CS130 : Computer Graphics

Lecture 3: Rasterizing Lines and Triangles

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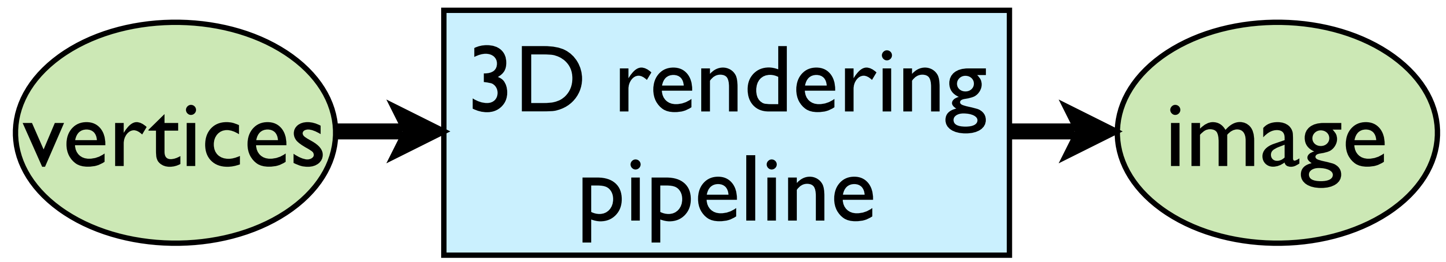
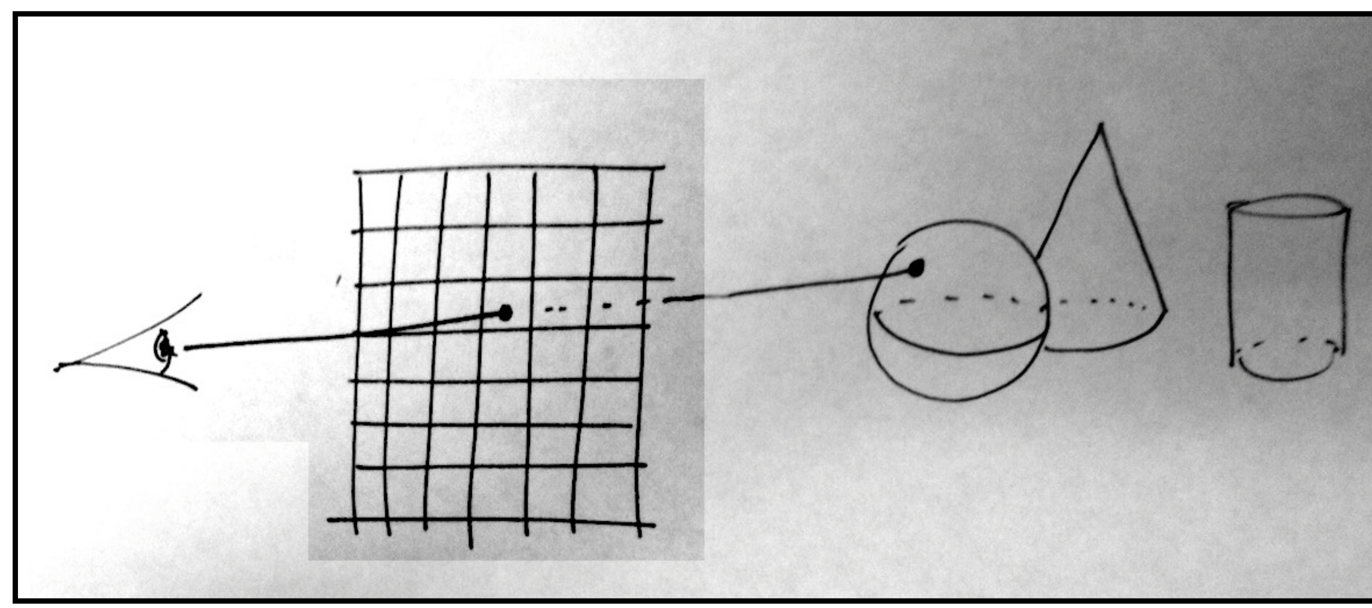
Rendering approaches

1. object-oriented



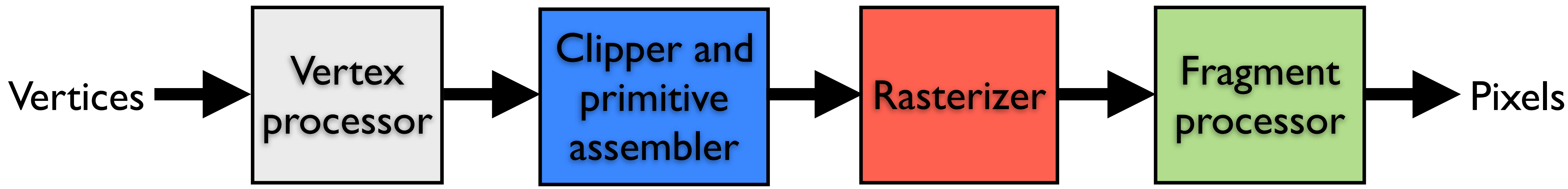
foreach object ...

2. image-oriented



foreach pixel ...

Outline

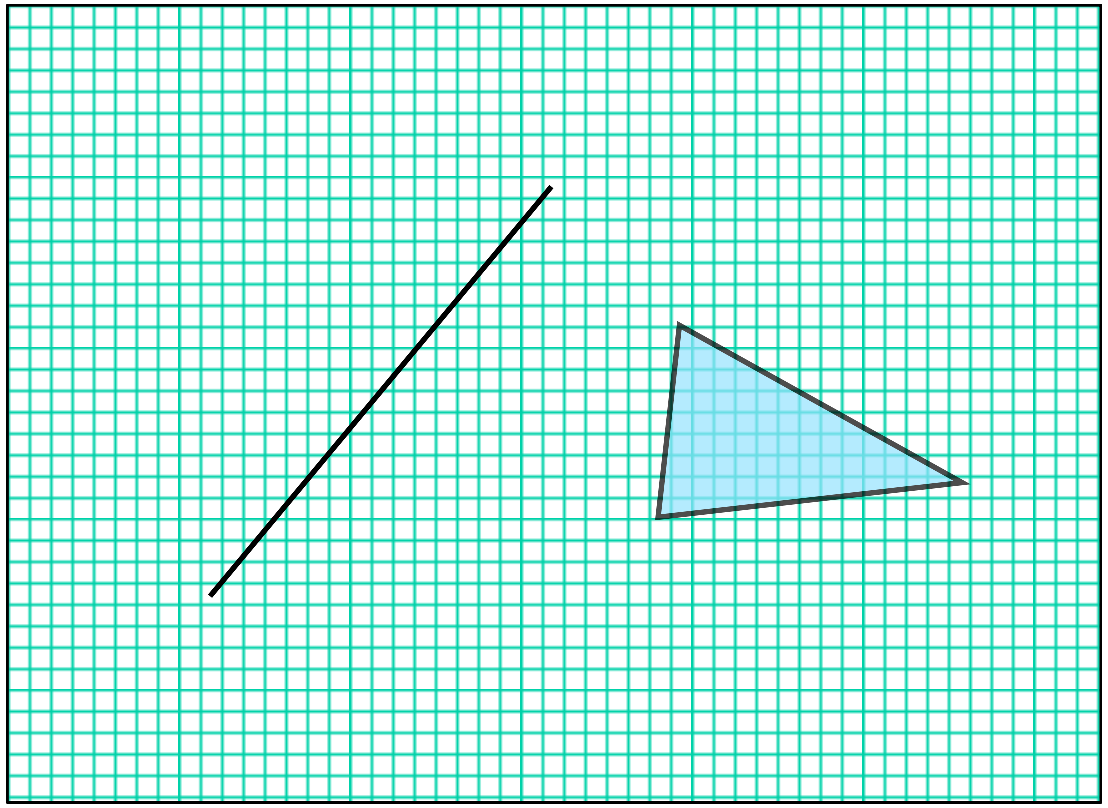


rasterization - make fragments from clipped objects

clipping - clip objects to viewing volume

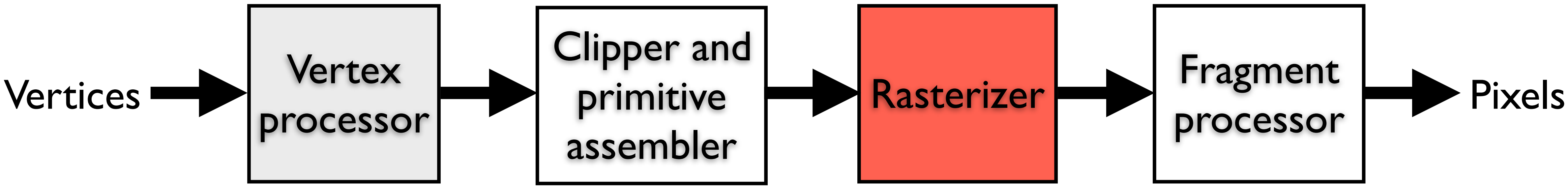
hidden surface removal - determine visible fragments

What is rasterization?



Rasterization is the process of determining which pixels are "covered" by the primitive

What is rasterization?



input: primitives **output**: fragments

enumerate the pixels covered by a primitive

interpolate attributes across the primitive

Rasterization

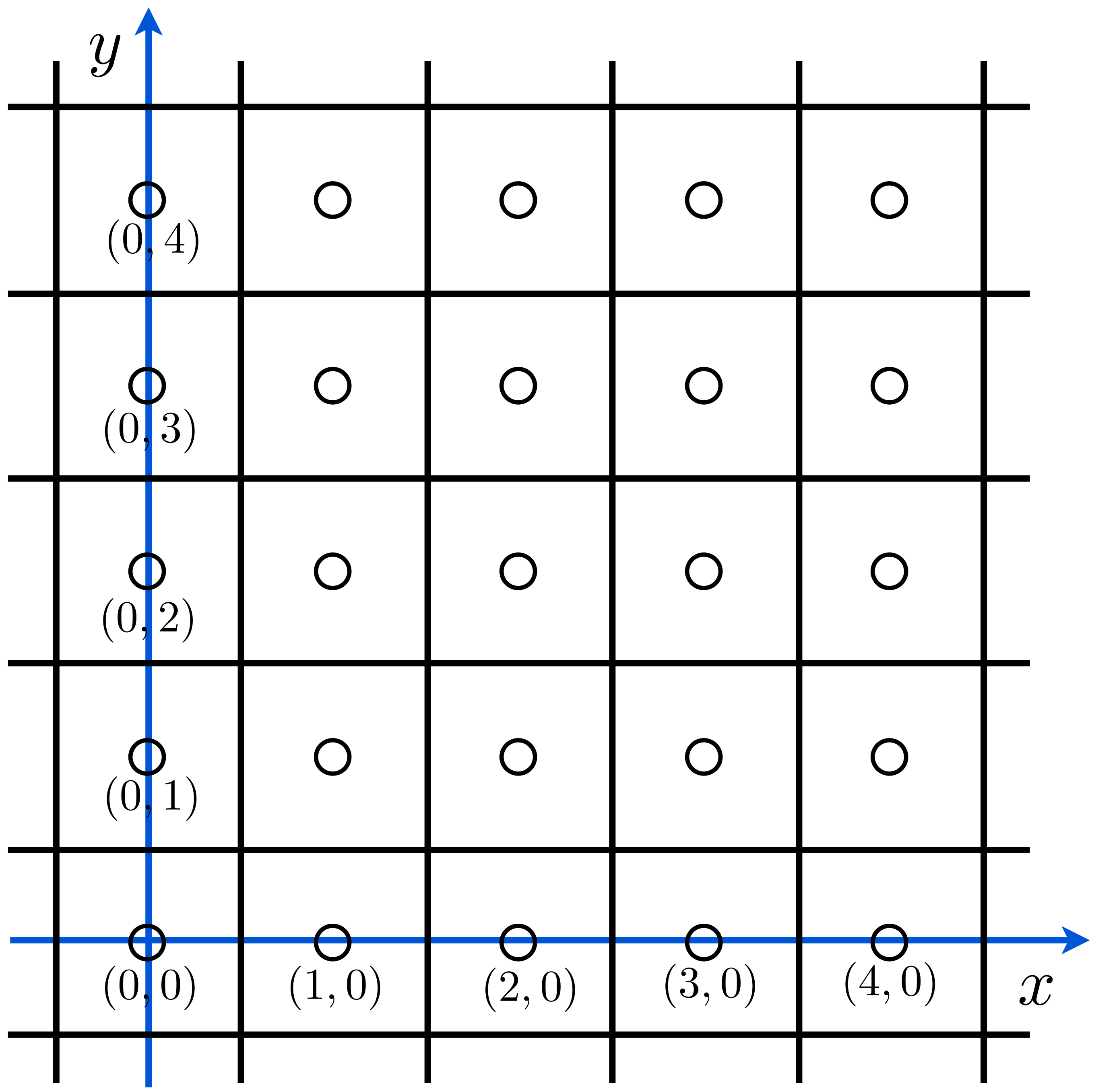
Compute integer coordinates for pixels covered by the 2D primitives

Algorithms are invoked many, many times and so must be efficient

Output should be visually pleasing, for example, lines should have constant density

Obviously, they should be able to draw all possible 2D primitives

Screen coordinates

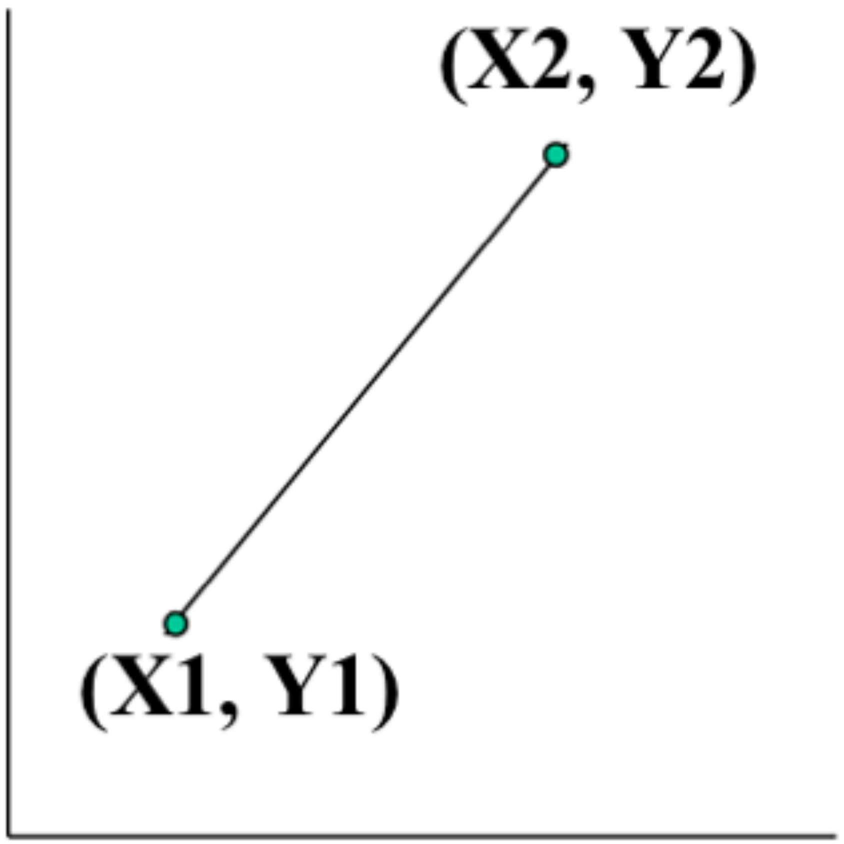


Line Representation

Math Review

- 2D math for lines

How do we determine the equation of the line?



Math Review

- 2D math for lines

  Slope-Intercept formula for a line

  Slope = (Y2 – Y1)/(X2 – X1)

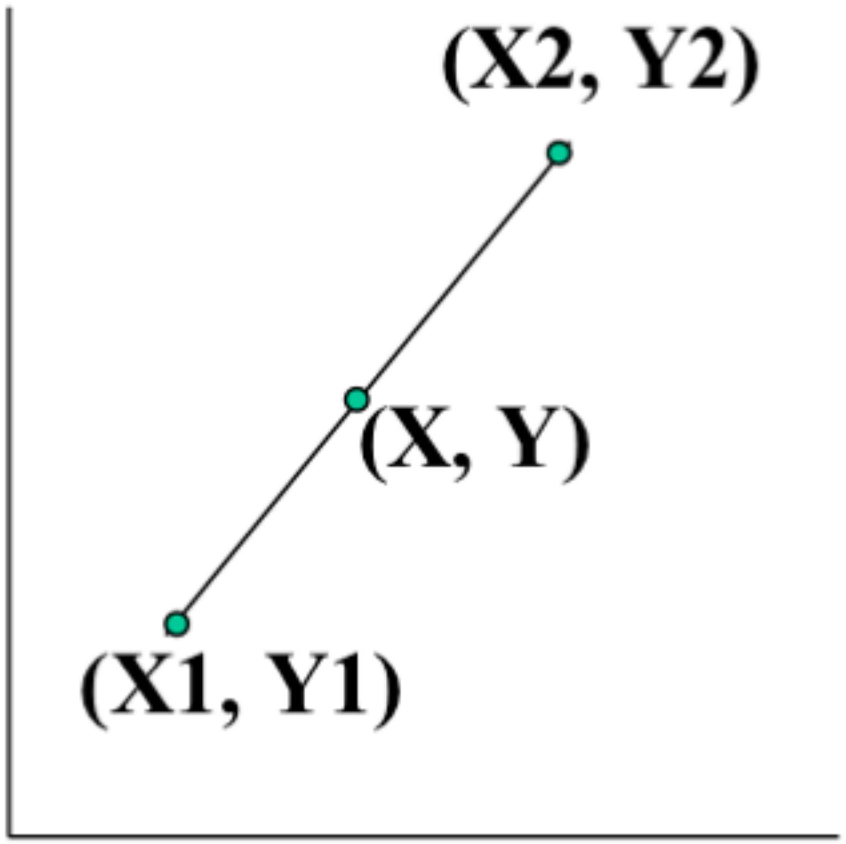
  (Y - Y1)/(X - X1)

  Solving For Y

  Y = [(Y2 – Y1)/(X2 – X1)]X

  + [-(Y2 – Y1)/(X2 - X1)]X1 + Y1 or

  Y = m X + b



Math Review

- Explicit (functional) representation

  $y = f(x)$

  y is the dependent, x independent variable

  Find value of y from value of x

  Example, for a line: for a circle:

  $y = mx + b$ $x^2 + y^2 = r^2$

Math Review

- Parametric Representation

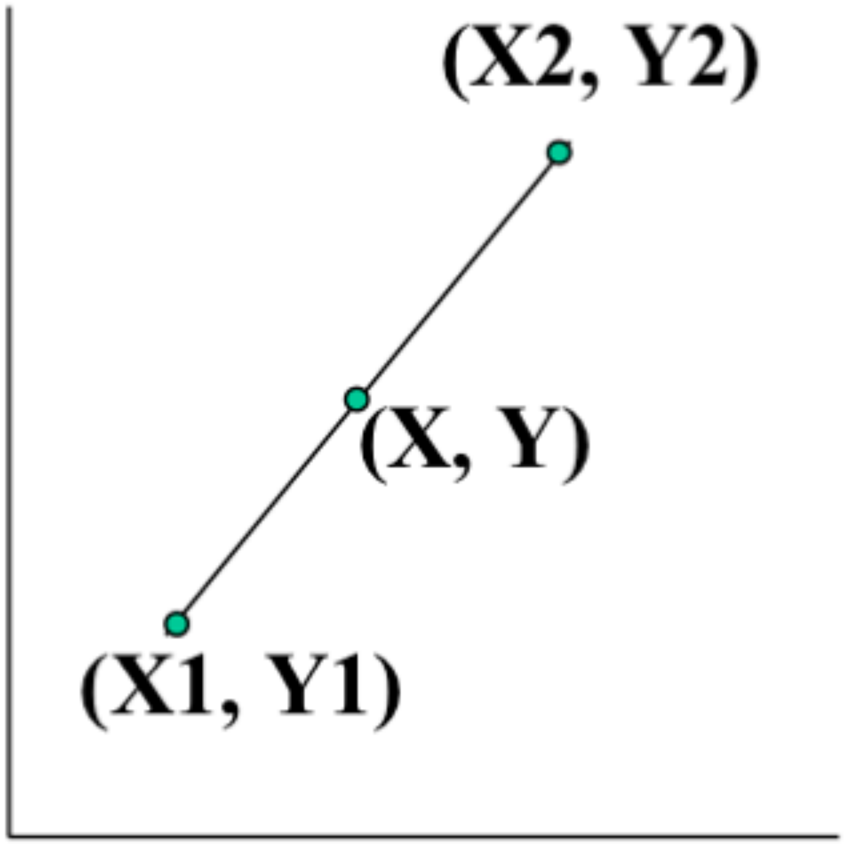
$$x = x(u),\ y = y(u)$$

where new parameter u (or often t) determines the value of x and y (and possibly z) for each point

x,y treated the same, axis invariant

Math Review

Parametric formula for a line



$X = X1 + t(X2 - X1)$

$Y = Y1 + t(Y2 - Y1)$

for parameter t from 0 to 1

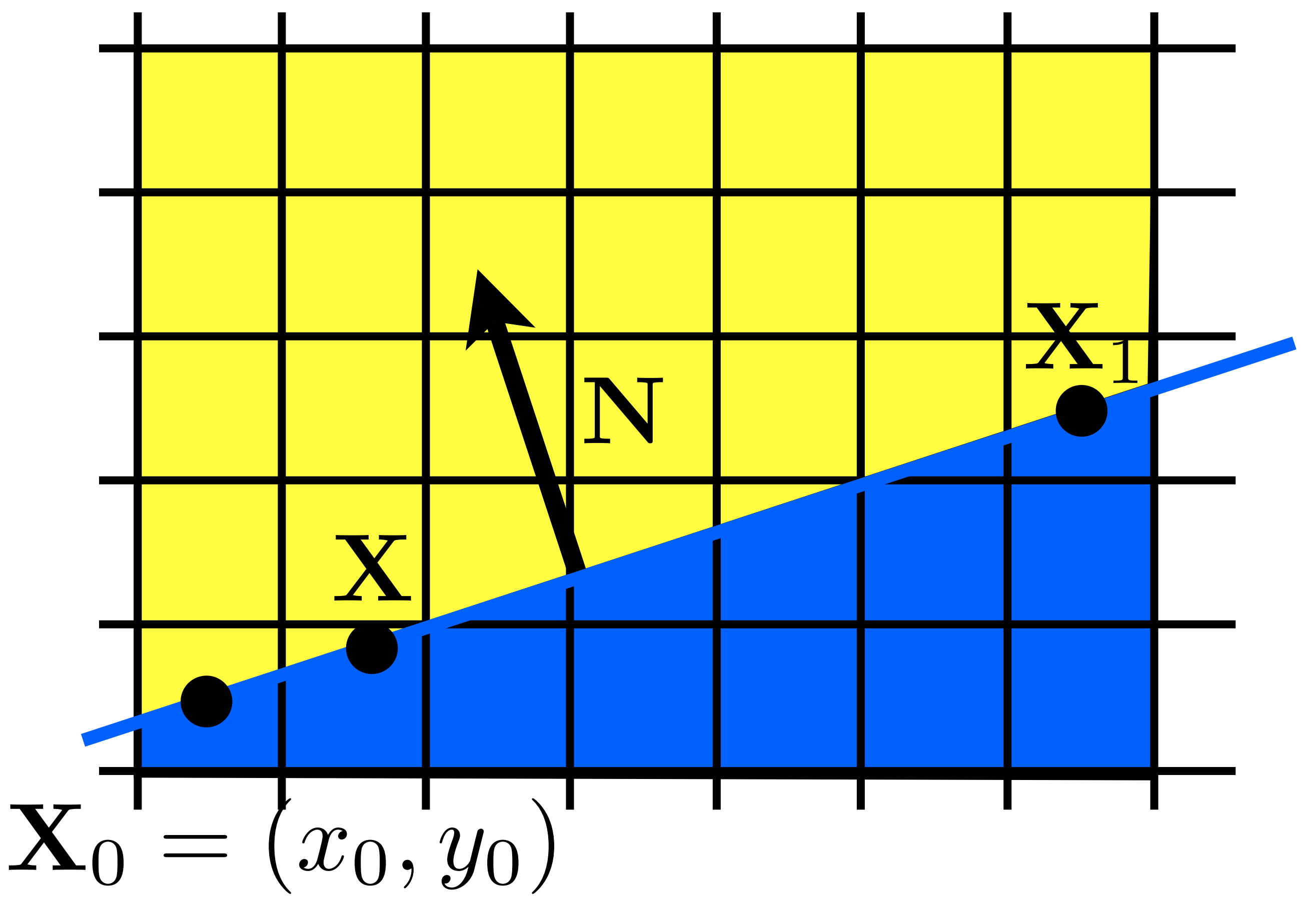
Therefore, when

$t = 0$ we get (X1,Y1)

$t = 1$ we get (X2,Y2)

Varying t gives the points along the line segment

Implicit Line Equation



$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

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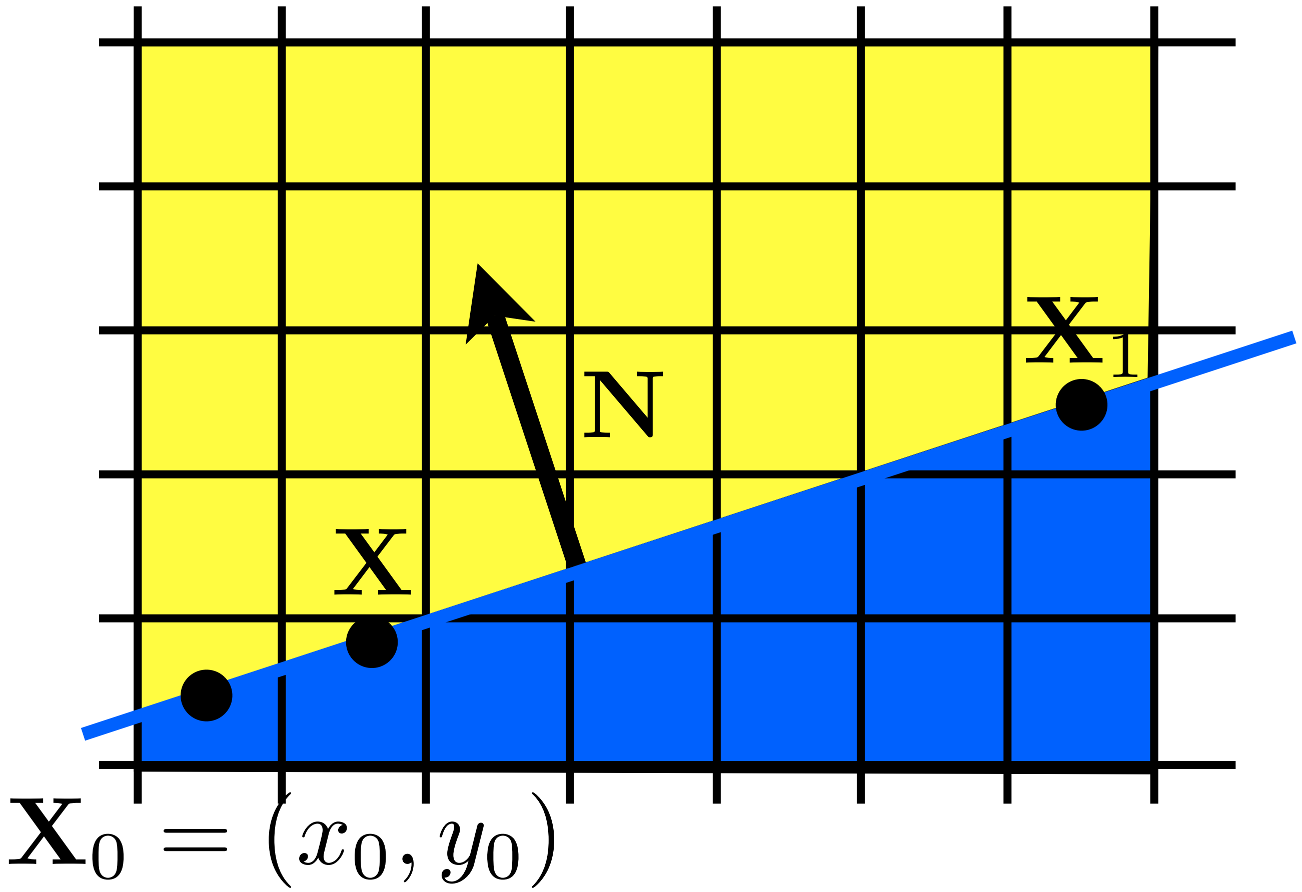
Implicit Line Equation



decision variable, d

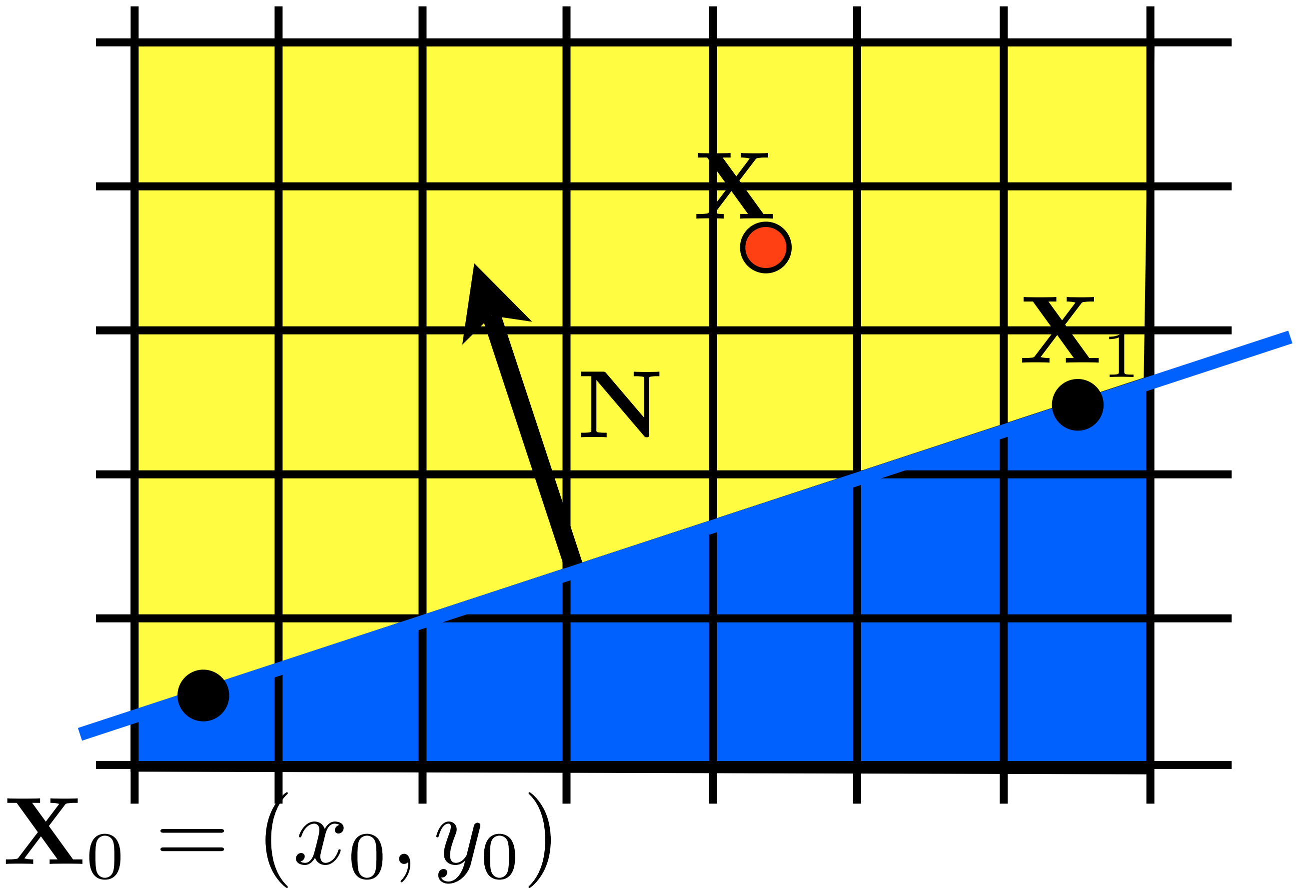
$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = d$$

$$d > 0$$

$$d < 0$$

$$d = 0$$

Implicit Line Equation



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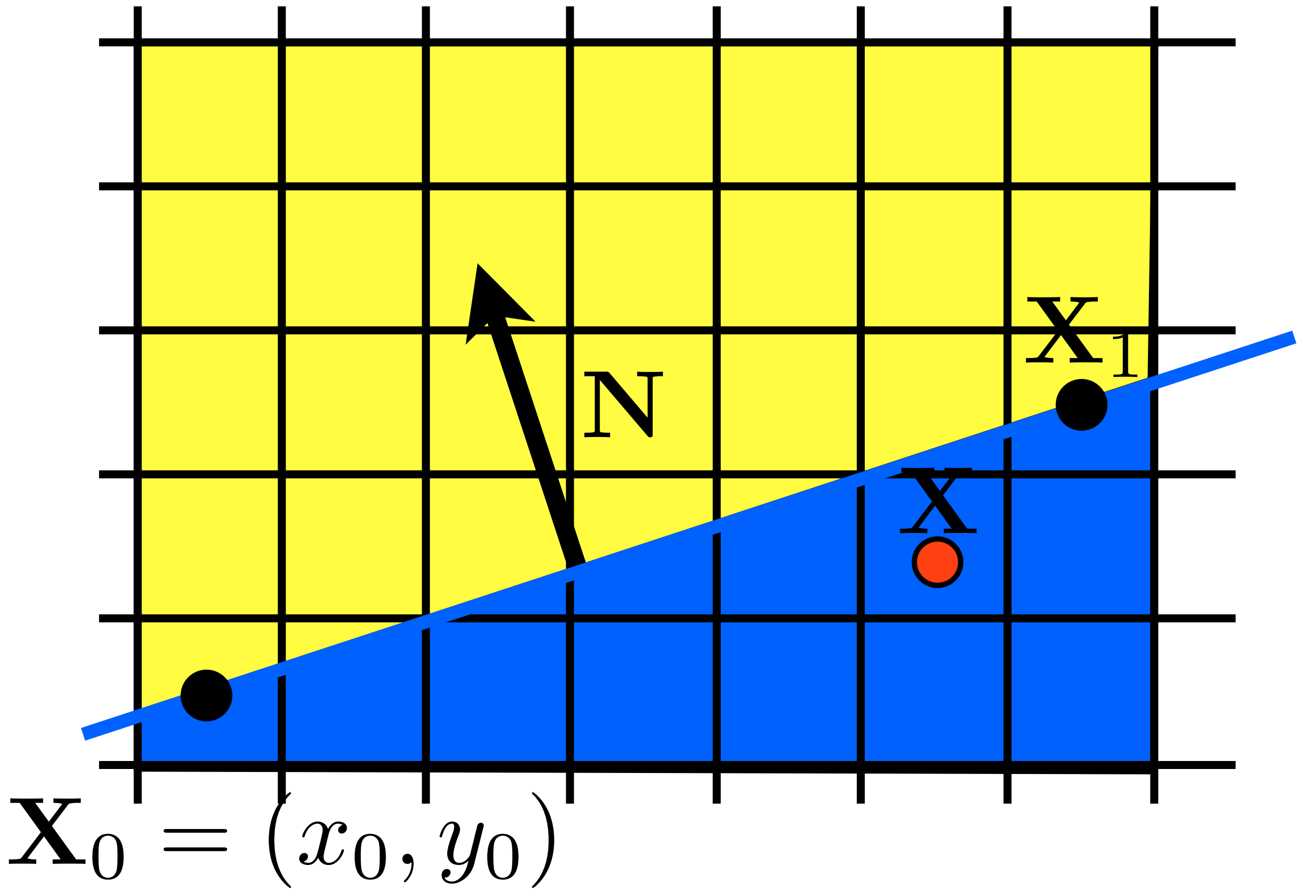
Implicit Line Equation



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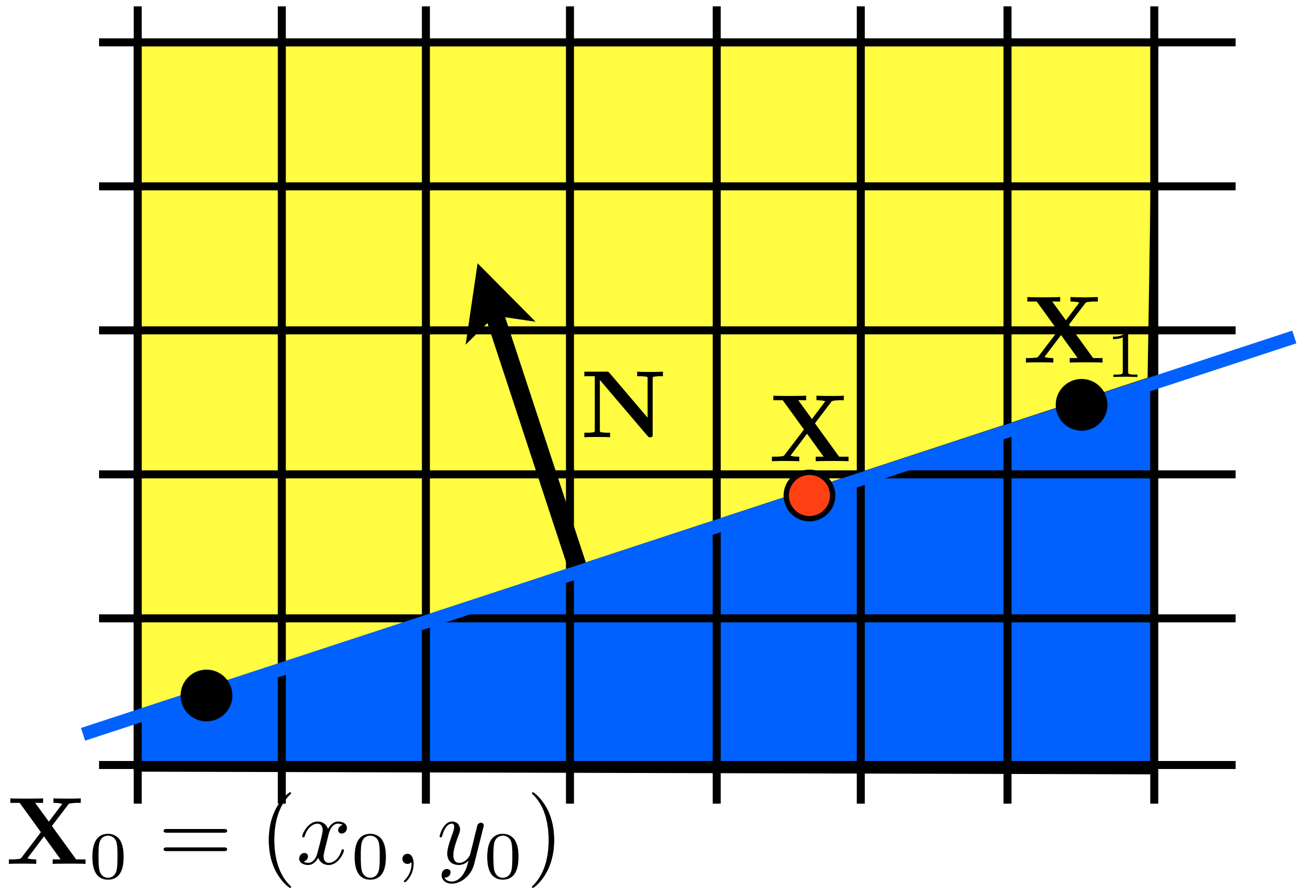
Implicit Line Equation



decision variable, d

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = d$$

$$d > 0$$

$$d < 0$$

$$\boxed{d = 0}$$

Line Drawing

DDA algorithm for lines

Parametric Lines: the DDA algorithm
(digital differential analyzer)

$Y_{i+1} = m\, x_{i+1} + B$

$= m(x_i + \Delta x) + B \qquad \Delta x = (x_{i+1} - x_i)$

$= y_i + m(\Delta x)$ **<- must round to find int**

If we increment by 1 pixel in X, we turn on
[xi, Round(yi)] or same for Y if m > 1

Scan conversion for lines

DDA includes Round( ); and this is fairly slow

For Fast Lines, we want to do only integer math +,-

We do this using the **Midpoint Algorithm**

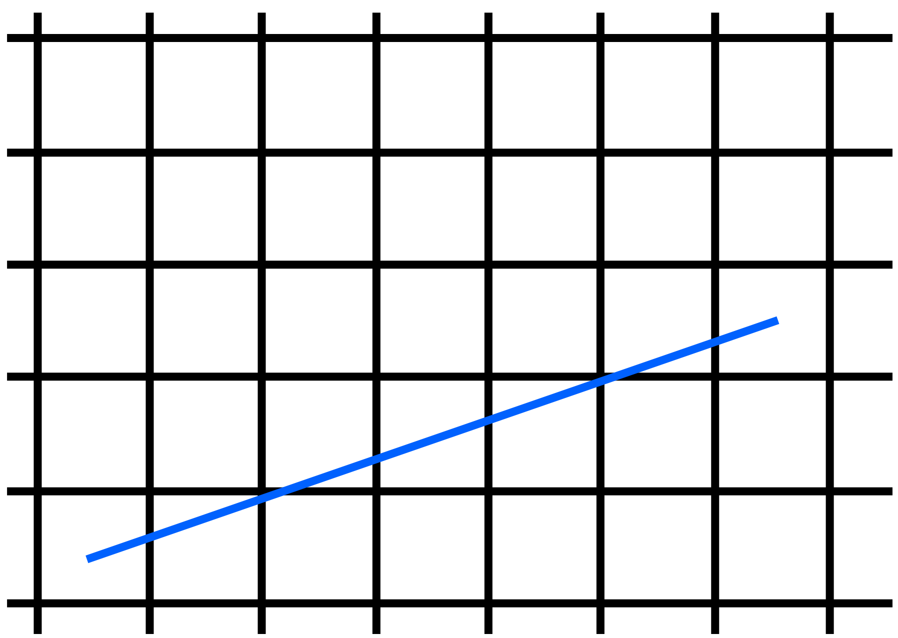
To do this, lets look at lines with y-intercept B and with slope between 0 and 1:

$$y = (dy/dx)x + B \quad \Longrightarrow$$

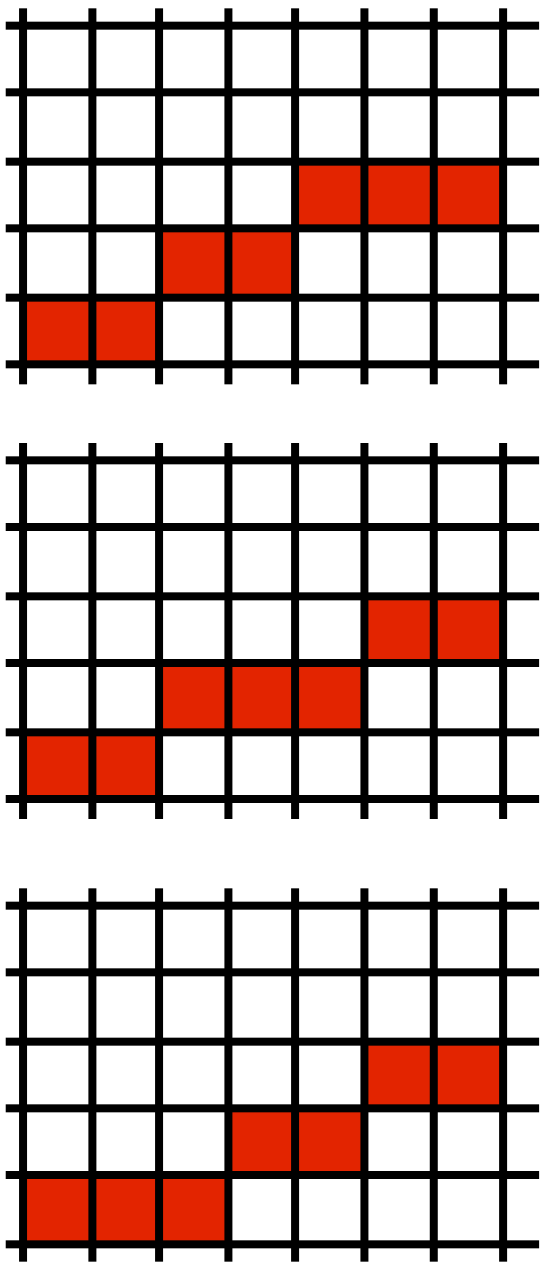
$$f(x,y) = (dy)x - (dx)y + B(dx) = 0$$

Removes the division => slope treated as 2 integers

Which pixels should be used to approximate a line?



Draw the thinnest possible line that has no gaps

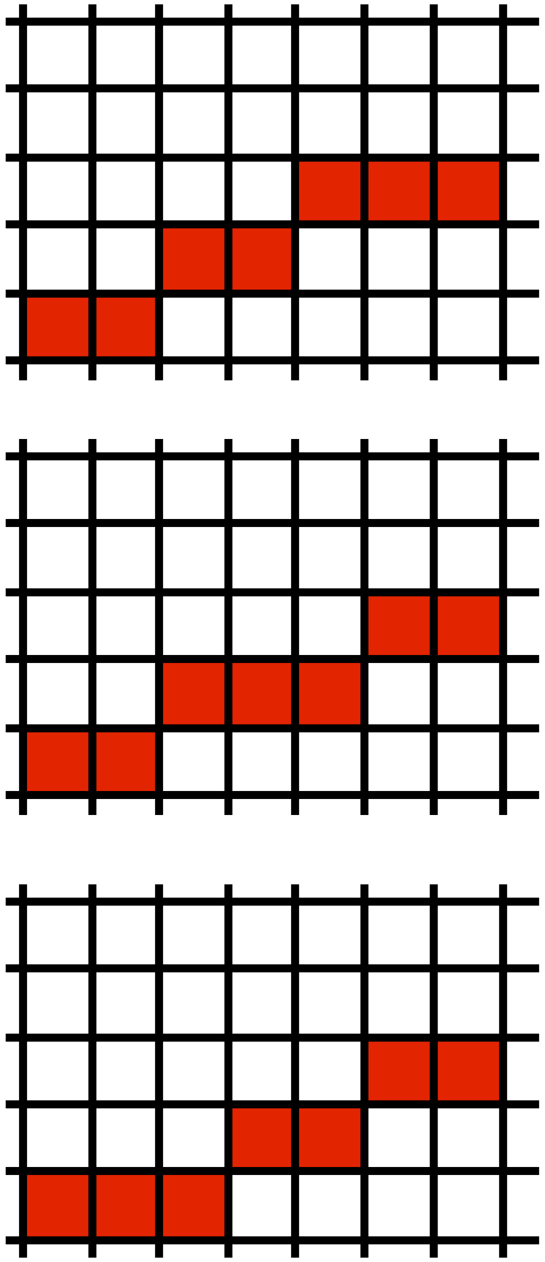


Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (<condition>) then
        y = y+1
```

- move from left to right
- choose between (x+1,y) and (x+1,y+1)



Line drawing algorithm

(case: $0 < m <= 1$)

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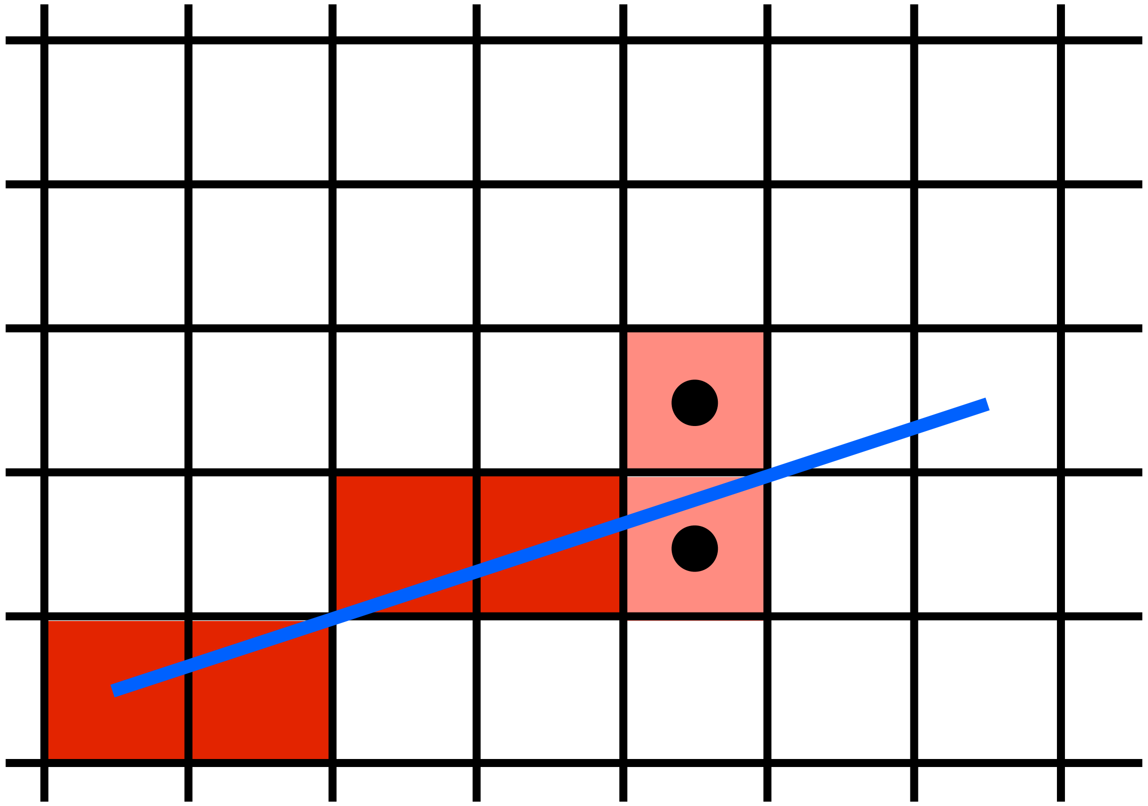
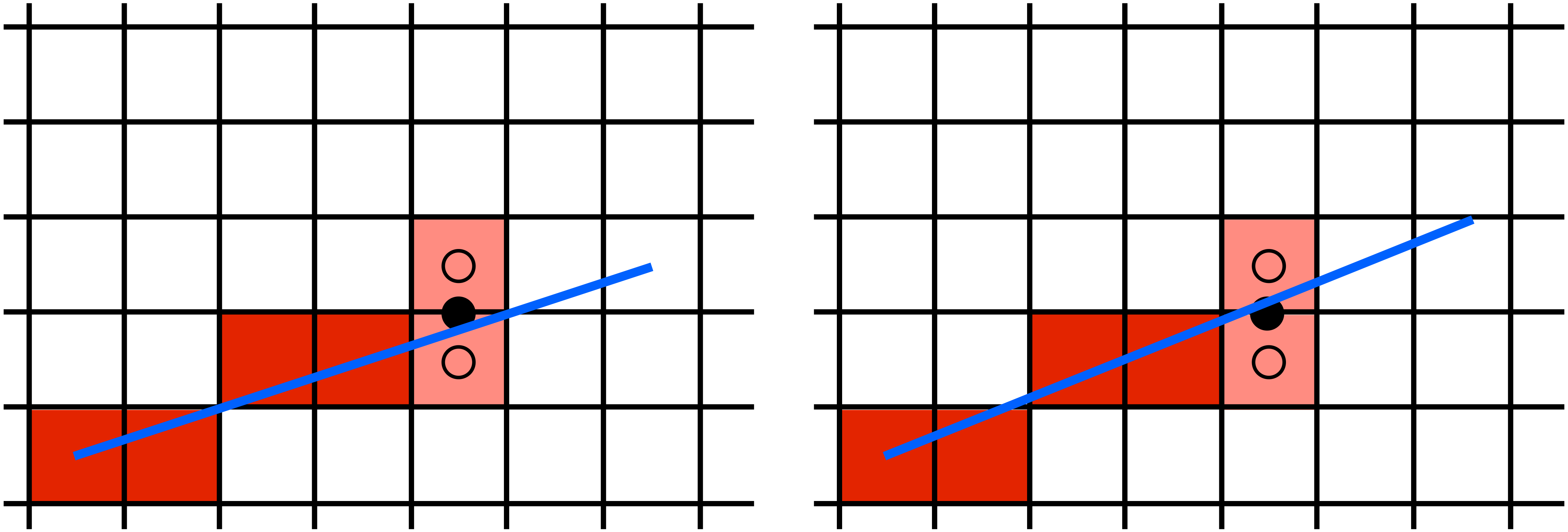
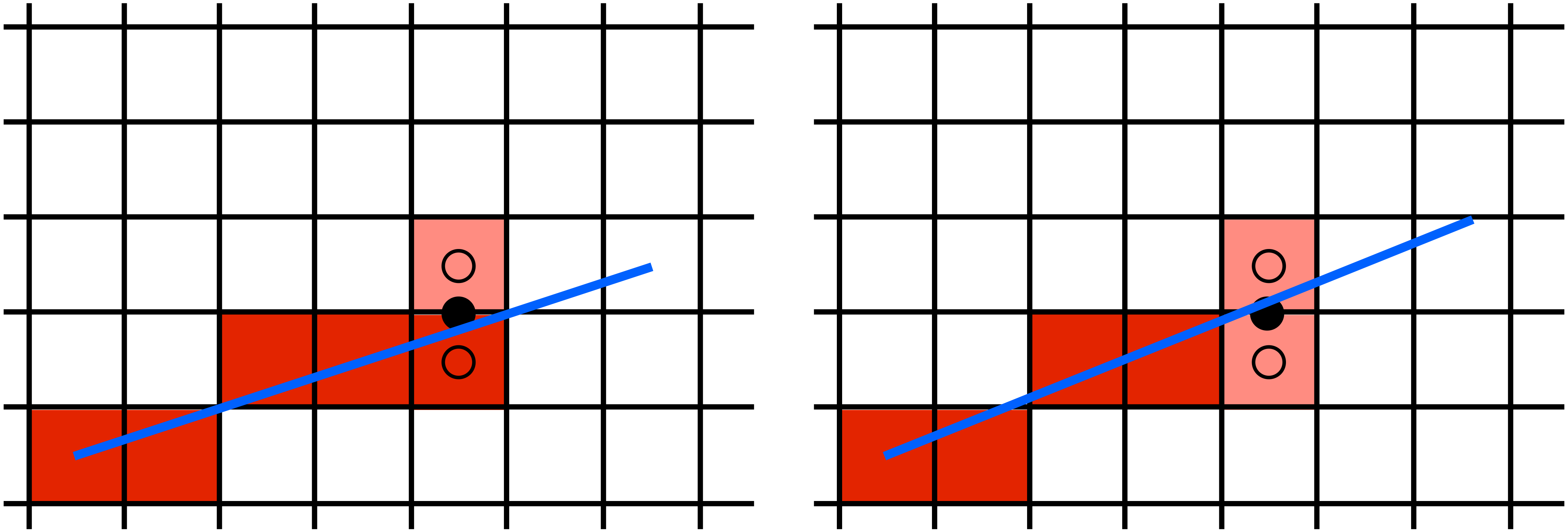


- move from left to right
- choose between $(x+1,y)$ and $(x+1,y+1)$

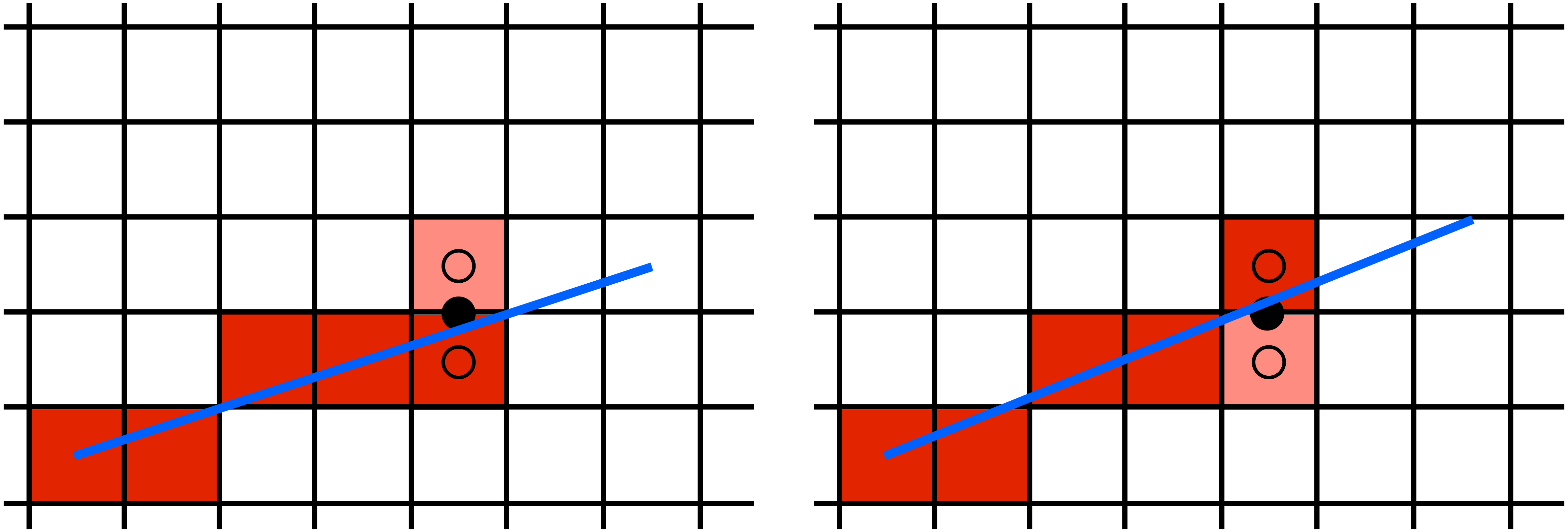
Use the midpoint between the two pixels to choose



Use the midpoint between the two pixels to choose



Use the midpoint between the two pixels to choose



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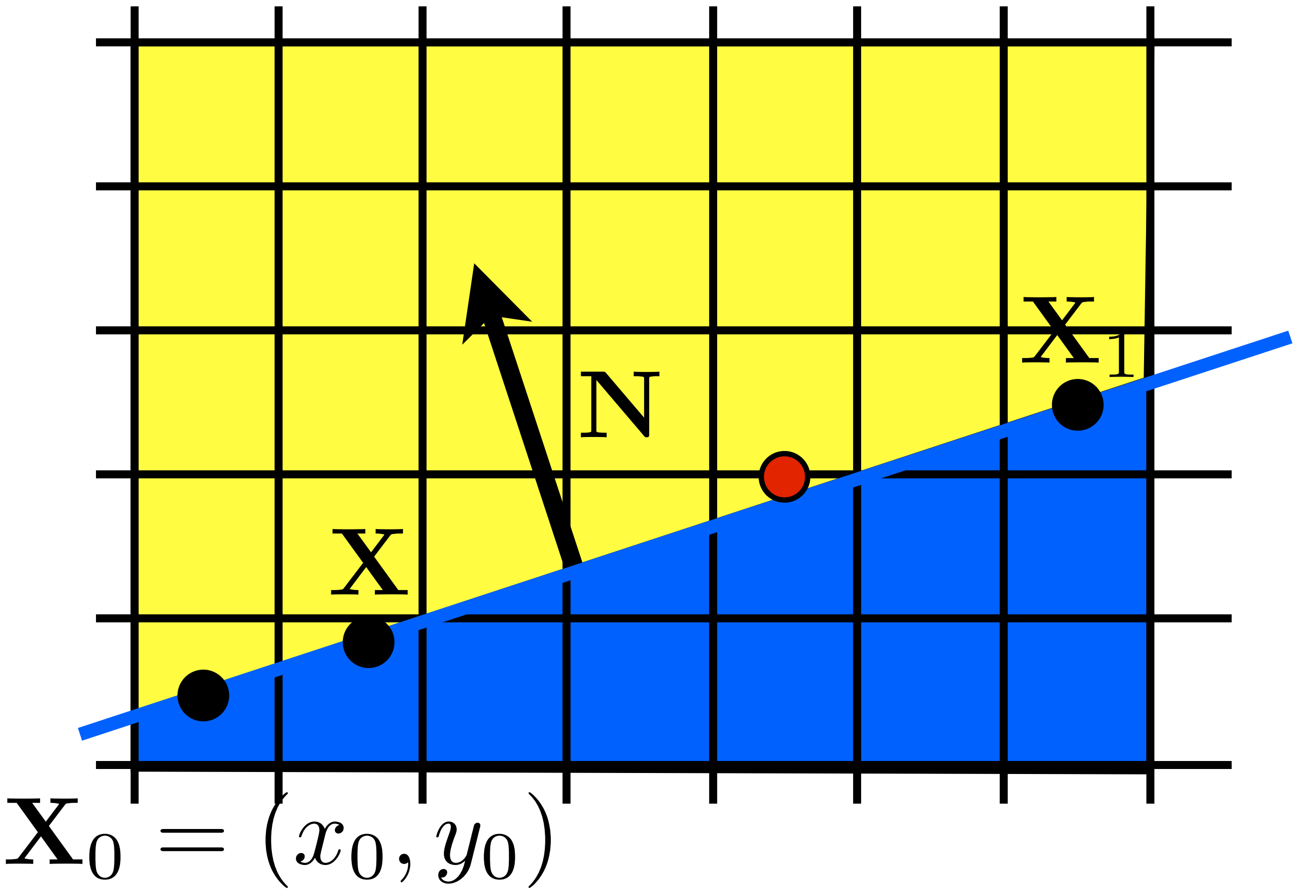

implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

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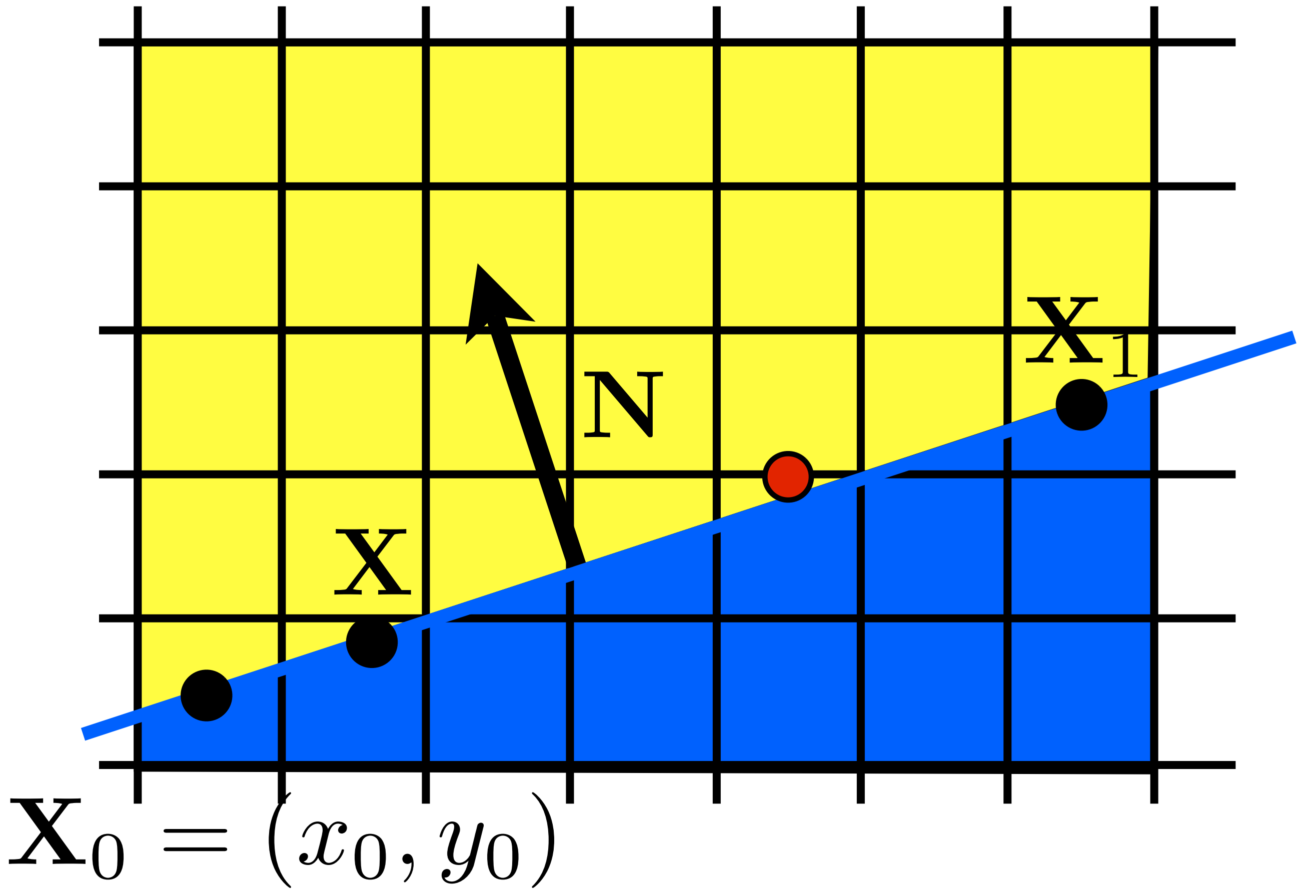
evaluate f at midpoint:

$$f(x, y + \frac{1}{2}) \ ? \ 0$$

Use the midpoint between the two pixels to choose



implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

evaluate f at midpoint:

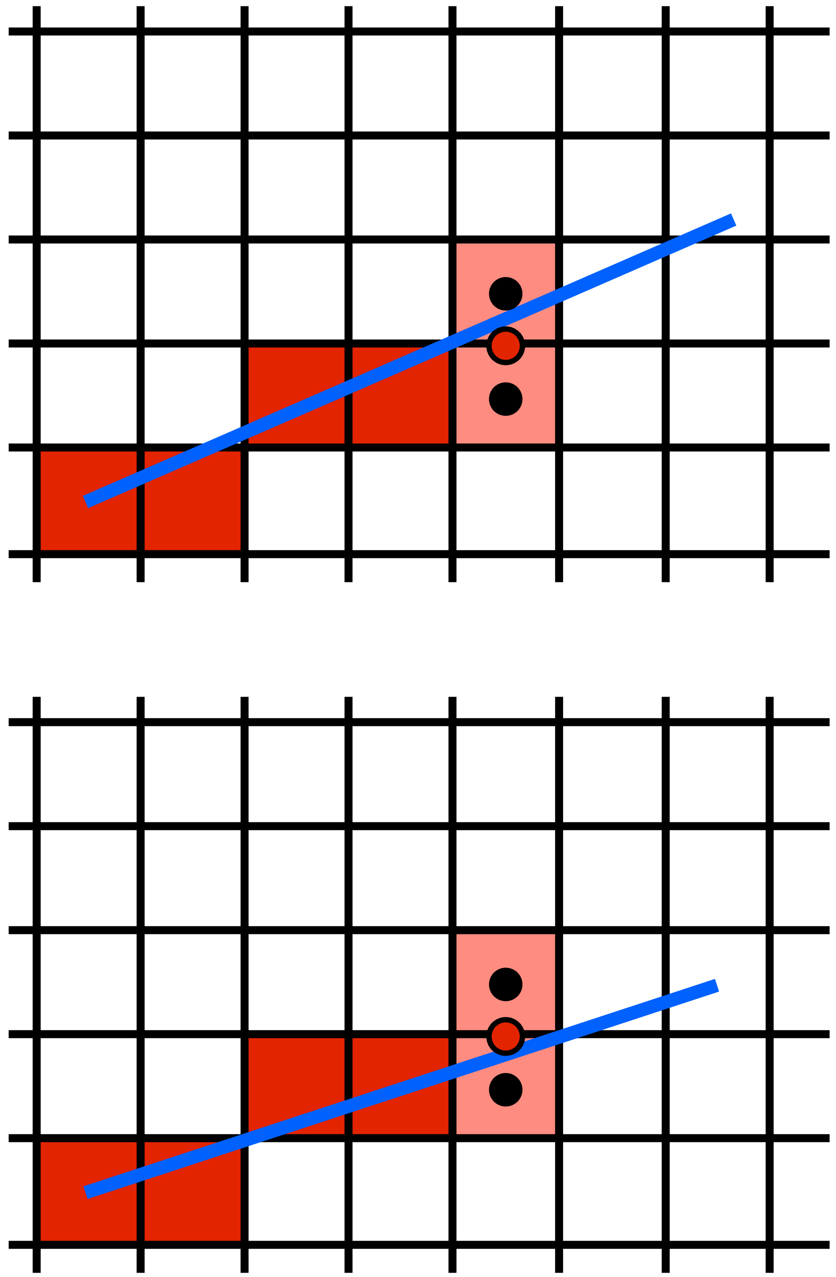
$$f(x, y + \frac{1}{2}) \; > \; 0$$

Line drawing algorithm

(case: 0 < m <= 1)

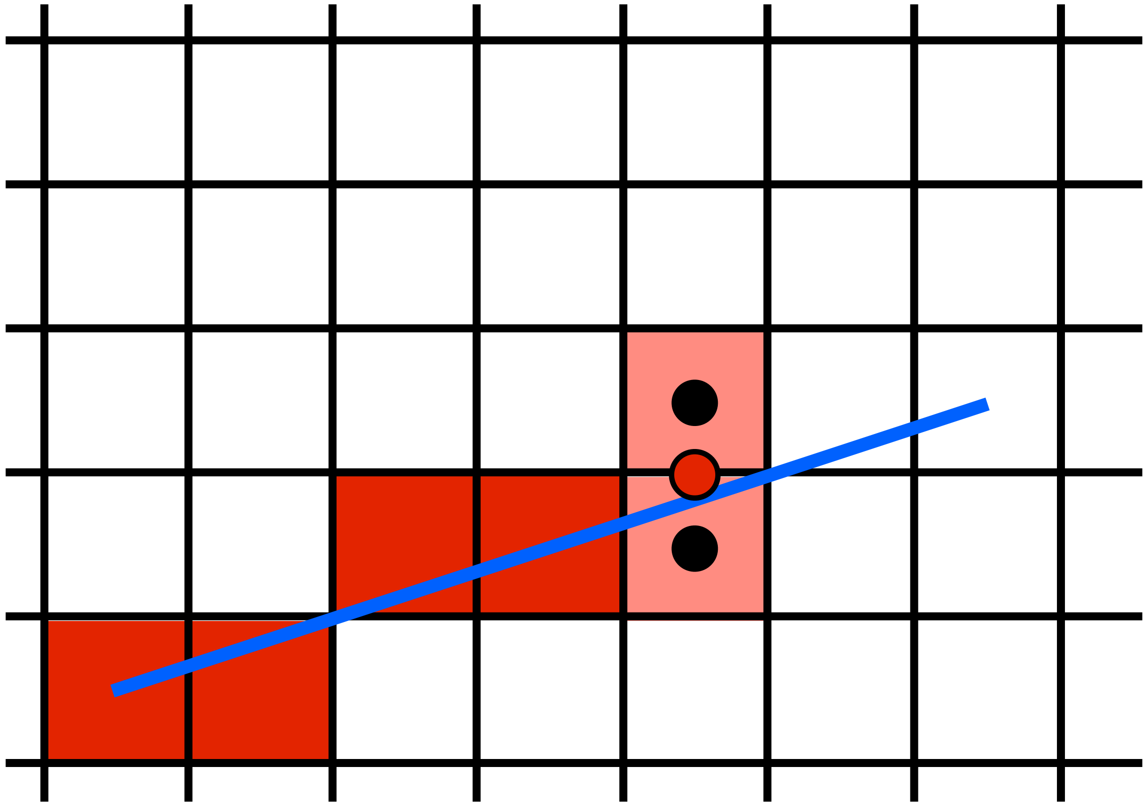
```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (f(x+1, y+1/2) < 0) then
        y = y+1
```

$$\text{if } \left(f(x+1, y+\tfrac{1}{2}) < 0 \right) \text{ then}$$



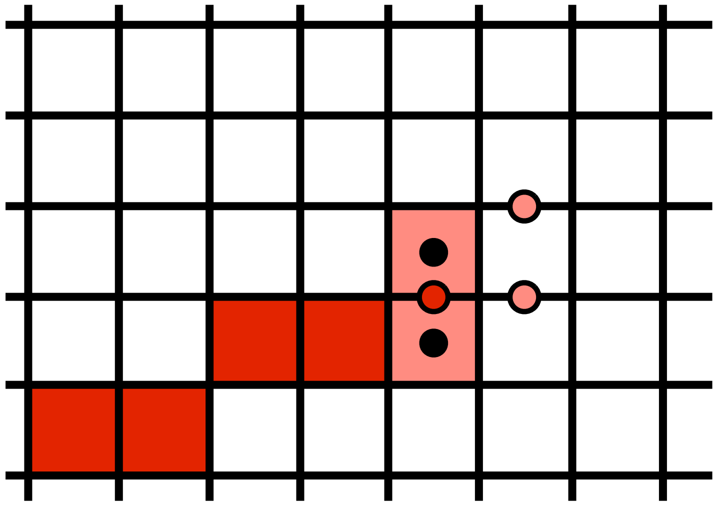
We can make the Midpoint Algorithm more efficient

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if ( f(x+1, y+1/2) < 0 ) then
        y = y+1
```



We can make the Midpoint Algorithm more efficient

by making it incremental!



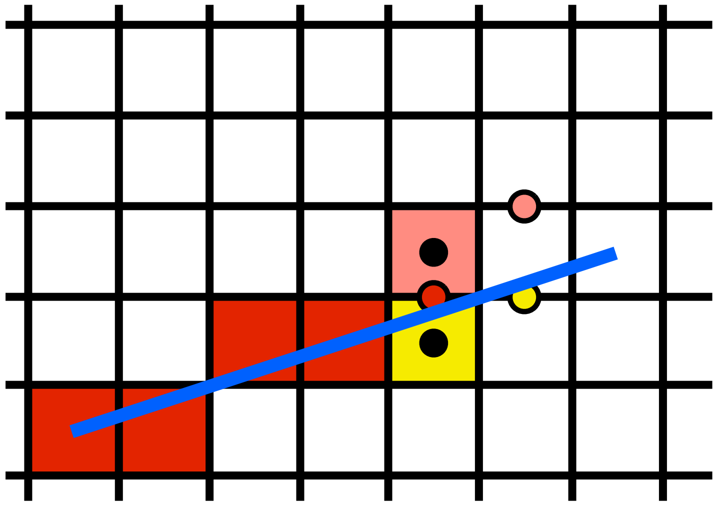
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) > 0$$



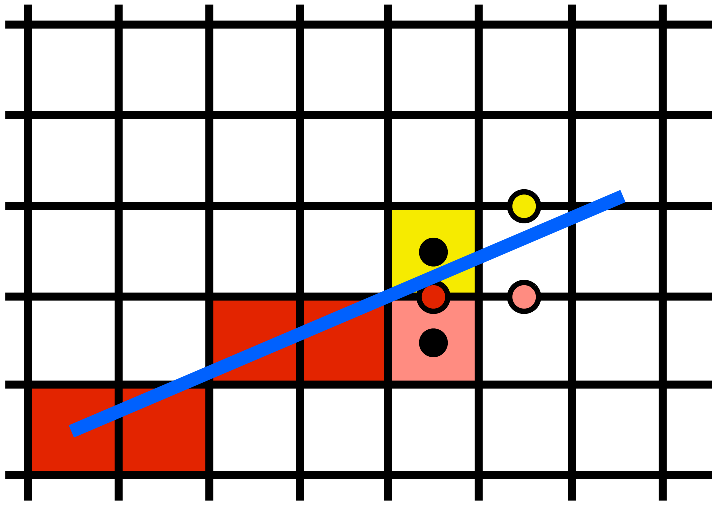
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0 = 0$$

$$\boxed{f(x+1, y) = f(x,y) + (y_0 - y_1)}$$

$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) < 0$$



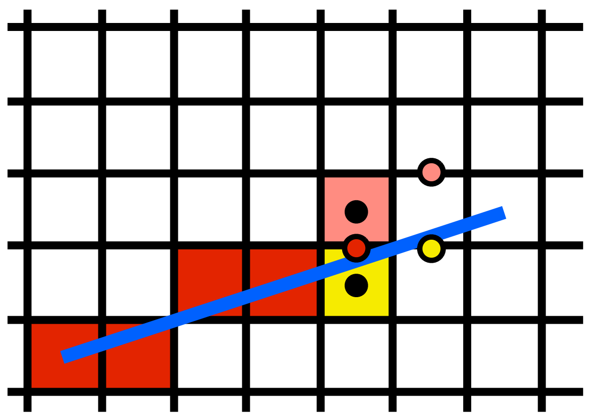
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

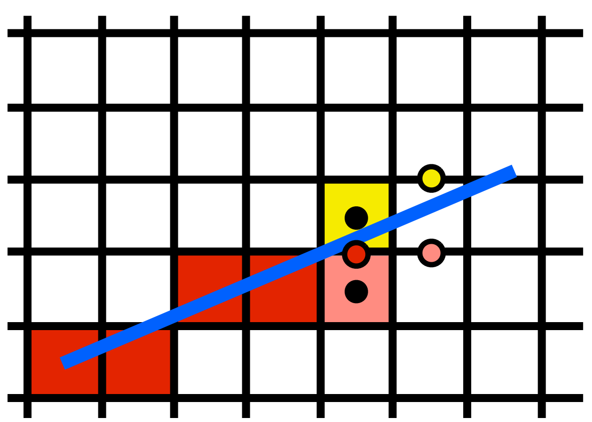
$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

```
y = y0
d = f(x0+1,y0+1/2)
for x = x0 to x1 do
    draw(x,y)
    if (d<0) then
        y = y+1
        d = d+(y0-y1)+(x1-x0)
    else
        d = d+(y0-y1)
```

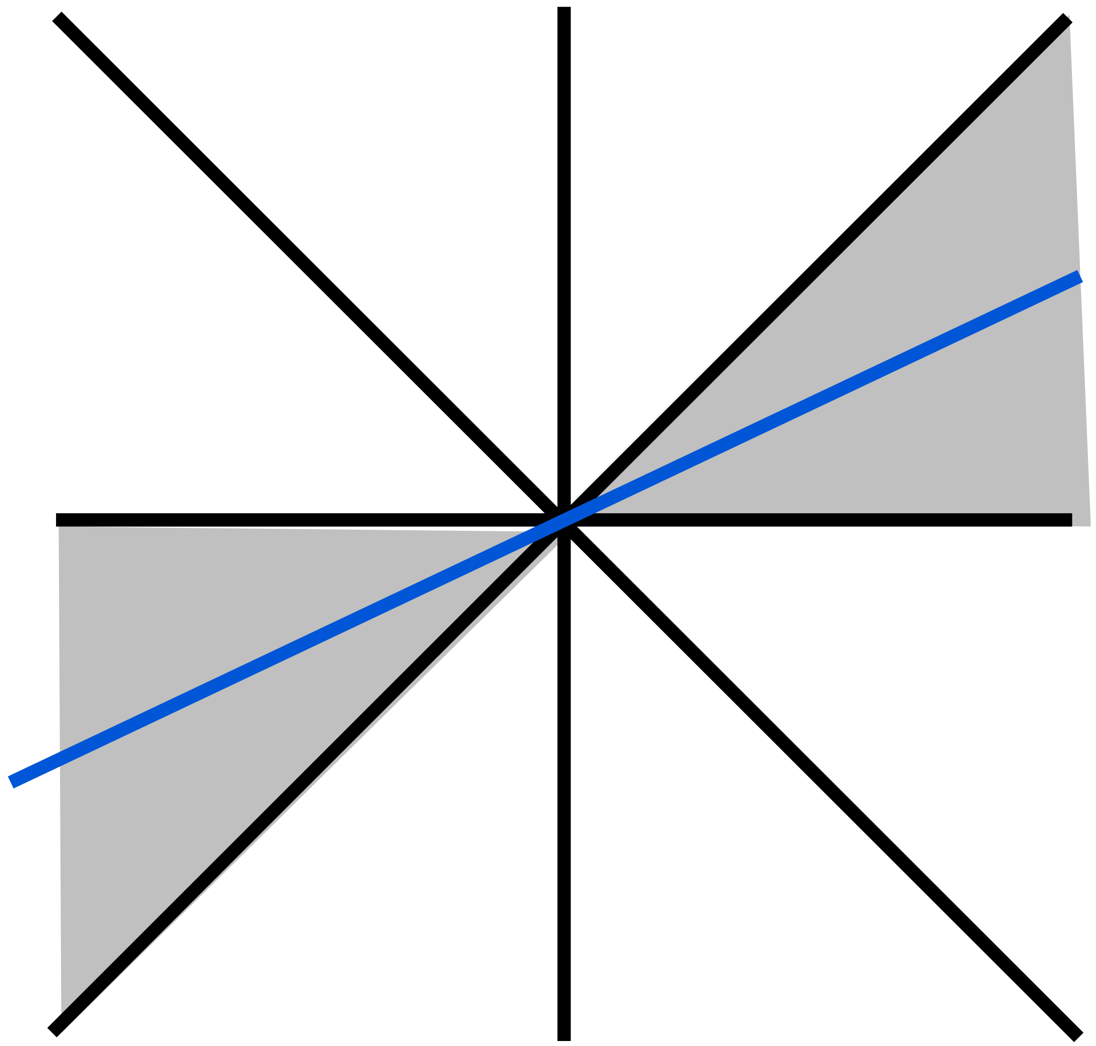


$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$

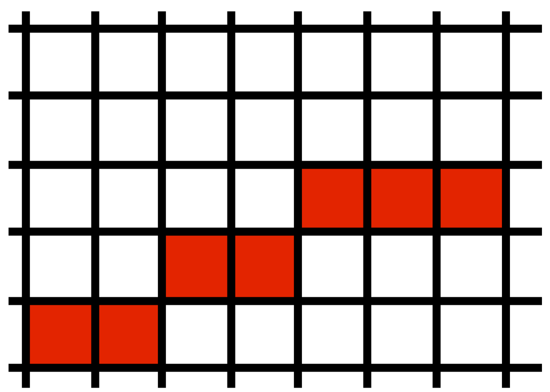


$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

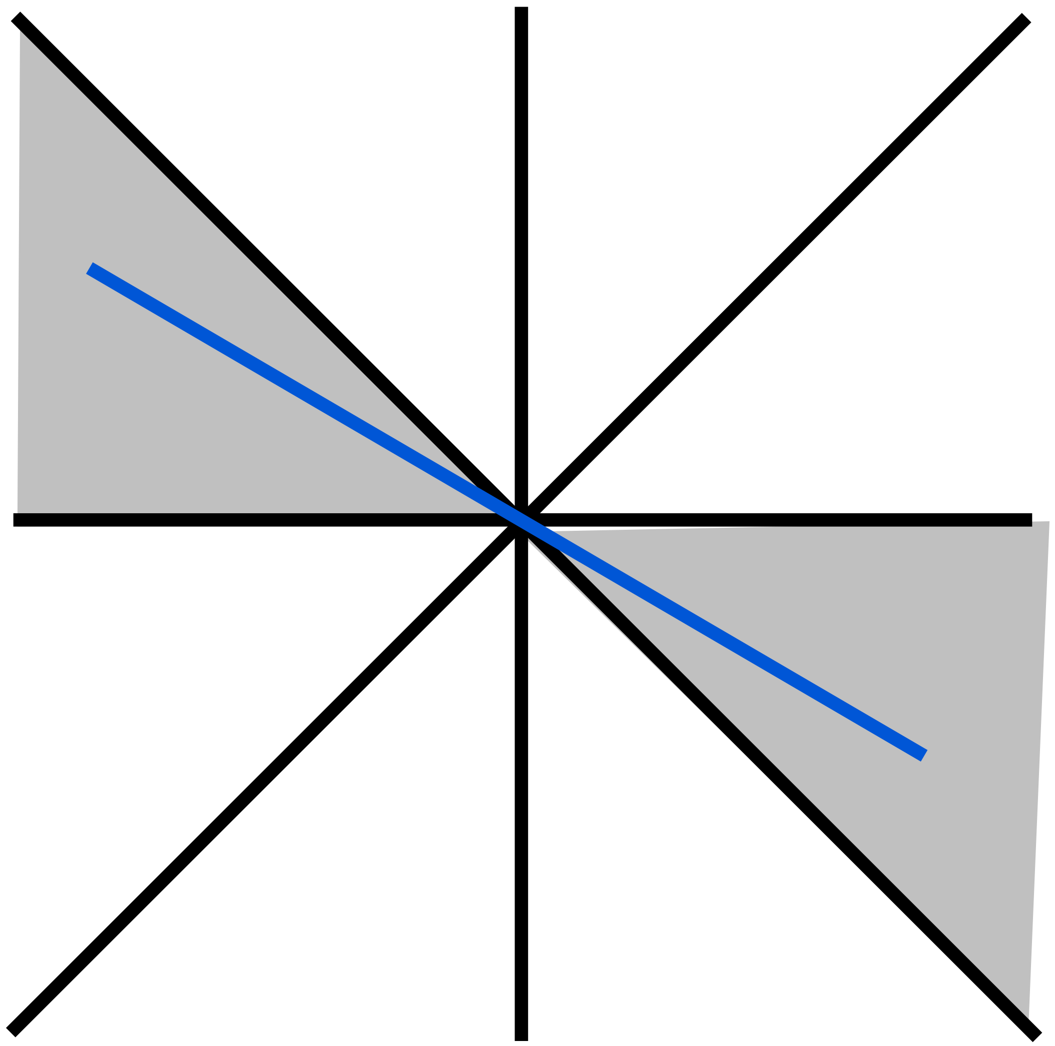
Adapt Midpoint Algorithm for other cases



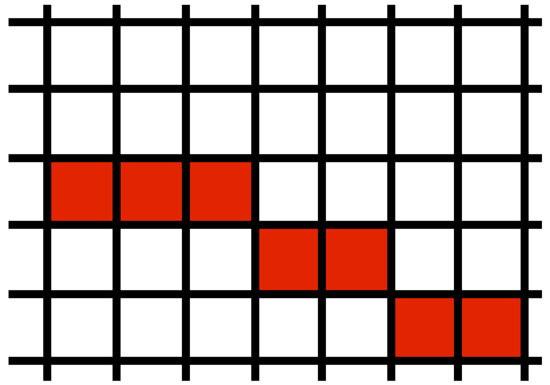
case: $0 < m <= 1$



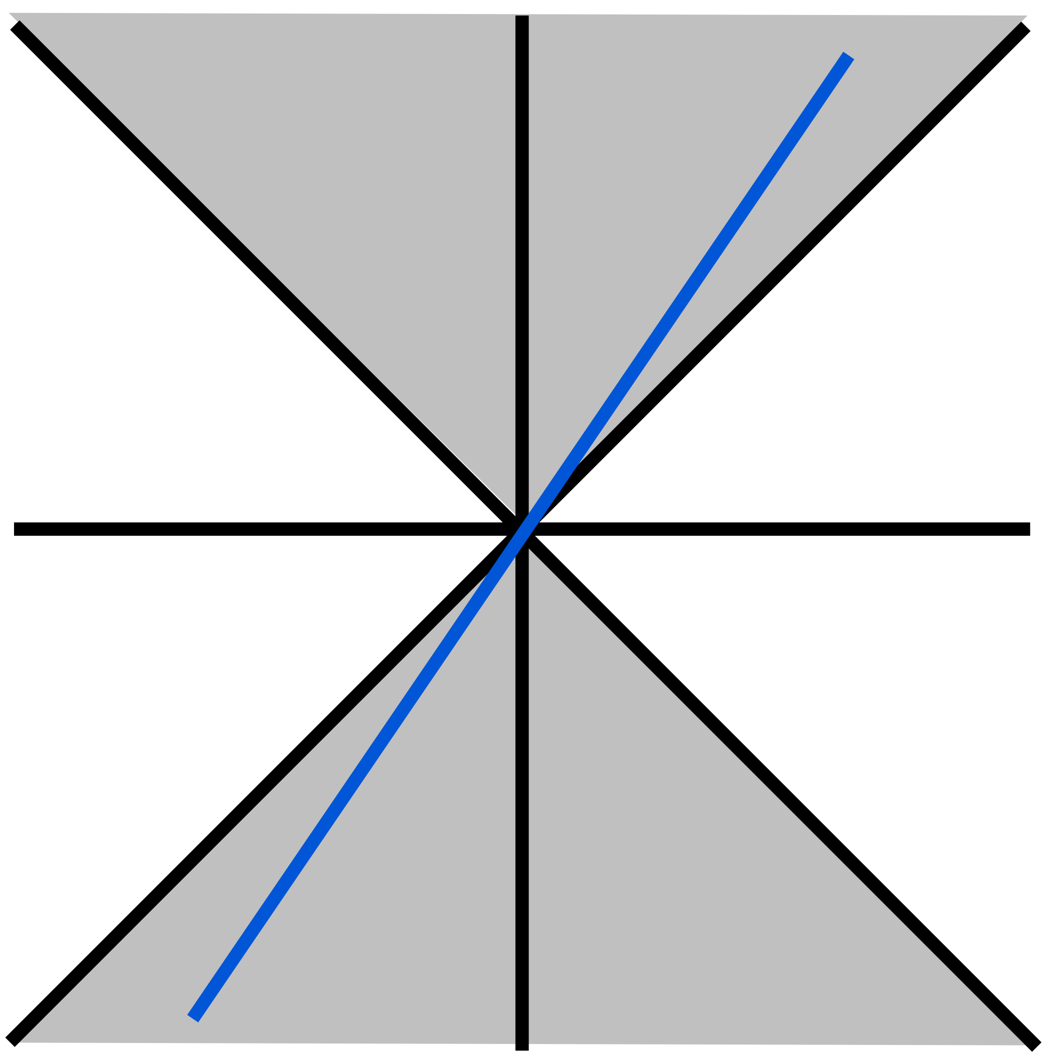
Adapt Midpoint Algorithm for other cases



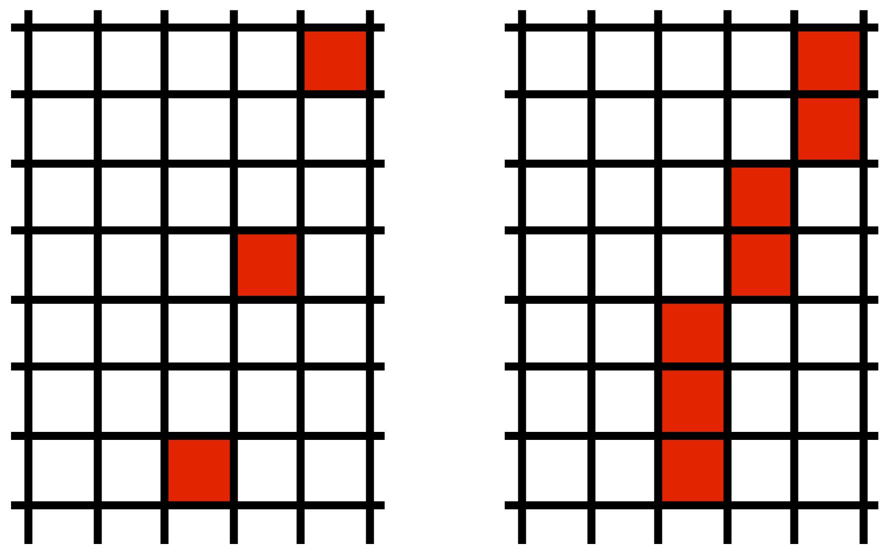
case: -1 <= m < 0



Adapt Midpoint Algorithm for other cases



case: 1 <= m
or m <= -1

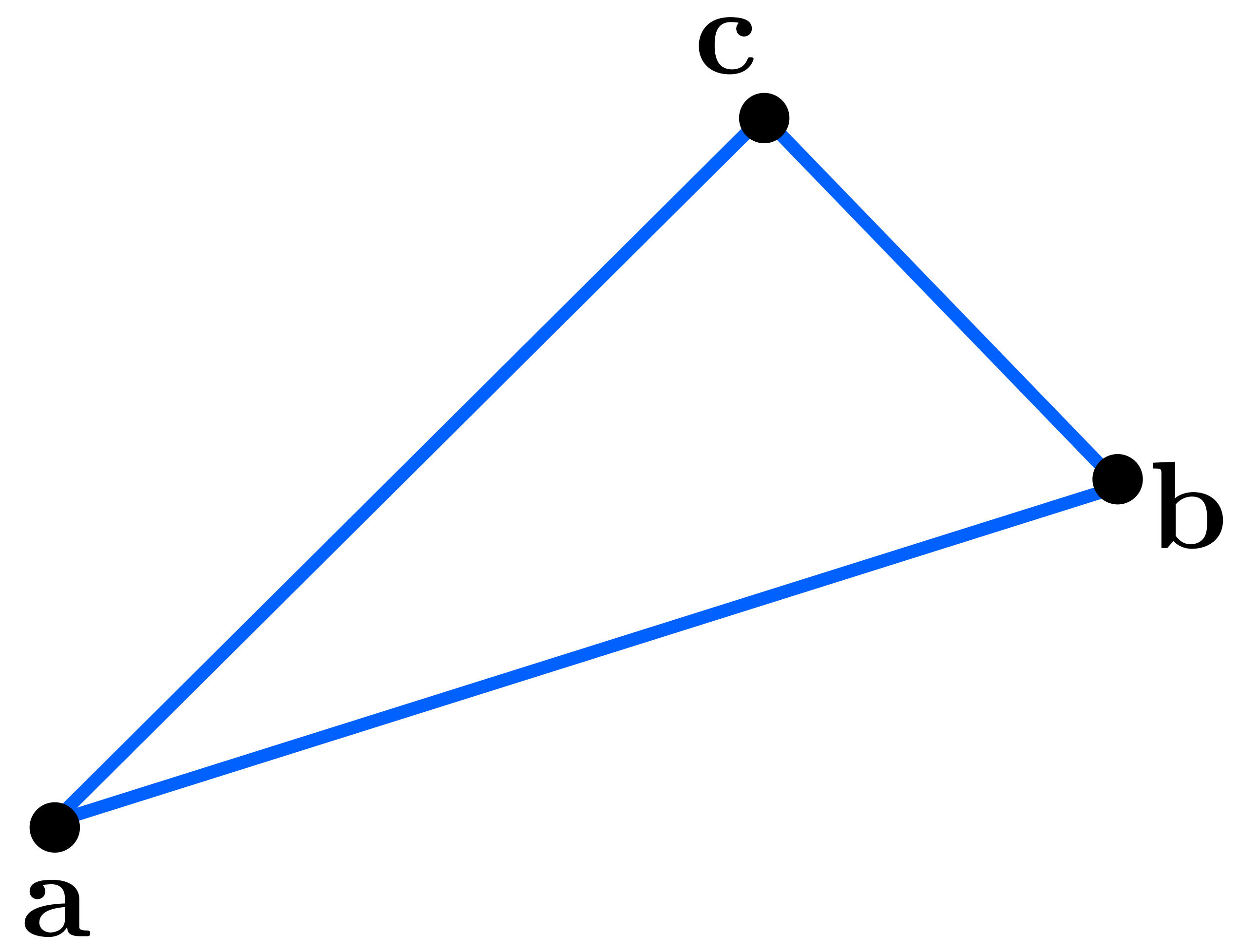


Line drawing references

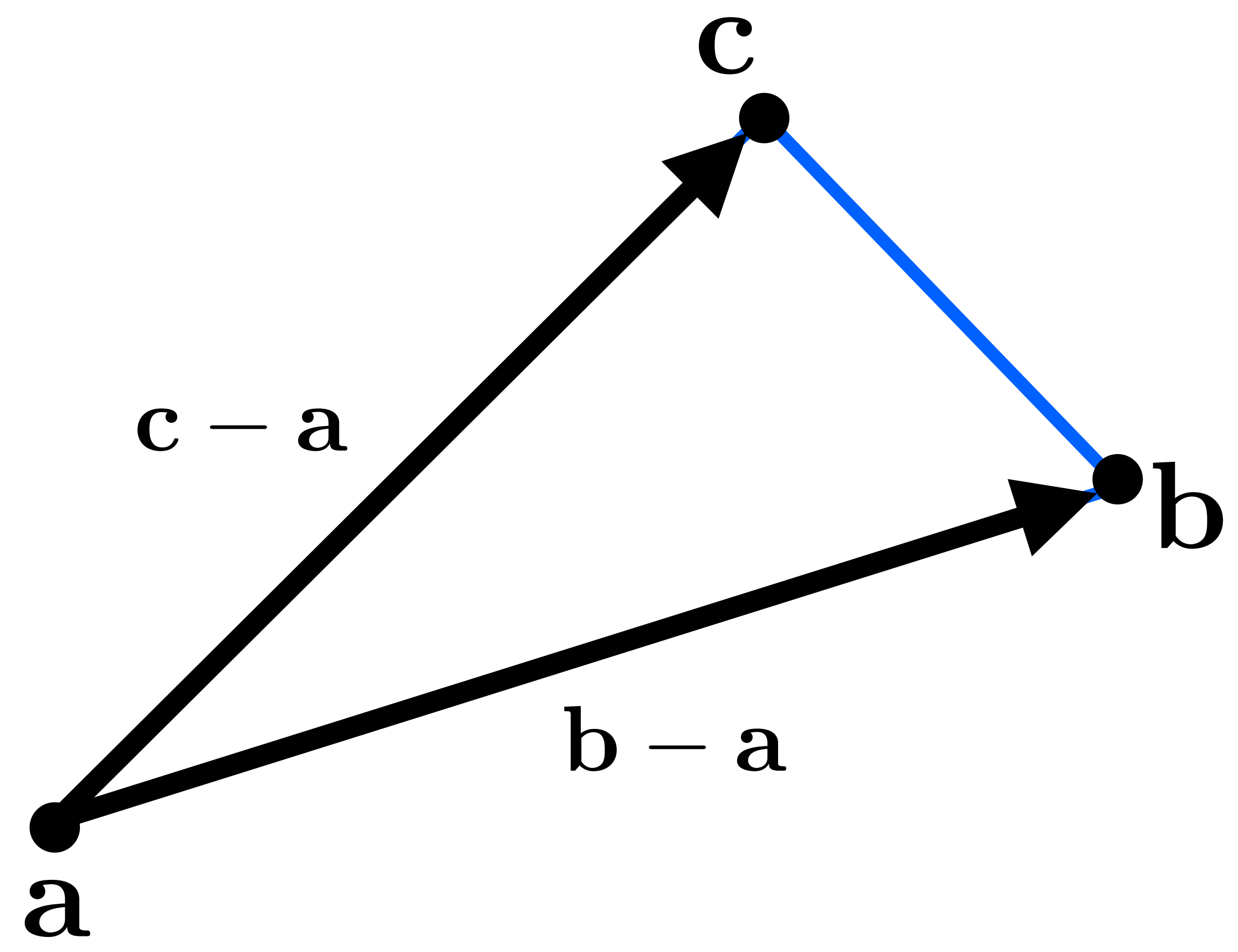
- the algorithm we just described is the *Midpoint Algorithm* (Pitteway, 1967), (van Aken and Novak, 1985)
- draws the same lines as the *Bresenham Line Algorithm* (Bresenham, 1965)

Triangles

barycentric coordinates



barycentric coordinates



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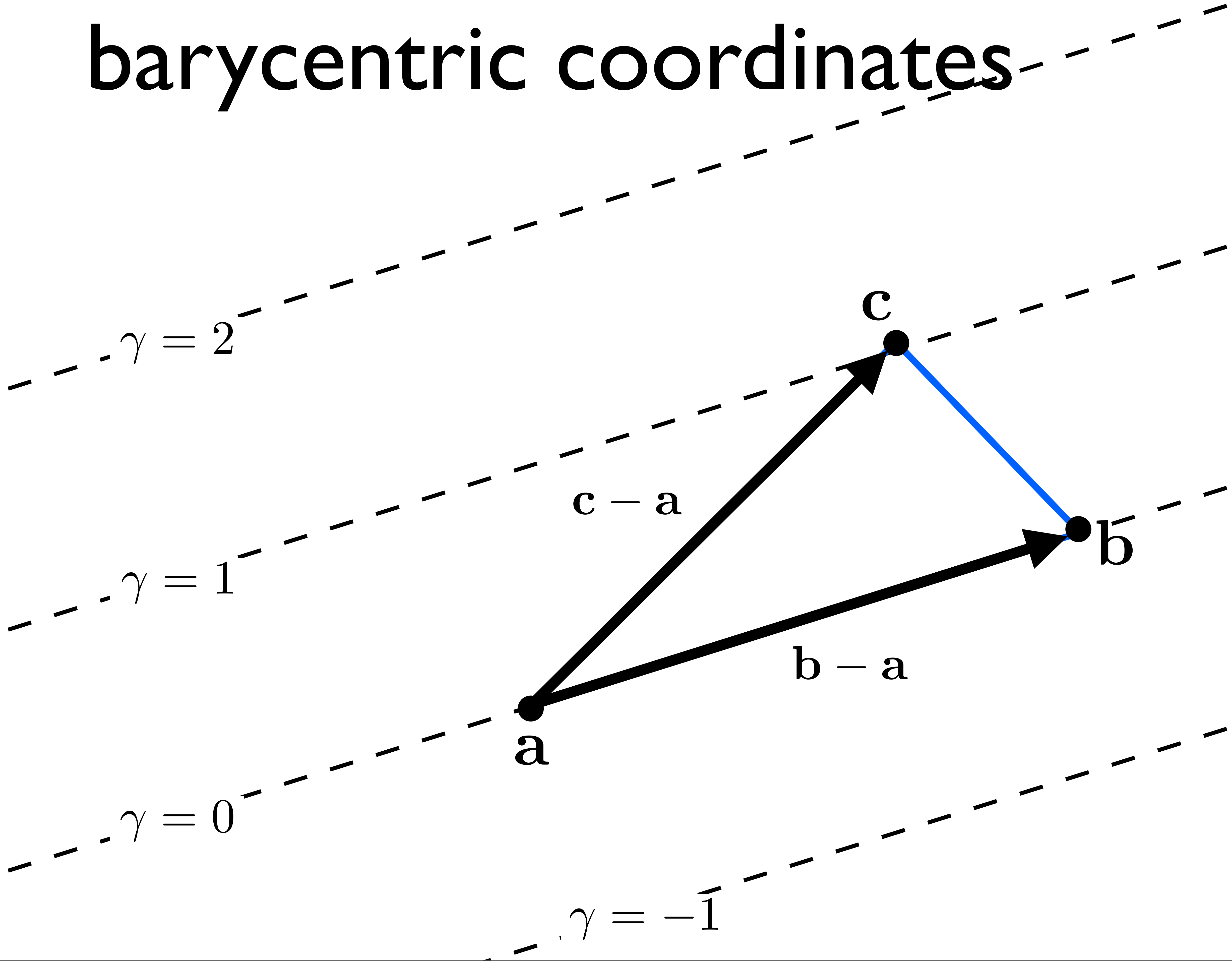

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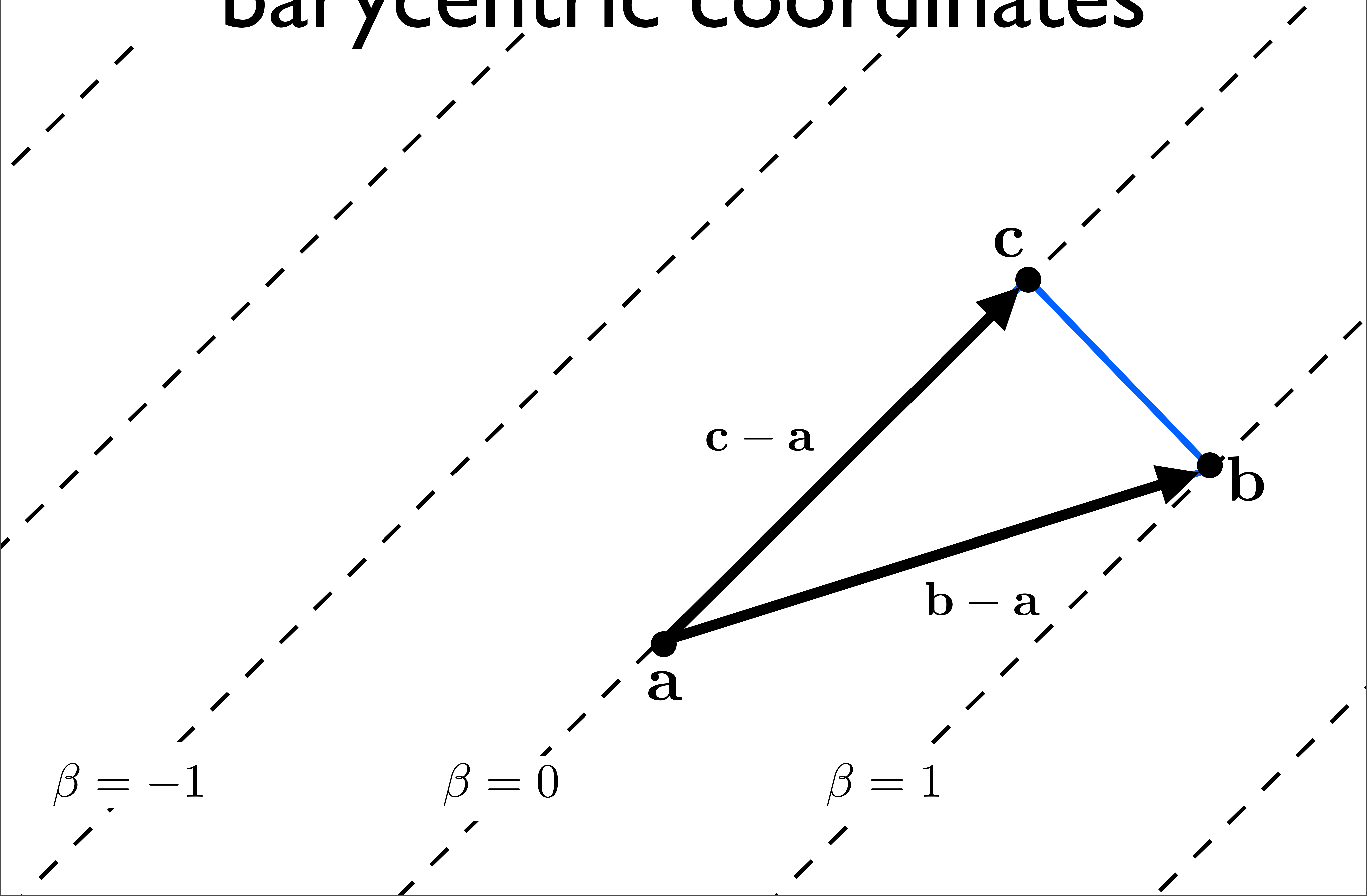
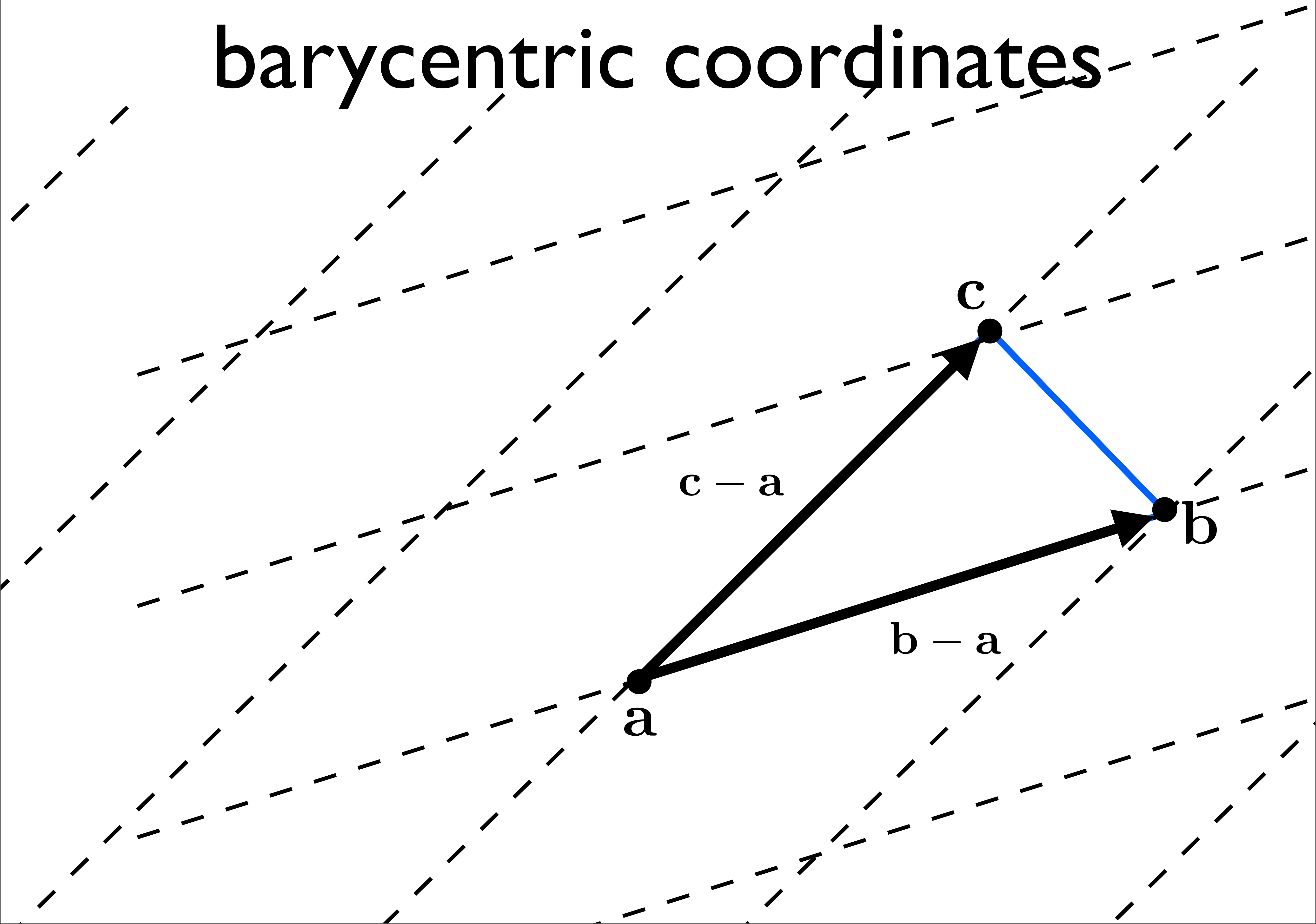
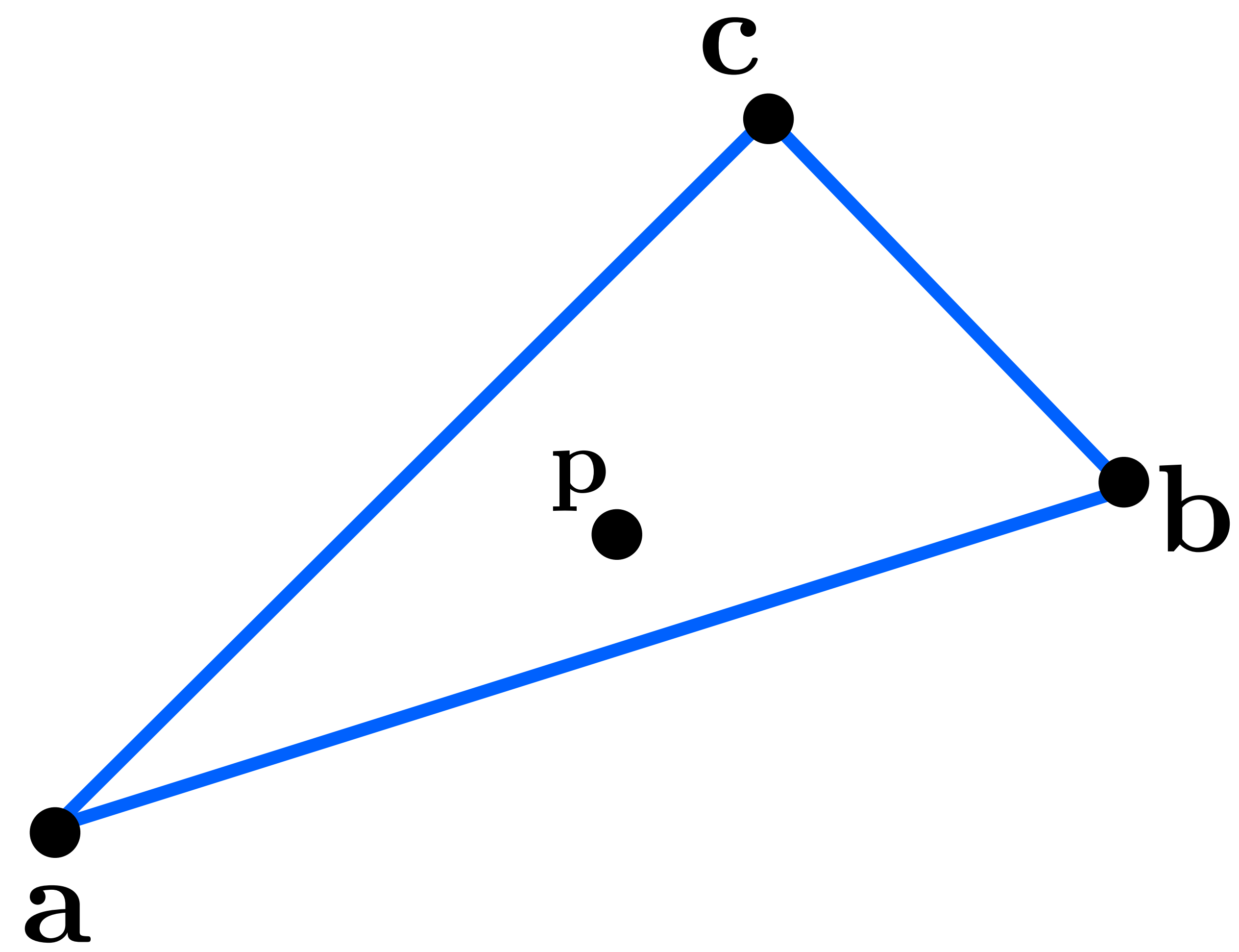


barycentric coordinates



barycentric coordinates

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$



What are (α, β, γ) ?

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