Design considerations

• local control of shape
• smoothness and continuity
• ability to evaluate derivatives
• stability
• ease of rendering

- local control – design each segment independently
- stability – small change in input values leads to small change in output
Parametric curve

\[ x = x(u) \]
\[ y = y(u) \]
\[ z = z(u) \]
Parametric curve example

\[ p(u) = c_0 + c_1 u + c_2 u^2 \]

\[ x(u) = 3u^2 \]
\[ y(u) = 2u + 3 \]

\[ c_0 = ?, \quad c_1 = ?, \quad c_2 = ? \]

- this is a curve in 2D
- for a curve in 3D, we would also have \( z(u) = \ldots \)
Parametric curve -

tangent vector

\[ x = x(u) \]
\[ y = y(u) \]
\[ z = z(u) \]

\[ t = \begin{pmatrix} x'(u) \\ y'(u) \\ z'(u) \end{pmatrix} \]

- tangent vector
Piecewise Parametric Representations

\[ f(u) = \begin{cases} 
  f_1(2u) & u \leq 0.5 \\
  f_2(2u - 1) & u > 0.5 
\end{cases} \]

continuity

\[ f_1(1) = f_2(0) \]

right: use simpler curves, but more of them to get the accuracy
Continuity

Top
C0: the curves are continuous, but have discontinuous first derivatives

Bottom
Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative
Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude
These images demonstrate problems with using higher order polynomials:
- overshoots
- non-local effects (in going from the 4th order polynomial in grey to the 5th order polynomial in black)