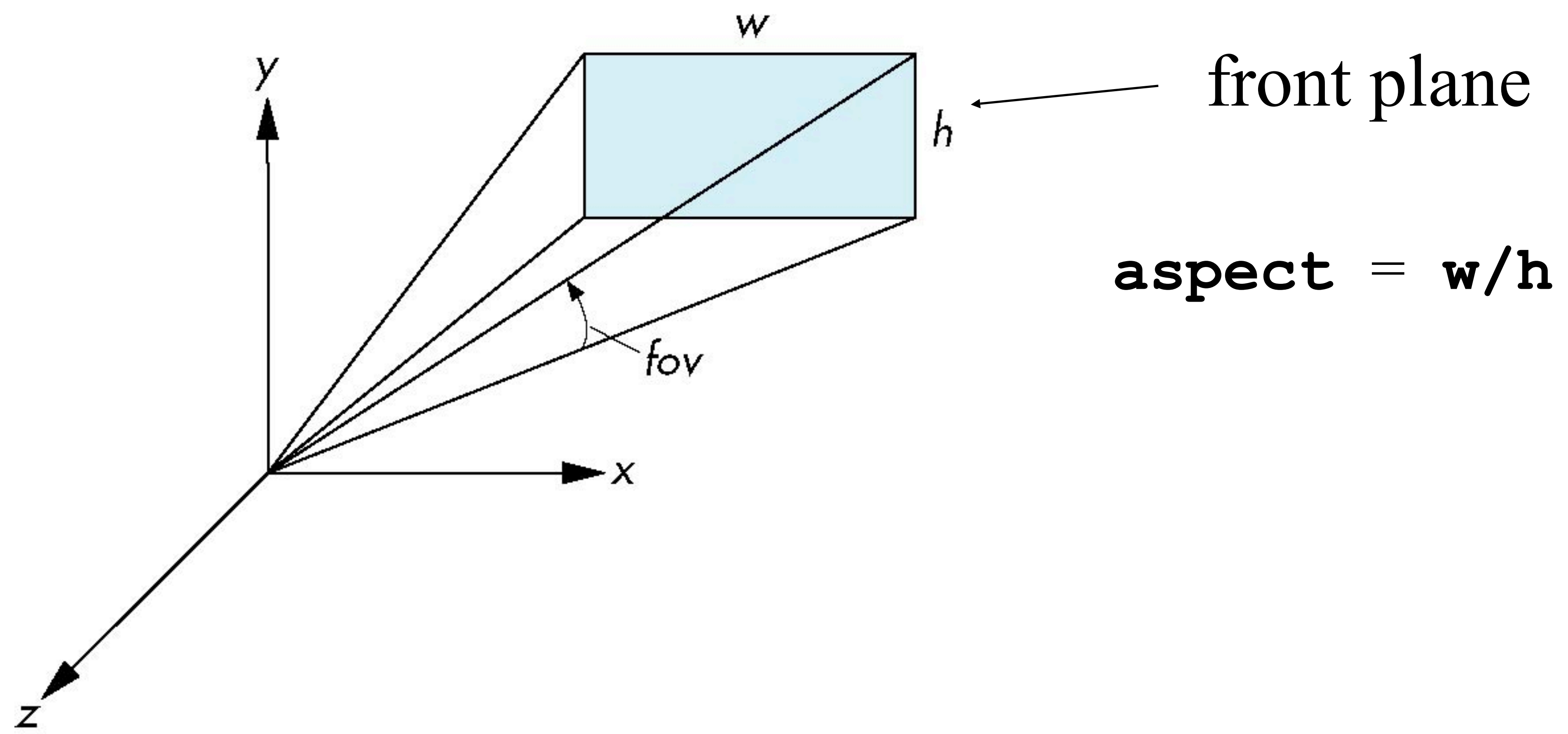
Clipping volume (frustrum) for a perspective projection



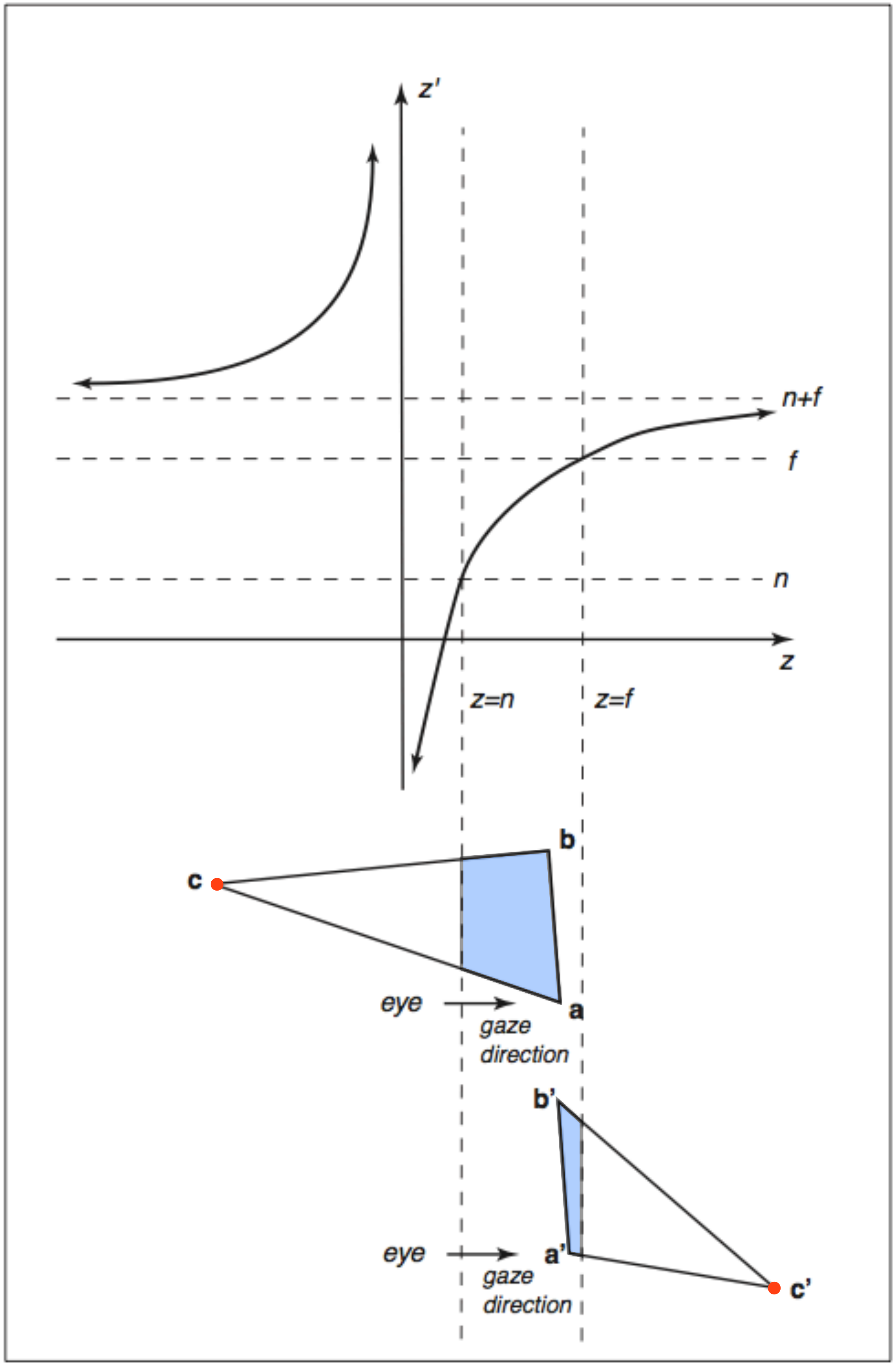
Here's how you set up a perspective view in OpenGL. Note that near and far are both negative, but you pass their absolute values to OpenGL.

Using Field of View

With **`glFrustum`** it is often difficult to get the desired view **`gluPerpective(fovy, aspect, near, far)`** often provides a better interface

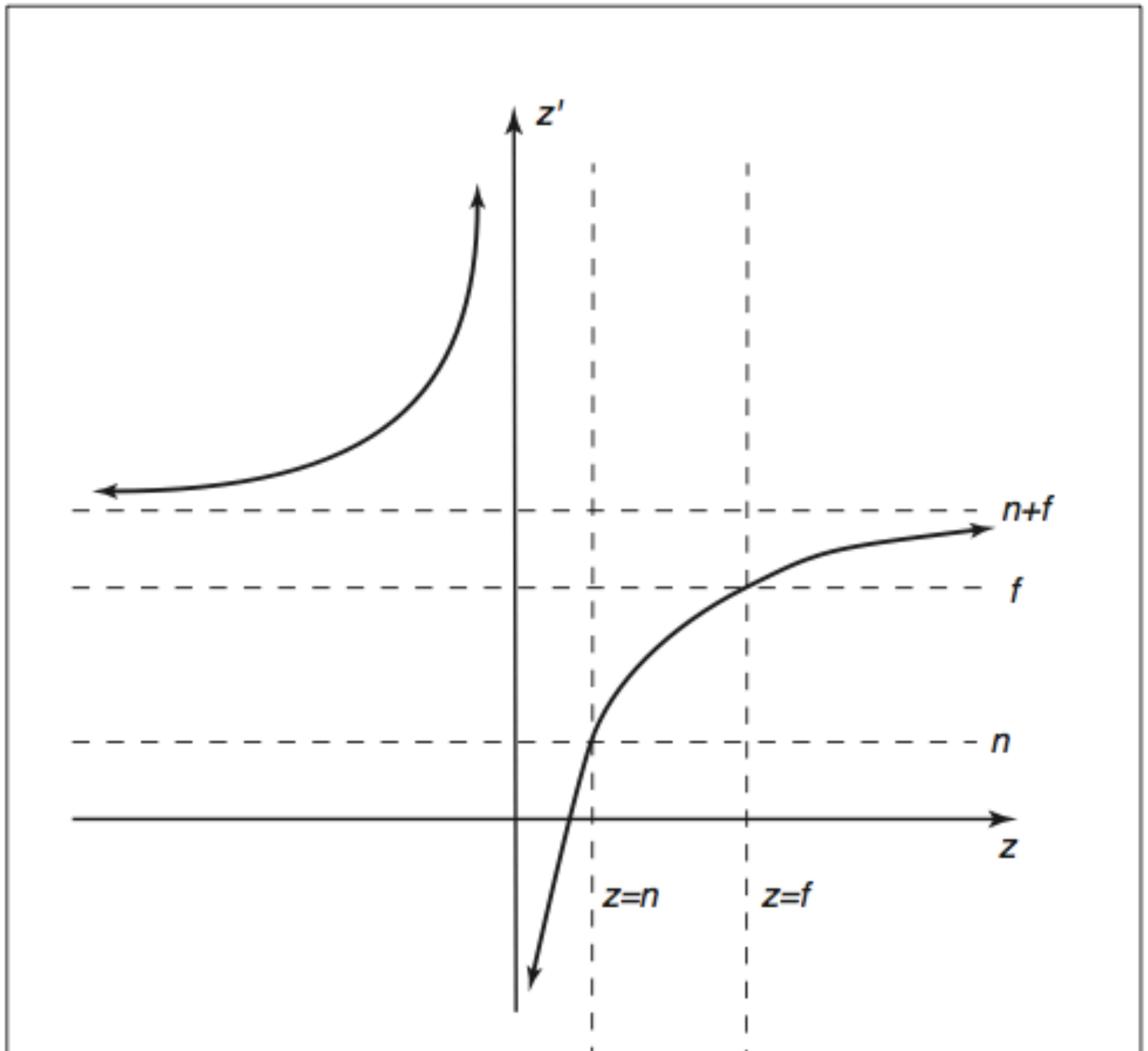


Sometimes it's more convenient to just give an angle, the field-of-view, and an aspect ratio, instead of l, r, t, b. The glu library provides such a function. It will figure out l, r, t, b, and call glFrustrum for you.



[Shirley, Marschner]

Clipping after the perspective transformation can cause problems



[Shirley, Marschner]

OpenGL clips **after** projection and **before** perspective division

$$-w \leq x \leq w$$

$$-w \leq y \leq w$$

$$-w \leq z \leq w$$