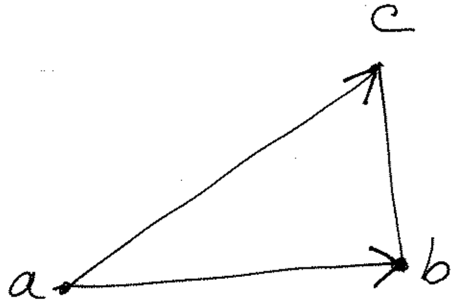
lecture 5

2.7 Triangles



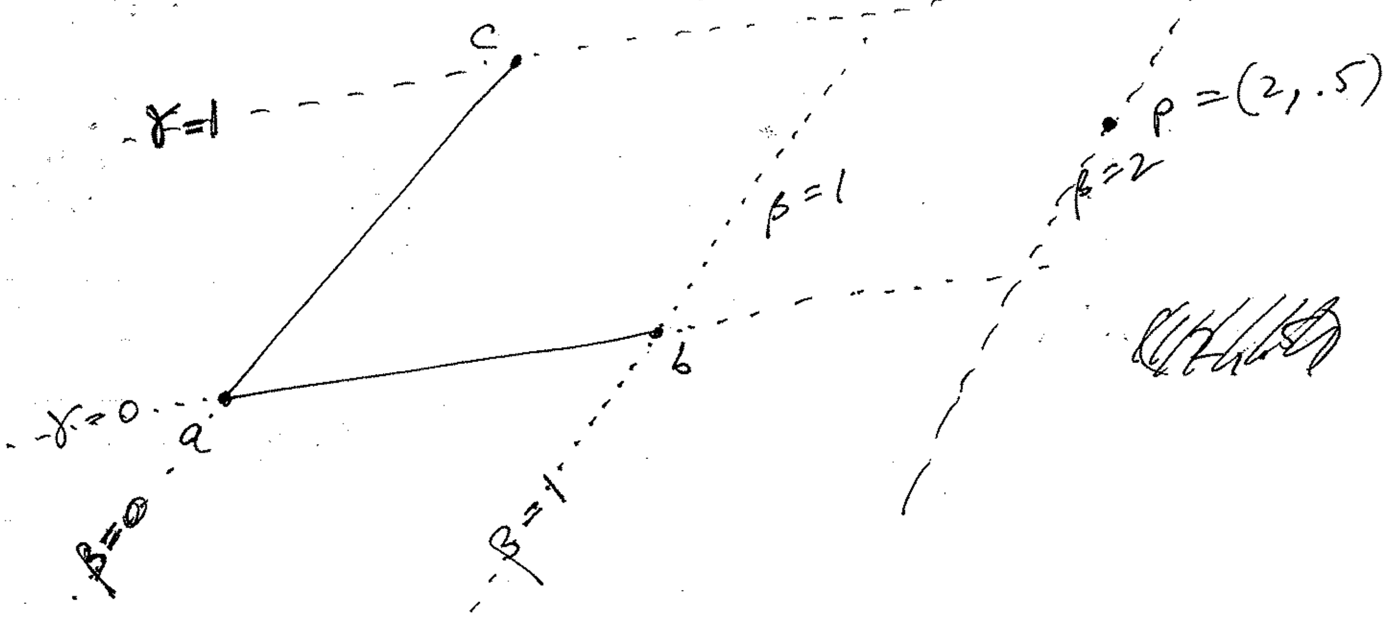
$$\text{area} = \frac{1}{2}\left| \begin{matrix} \mid & \mid \\ b-a & c-a \\ \mid & \mid \end{matrix} \right|$$

$$= \frac{1}{2}\begin{vmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{vmatrix}$$

barycentric coordinates

assign a color at each vertex & smoothly interpolate.

non-orthogonal coordinate system.



$$\boxed{p = a + \beta(b-a) + \gamma(c-a)}$$

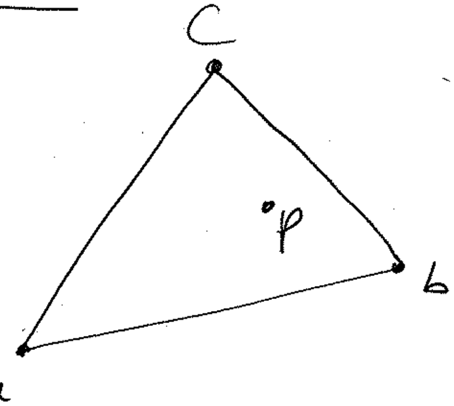
$$p = a - \beta a + \beta b + \gamma c - \gamma a$$

$$p = a - \beta a - \gamma a + \beta b + \gamma c$$

$$= (1 - \beta - \gamma) a + \beta b + \gamma c$$

$$\boxed{p = \alpha a + \beta b + \gamma c}$$

Inside test



$$p = \alpha a + \beta b + \gamma c$$

$\boxed{p \text{ inside triangle}}$ $\iff$ $\boxed{0 < \alpha, \beta, \gamma < 1}$

$(\alpha + \beta + \gamma = 1)$

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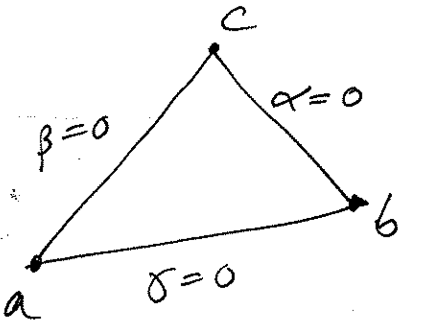
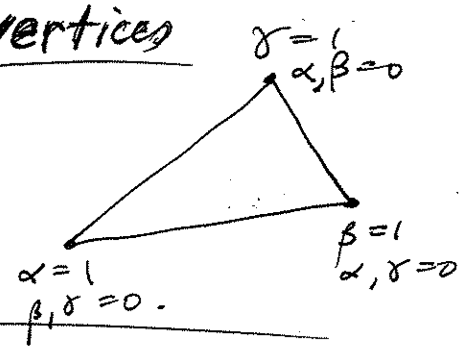
edges



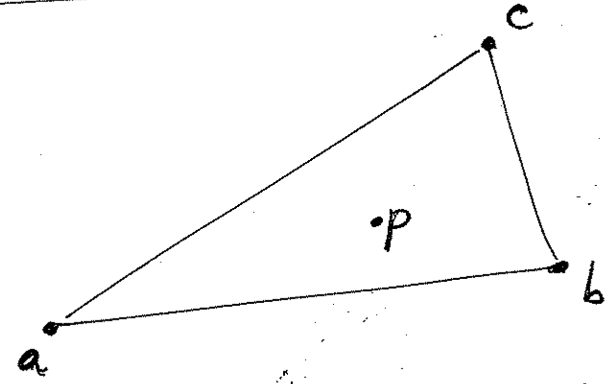
Vertices



Interpolation

use α, β, γ to interpolate properties.

Find α, β, γ given a point p

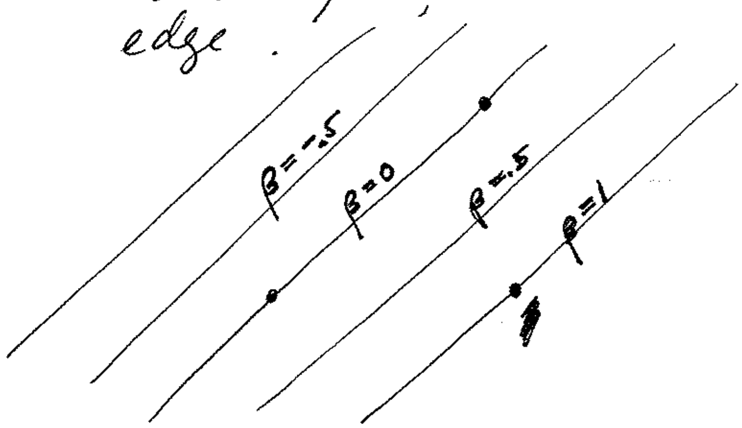


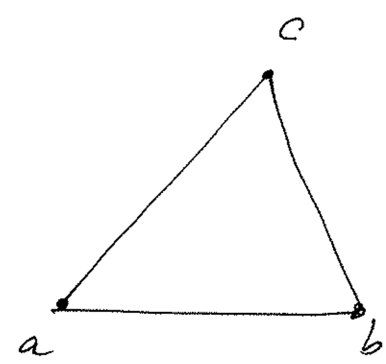
$$p = a + \beta(b-a) + \gamma(c-a)$$

Solve for β, γ

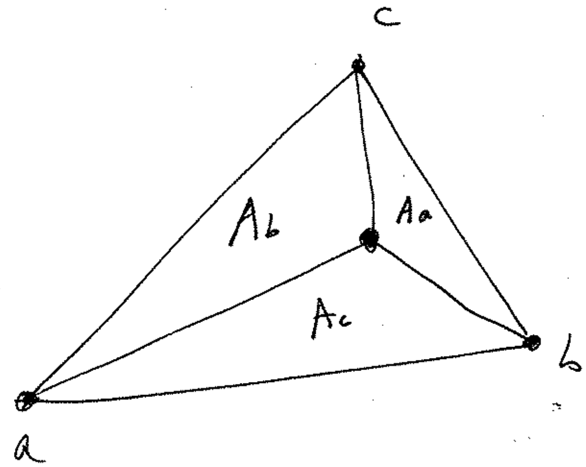
$$\begin{bmatrix} | & | \\ b-a & c-a \\ | & | \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} | \\ p-a \\ | \end{bmatrix}$$

Or use geometric property: bary. coords. are signed, scale distance from corresponding edge.





$$\beta = \frac{f_{ac}(p)}{f_{ac}(b)}$$



$$\alpha = \frac{A_a}{A}, \quad \beta = \frac{A_b}{A}, \quad \gamma = \frac{A_c}{A}$$