**Blending functions** are more convenient basis than monomial basis

- **blending functions**

\[ f(u) = b_0(u)p_0 + b_1(u)p_1 + b_2(u)p_2 + b_3(u)p_3 \]

- **monomial basis**

\[ f(u) = a_0 + a_1u + a_2u^2 + a_3u^3 \]

- geometric form is more intuitive because it combines control points with blending functions
Stitching curve segments together: \textbf{continuity}

Top
C0: the curves are continuous, but have discontinuous first derivatives

Bottom
Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative
Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude
Cubics

\[ f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 \]

- Allow up to \( C^2 \) continuity at knots
- Symmetry: specify position and derivative at the beginning and end
- Good smoothness and computational properties

Need 4 control points: might be 4 points on the curve, combination of points and derivatives, ...
Cubic Hermite Curves
Cubic Hermite Curves

Specify endpoints and derivatives

construct curve with \( C^1 \) continuity
Hermite blending functions

\[ b_0(u) = 2u^3 - 3u^2 + 1 \]
\[ b_1(u) = -2u^3 + 3u^2 \]
\[ b_2(u) = u^3 - 2u^2 + u \]
\[ b_3(u) = u^3 - u^2 \]
Example: keynote curve tool
Control points

Interpolating

Approximating (non-interpolating)
Cubic Bezier Curves
Bezier Curve Examples
Bezier blending functions

\[ b_0(u) = (1 - u)^3 \]
\[ b_1(u) = 3u(1 - u)^2 \]
\[ b_2(u) = 3u^2(1 - u) \]
\[ b_3(u) = u^3 \]
Bernstein Polynomials

• The blending functions are a special case of the Bernstein polynomials

\[ b_{kd}(u) = \frac{d!}{k!(d-k)!} u^k (1-u)^{d-k} \]

• These polynomials give the blending polynomials for any degree Bezier form
  - All roots at 0 and 1
  - For any degree they all sum to 1
  - They are all between 0 and 1 inside (0,1)
Beziers Curve Properties

- Curve lies in the convex hull of the data
- Variation diminishing
- Symmetry
- Affine invariant
- Efficient evaluation and subdivision
Bezier subdivision
Recursive Subdivision
Surfaces
Parametric Surface

\[ x = x(u, v) \]
\[ y = y(u, v) \]
\[ z = z(u, v) \]
Parametric Surface - tangent plane

\[ t_u = \left( \begin{array}{c} \frac{\partial x}{\partial u} \\ \frac{\partial x}{\partial y} \\ \frac{\partial z}{\partial u} \end{array} \right) \]

\[ t_v = \left( \begin{array}{c} \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial v} \end{array} \right) \]
Bicubic Surface Patch

\[ f(u, v) = \sum_i \sum_j b_i(u) b_j(v) p_{ij} \]
Bezíer Surface Patch

\[ f(u, v) = \sum_i \sum_j b_i(u)b_j(v)p_{ij} \]

Patch lies in convex hull