Name:

Student ID:

Homework 9

1 Multiple Choice (2pt each)

For each multiple choice choose, the one correct answer.

______1. How many degrees of freedom does a rigid body in three dimensions have? A) 3 B) 4 C) 6 D) 8 E) 9

______2. If a curve is C^0 continuous, then A) it can have sharp corners B) its tangent vectors are continuous C) A and B D) none of the above

______3. Given a ray tracing algorithm, if we add small random perturbations to each view ray, how will that change the resulting image? A) it will blur the image B) the image will be distorted beyond recognition C) it will appear grainy D) it will increase aliasing artifacts

2 True/False (1pt each)

You get 1 point for answering a question correctly. You get -0.25 points for answering the question incorrectly and 0.5 points for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60 percent confident that your answer is correct)

______1. (T/F) All rotation matrices are invertible.

______2. (T/F) Texture mapping is applied during the fragment processing step of the graphics pipeline.

______3. (T/F) Texture mapping with mipmapping consumes more memory than texture mapping without mipmapping.

______4. (T/F) A Bezier curve interpolates its control points.

______5. (T/F) Shadow rays point from an intersection point to a light source.

3 Written Response

1. (10 points) Hermite Curve. Consider the cubic curve below, which can be written in the monomial basis functions as

$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3$$

for some vector coefficients $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. You are given the endpoints of the curve, $\mathbf{p}_0 = \mathbf{f}(0), \mathbf{p}_3 = \mathbf{f}(1)$, and the derivatives at the endpoints $\mathbf{p}_1 = \mathbf{f}'(0), \mathbf{p}_2 = \mathbf{f}'(1)$.

(a) Set up a linear system that you could solve for $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ given $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$.

(b) Write down the inverse of the matrix you constructed in part (a). You may invert the matrix using a tool of your choice rather than by hand.

(c) Find the blending functions, $b_0(u), b_1(u), b_2(u), b_3(u)$, such that

$$\mathbf{f}(u) = \mathbf{p}_0 b_0(u) + \mathbf{p}_1 b_1(u) + \mathbf{p}_2 b_2(u) + \mathbf{p}_3 b_3(u).$$

2. (10 points) Consider a ray with endpoint $\mathbf{a}$ and a normalized direction $\mathbf{u}$,

$$\mathbf{p}(t) = \mathbf{a} + t\mathbf{u}, \quad t \geq 0,$$

and a sphere of radius r, centered at the origin. The implicit equation is given as follows:

$$\mathbf{p} \cdot \mathbf{p} - r^2 = 0$$

Describe geometrically in what ways can the ray intersect/not intersect with the sphere(when is there exactly one intersection, when is there two intersections, and when are there no intersections), and what does each of those cases says about the value of t. Come up with an algorithm that finds all of the intersection points of the ray and sphere, if any.