CS130 : Computer Graphics

Lecture 15: Curves

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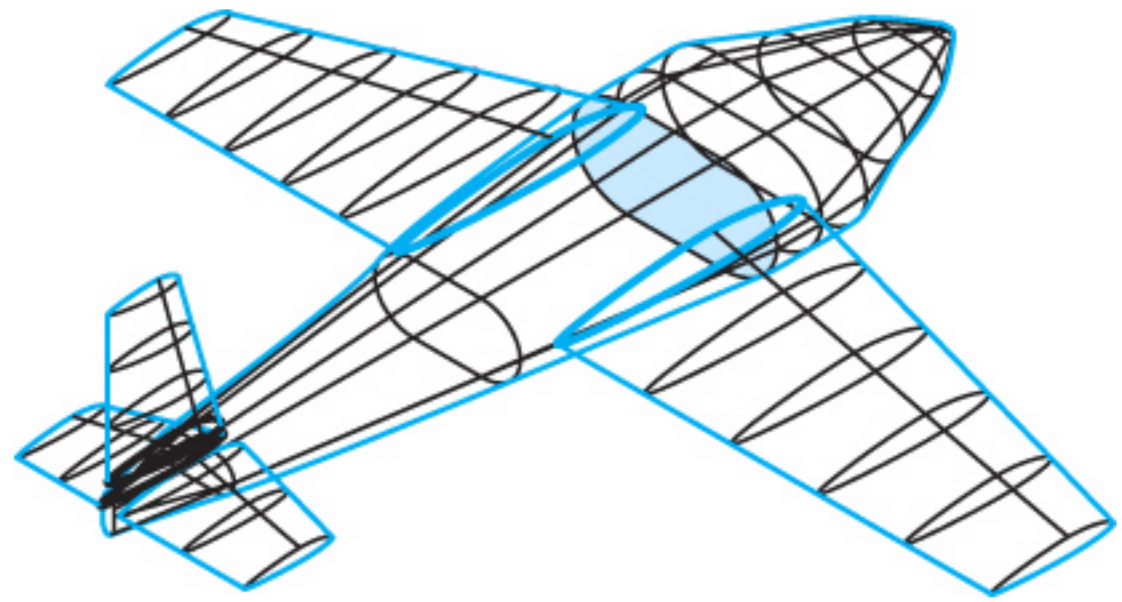
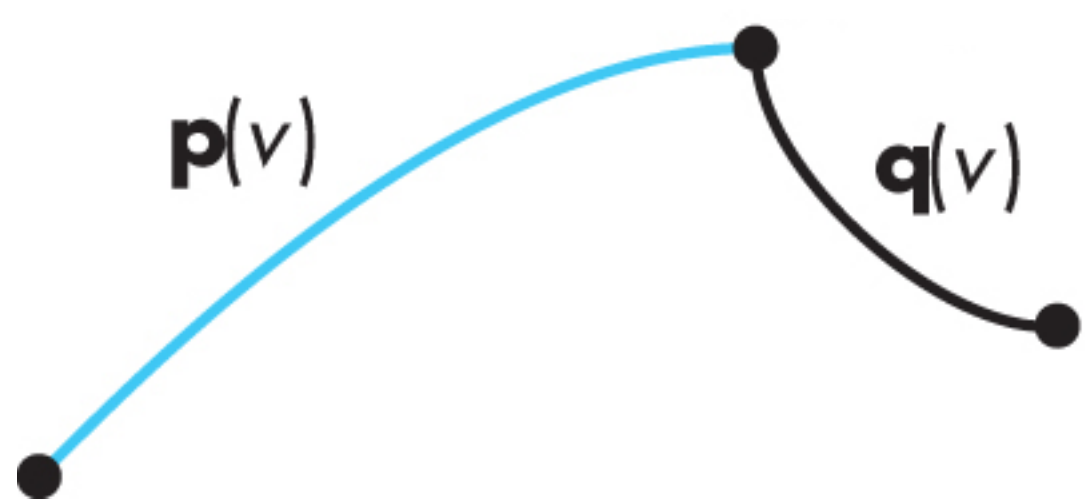
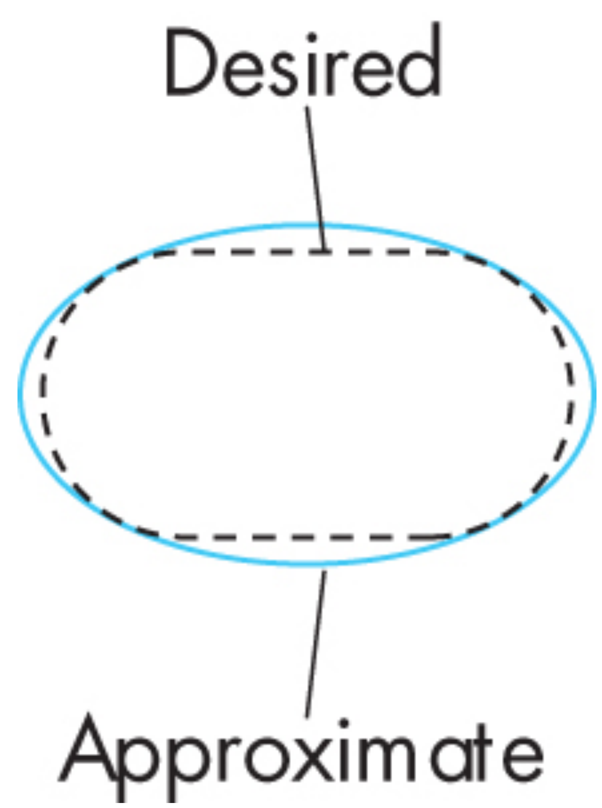
Design considerations

- local control of shape
- smoothness and continuity
- ability to evaluate derivatives
- stability
- ease of rendering



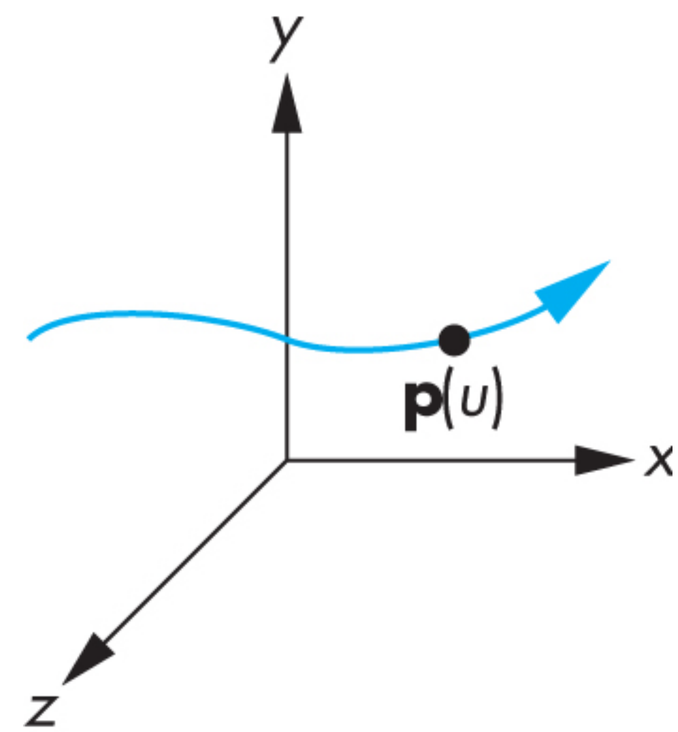


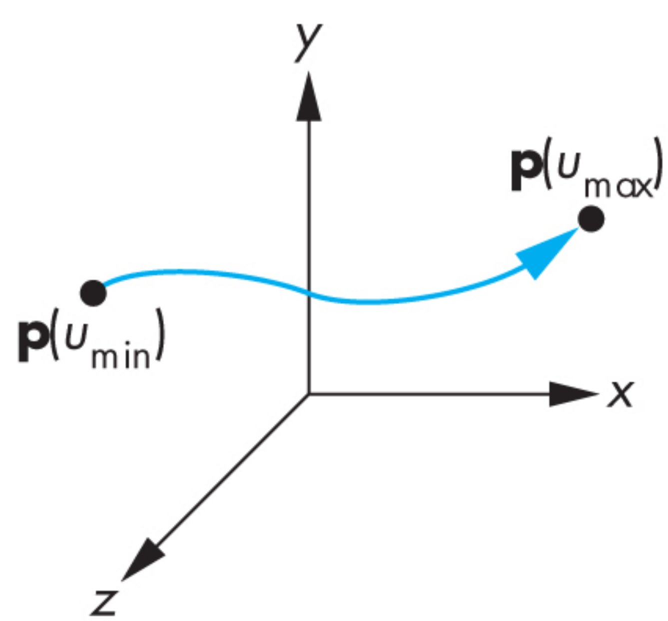


- local control - design each segment independently
- stability - small change in input values leads to small change in output

Parametric curve

$$x = x(u)$$
$$y = y(u)$$
$$z = z(u)$$

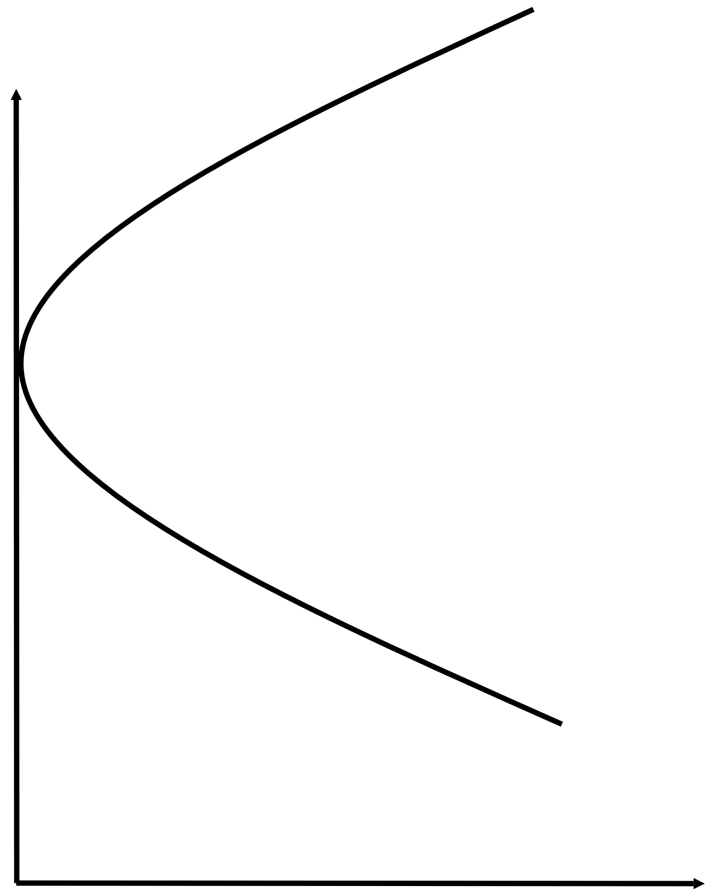




Curve segment

Parametric curve example

$$\mathbf{p}(u) = \begin{pmatrix} x(u) \\ y(u) \end{pmatrix} = \mathbf{c_0} + \mathbf{c_1} u + \mathbf{c_2} u^2$$



$$x(u) = 3u^2$$

$$y(u) = 2u + 3$$

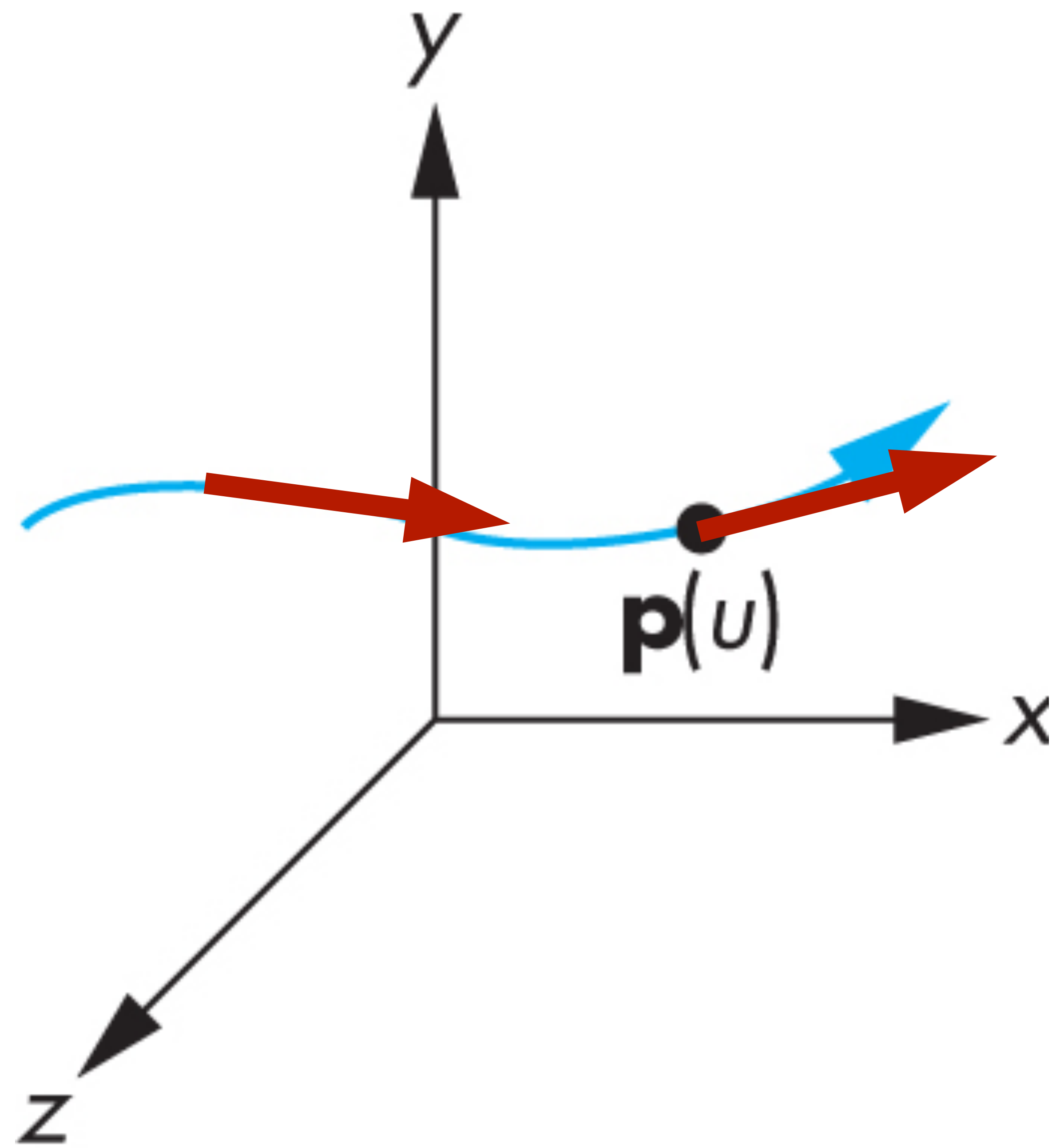
$$\mathbf{c_0} = ?, \quad \mathbf{c_1} = ?, \quad \mathbf{c_2} = ?$$

- this is a curve in 2D
- for a curve in 3D, we would also have z(u) = ...

Parametric curve - tangent vector

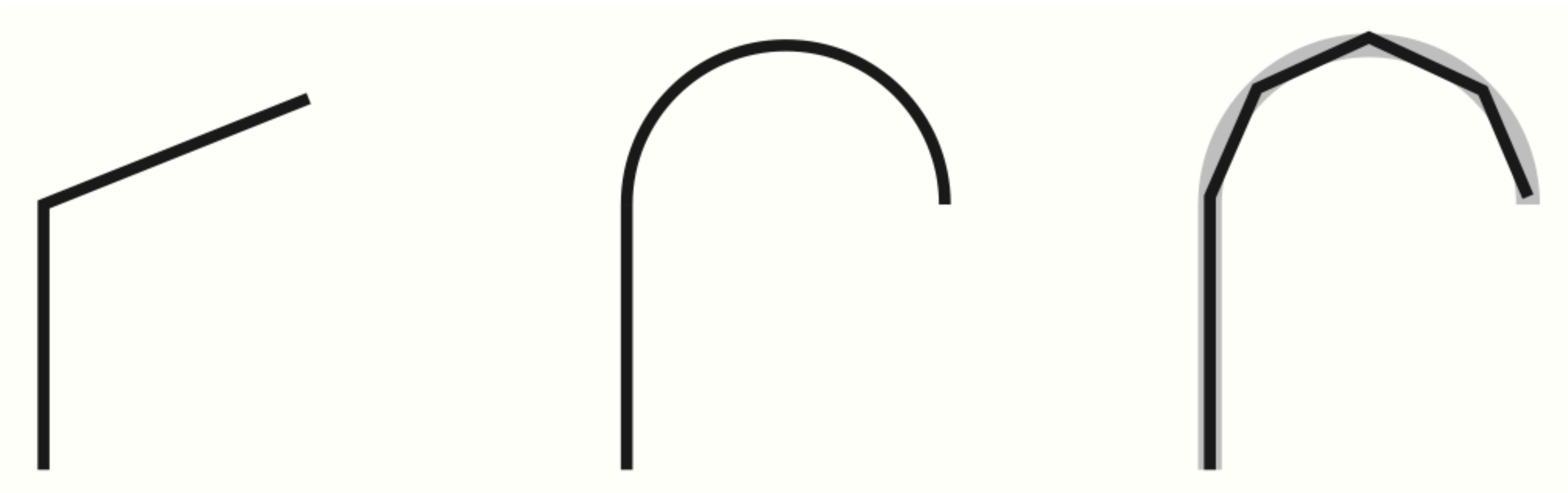
$$x = x(u)$$
$$y = y(u)$$
$$z = z(u)$$

$$\mathbf{t} = \begin{pmatrix} x'(u) \\ y'(u) \\ z'(u) \end{pmatrix}$$



– tangent vector

Piecewise Parametric Representations



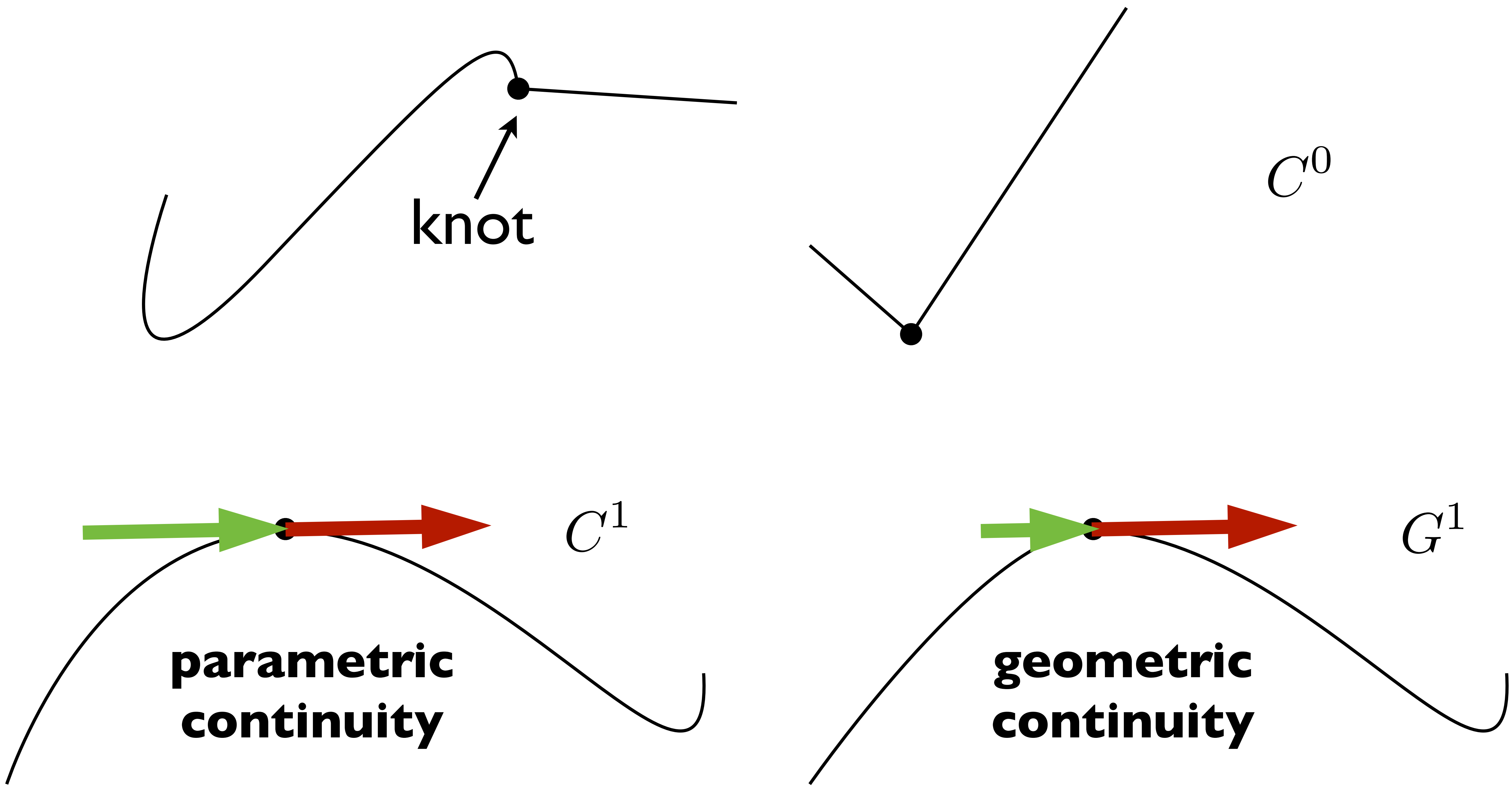
$$\mathbf{f}(u) = \begin{cases} \mathbf{f}_1(2u) & u \leq 0.5 \\ \mathbf{f}_2(2u-1) & u > 0.5 \end{cases}$$

continuity

$$\mathbf{f}_1(1) = \mathbf{f}_2(0)$$

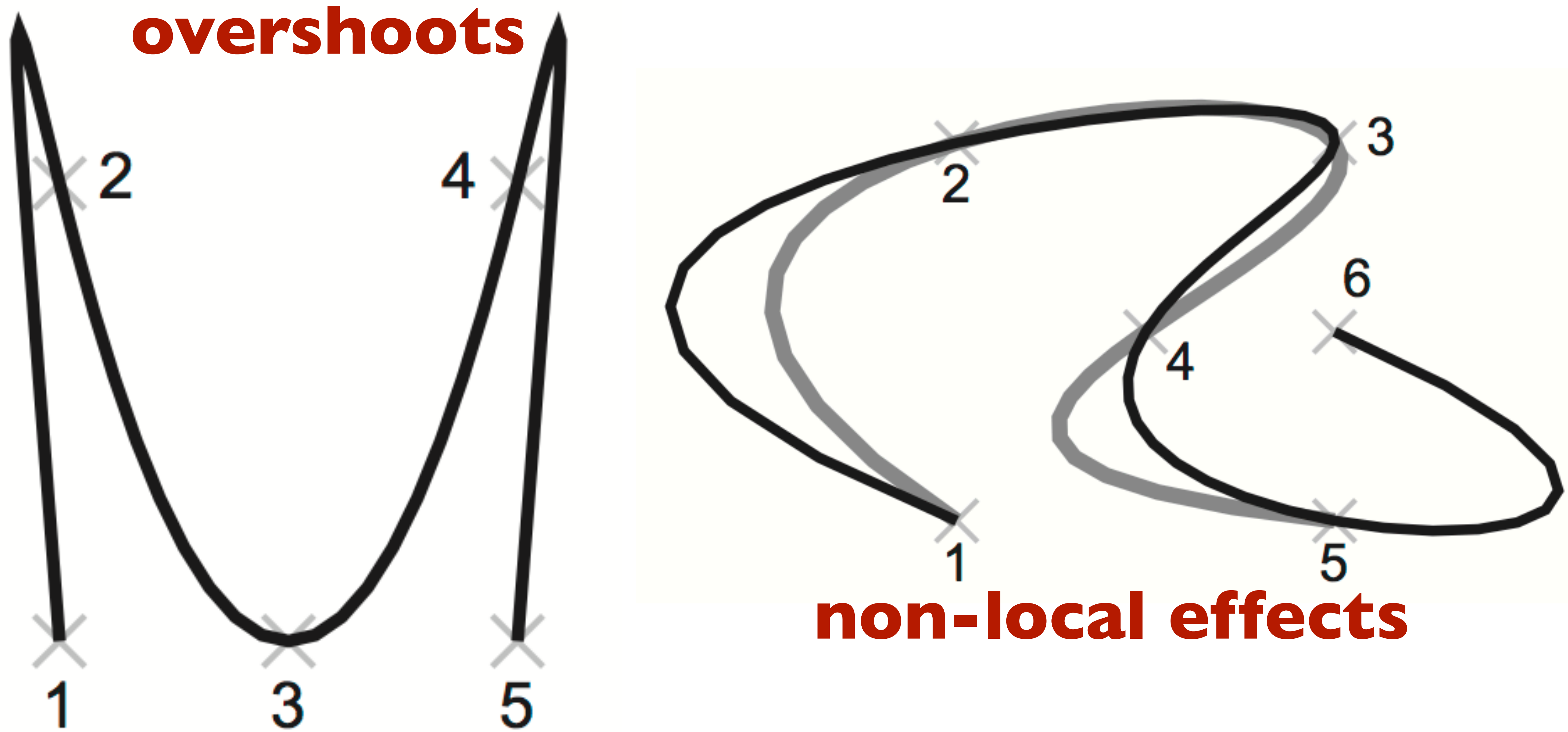
right: use simpler curves, but more of them to get the accuracy

Stitching curve segments together: **continuity**



Top
C0: the curves are continuous, but have discontinuous first derivatives
Bottom
Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative
Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude

higher order interpolating polynomials



These images demonstrate problems with using higher order polynomials:
- overshoots
- non-local effects (in going from the 4th order polynomial in grey to the 5th order polynomial in black)

Blending Functions

Blending functions are more convenient basis than monomial basis

- "canonical form" (monomial basis)

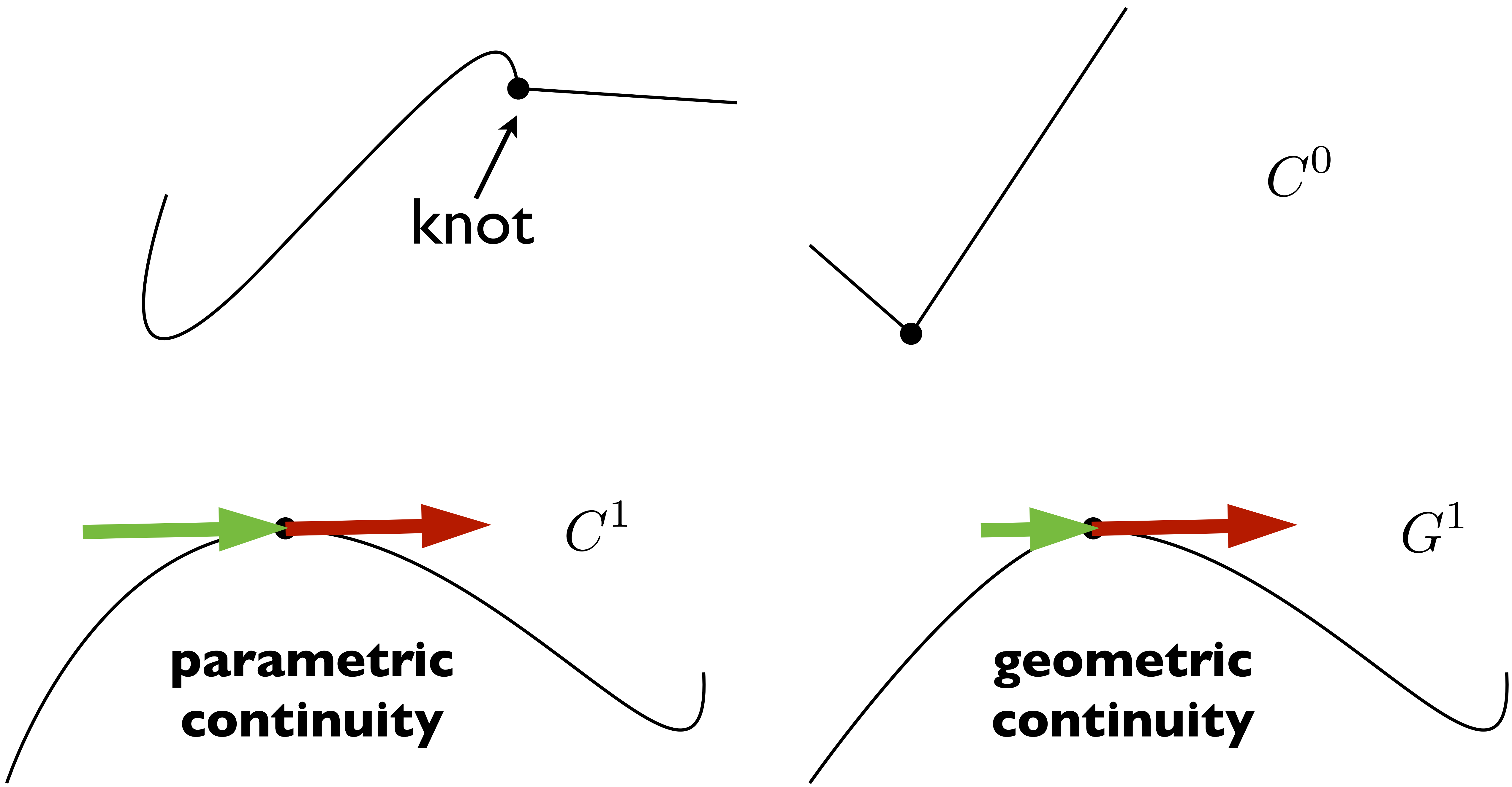
$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3$$

- "geometric form" (blending functions)

$$\mathbf{f}(u) = b_0(u)\mathbf{p}_0 + b_1(u)\mathbf{p}_1 + b_2(u)\mathbf{p}_2 + b_3(u)\mathbf{p}_3$$

– geometric form (bottom) is more intuitive because it combines control points with blending functions
[see Shirley Section 15.3]

Stitching curve segments together: **continuity**



Top

C0: the curves are continuous, but have discontinuous first derivatives

Bottom

Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative

Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude

Cubics

$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3$$

- Allow up to C^2 continuity at knots
- Symmetry: specify position and derivative at the beginning and end
- good smoothness and computational properties

need 4 control points: might be 4 points on the curve, combination of points and derivatives, ...