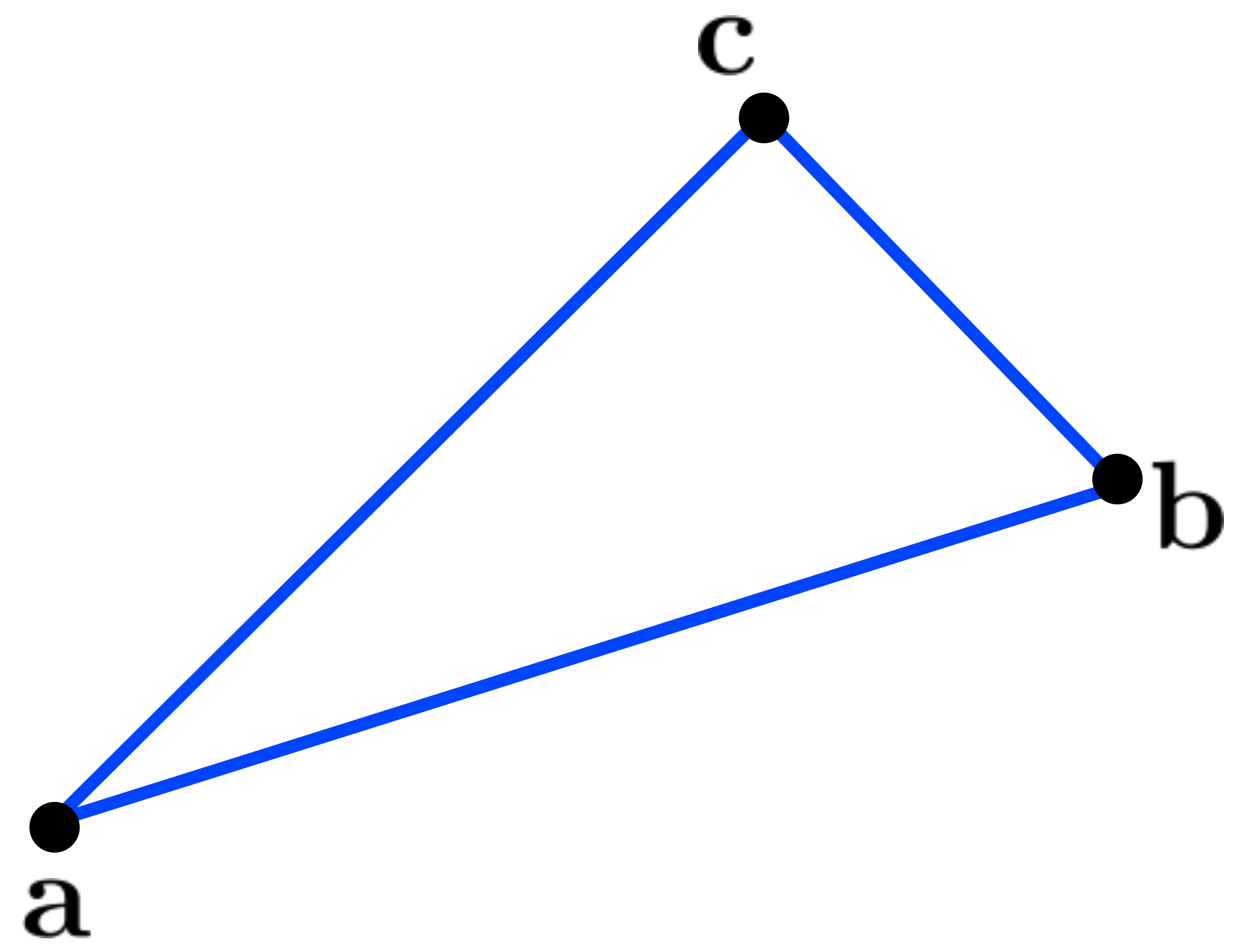
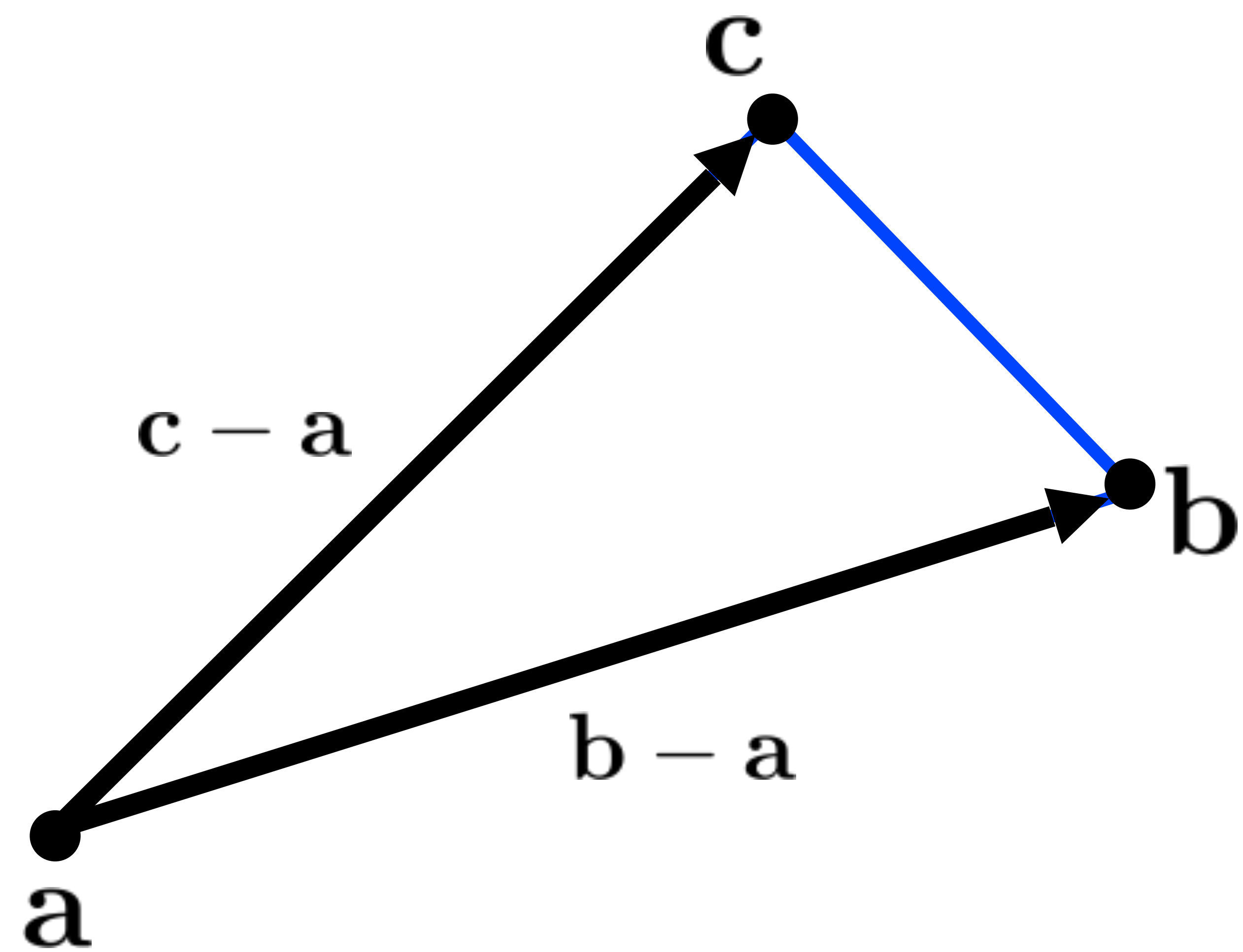
Triangles

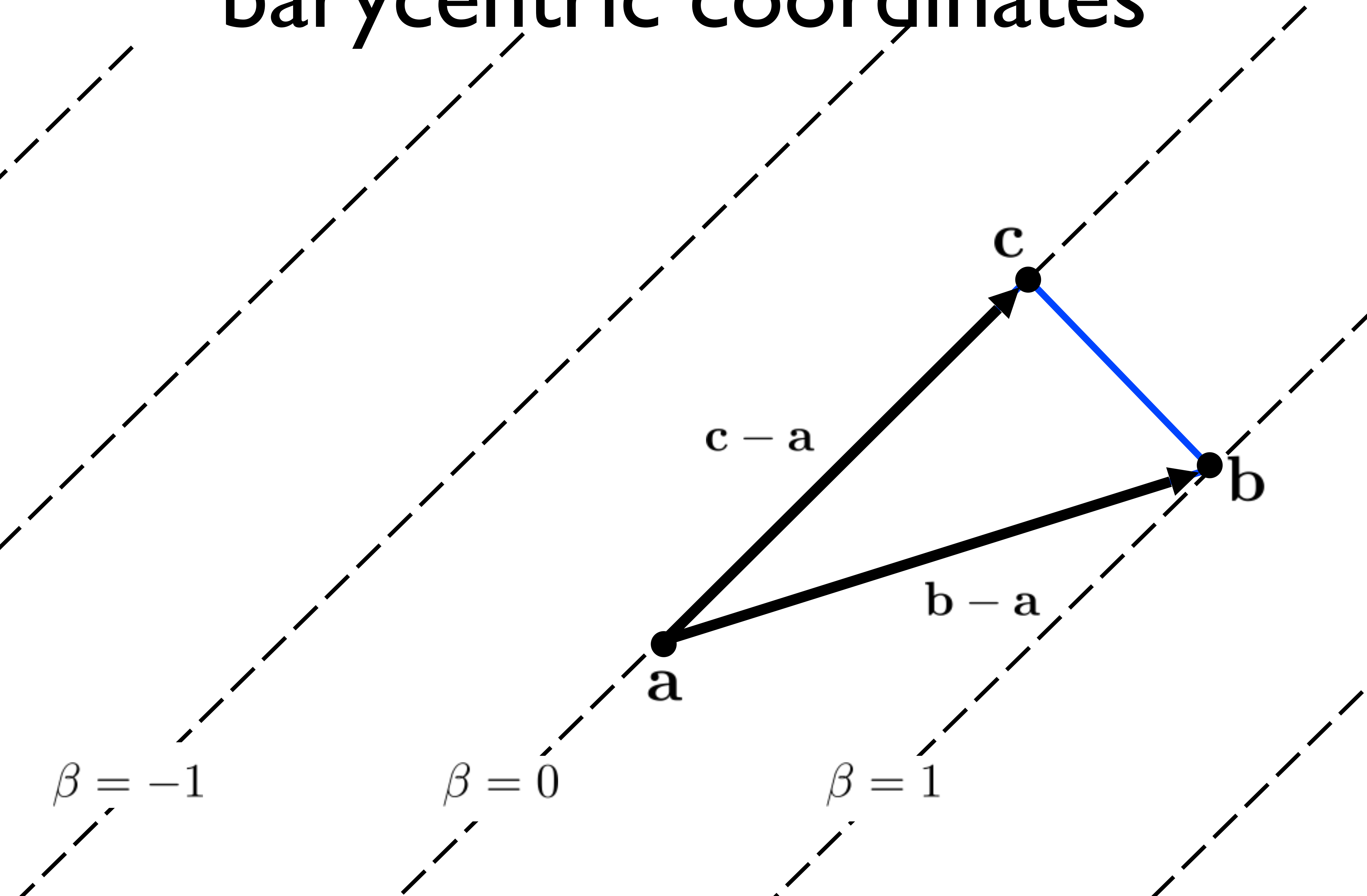
barycentric coordinates



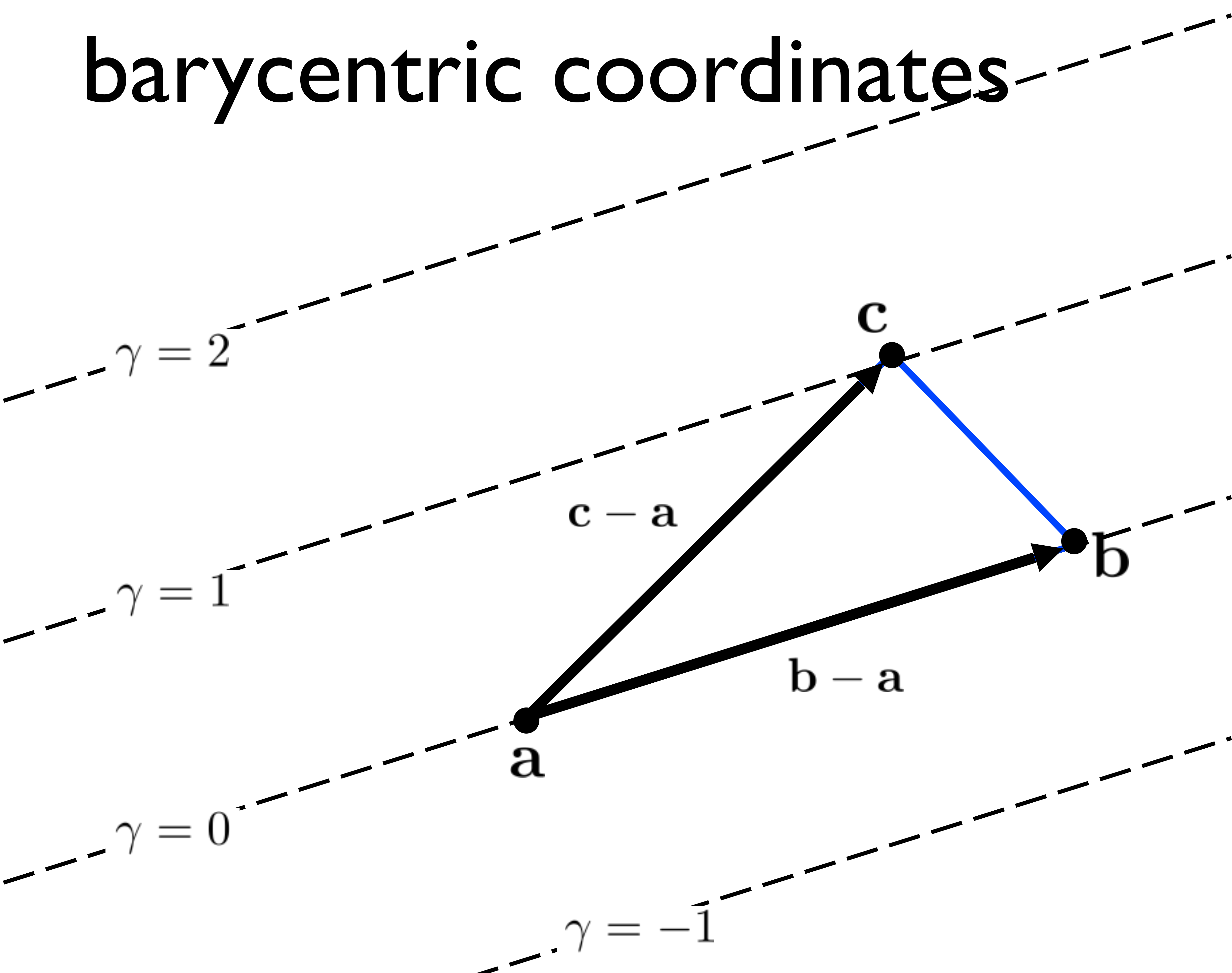
barycentric coordinates



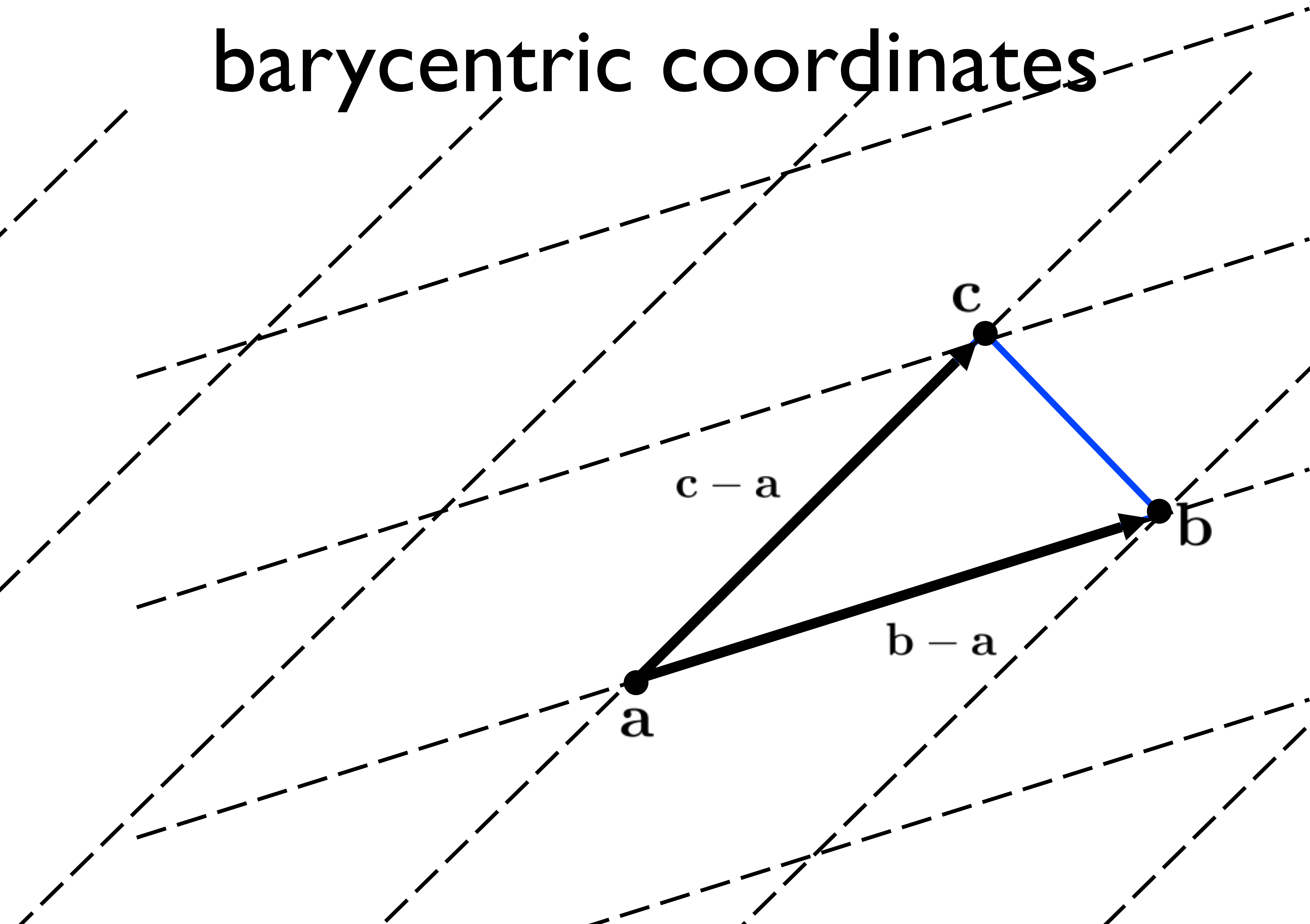
barycentric coordinates



barycentric coordinates



barycentric coordinates

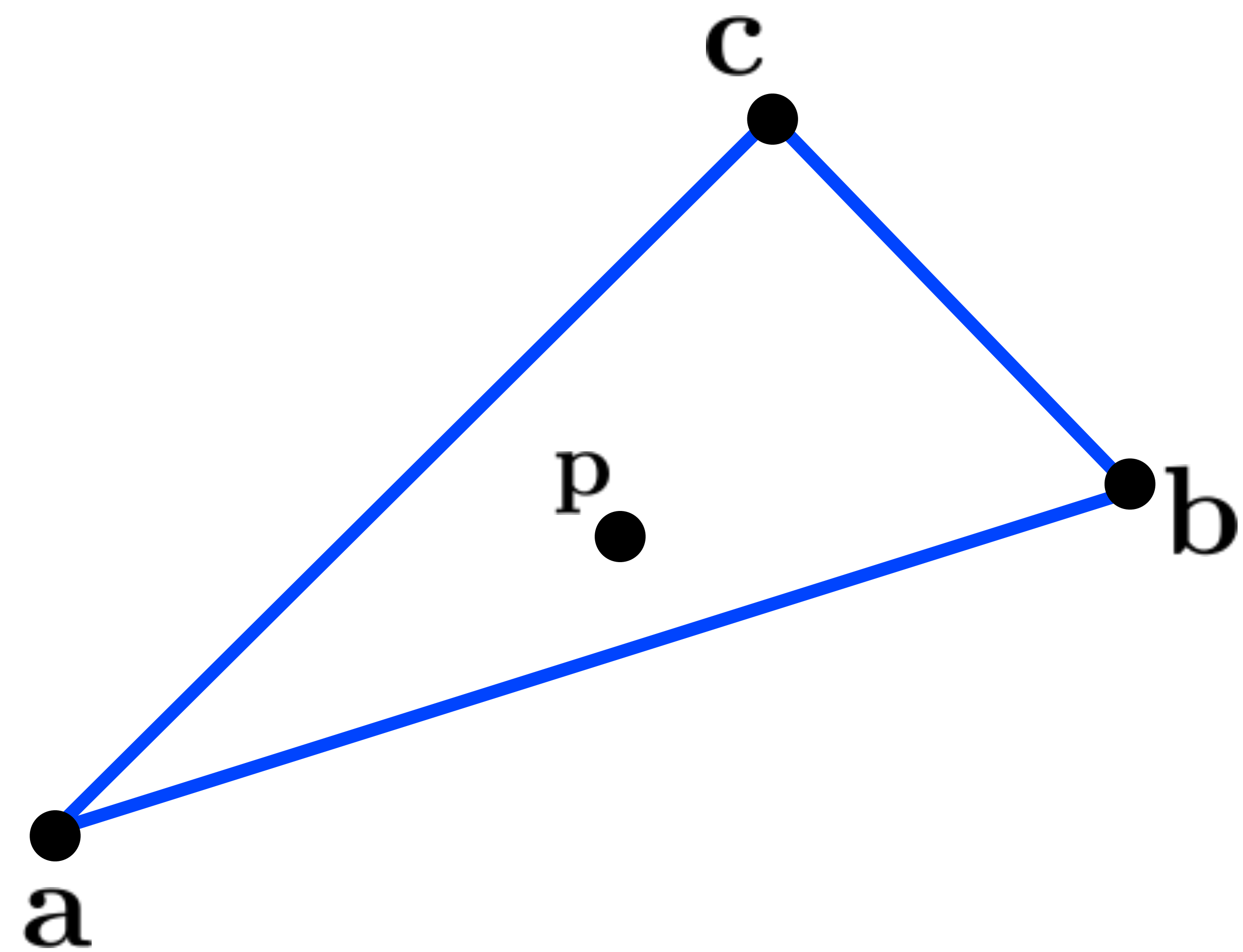


barycentric coordinates

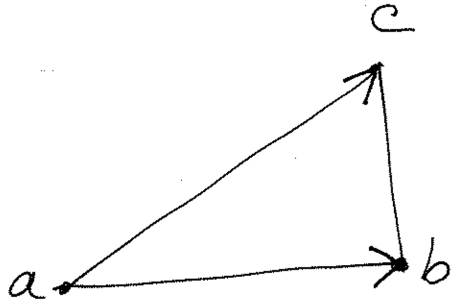
$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$

What are (α, β, γ) ?

<whiteboard>



2.7 Triangles



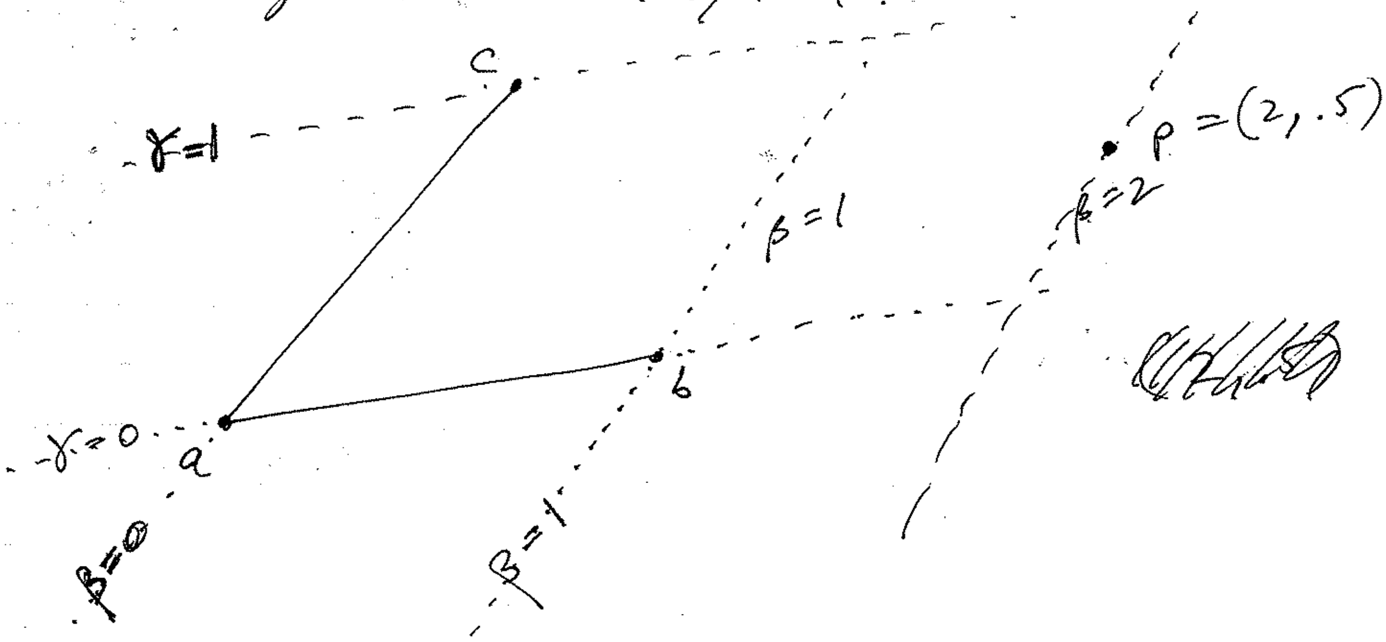
$$\text{area} = \frac{1}{2}\begin{vmatrix} b-a & c-a \\ 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{vmatrix}$$

barycentric coordinates

assign a color at each vertex & smoothly interpolate.

non-orthogonal coordinate system.



$$p = a + \beta(b-a) + \gamma(c-a)$$

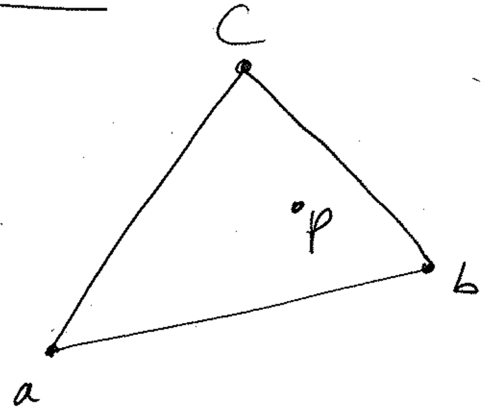
$$p = a - \beta a + \beta b + \gamma c - \gamma a$$

$$p = a - \beta a - \gamma a + \beta b + \gamma c$$

$$= (1-\beta-\gamma)a + \beta b + \gamma c$$

$$p = \alpha a + \beta b + \gamma c$$

Inside test



$p = \alpha a + \beta b + \gamma c$

p inside triangle $(\alpha + \beta + \gamma = 1)$ $\Longleftrightarrow$ $0 < \alpha, \beta, \gamma < 1$

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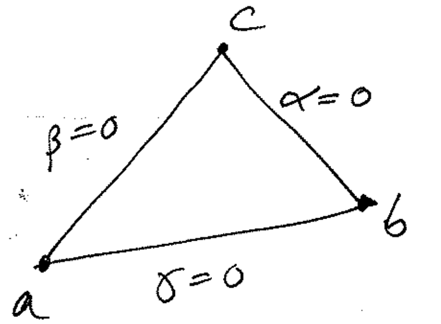
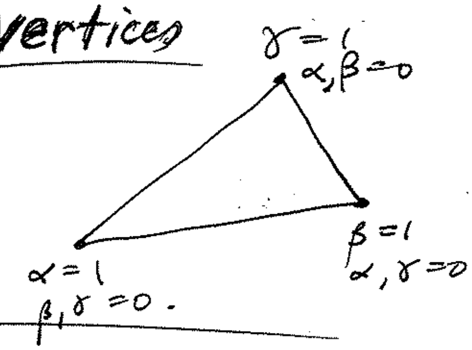
edges



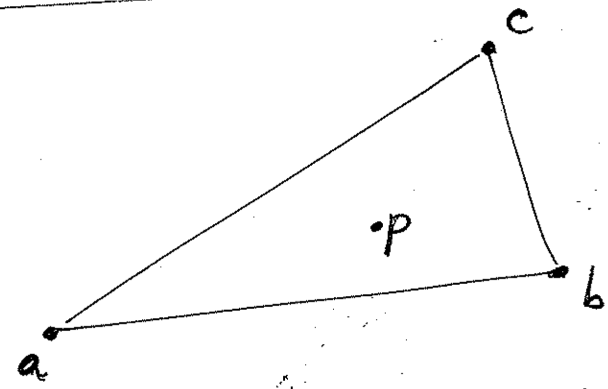
Vertices



Interpolation

use α, β, γ to interpolate properties.

Find α, β, γ given a point p

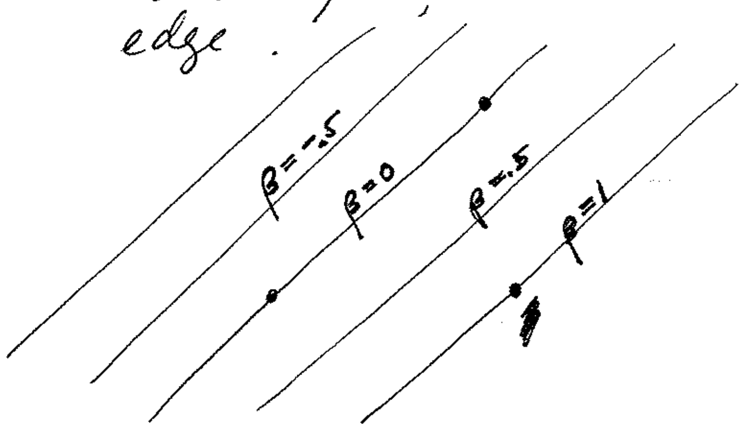


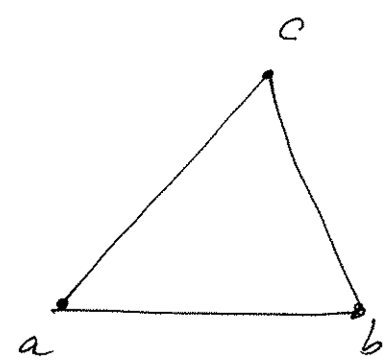
$$p = a + \beta(b-a) + \gamma(c-a)$$

Solve for β, γ

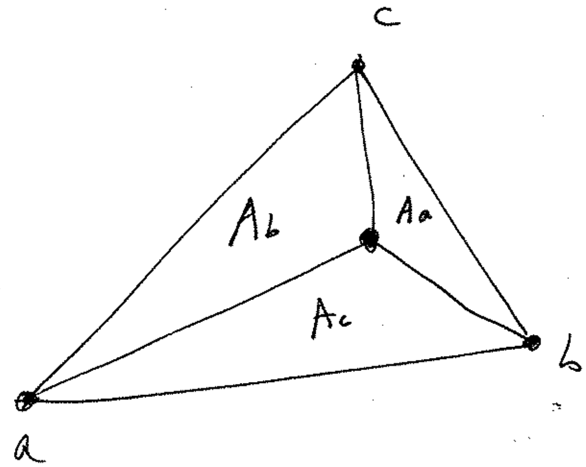
$$\begin{bmatrix} | & | \\ b-a & c-a \\ | & | \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} | \\ p-a \\ | \end{bmatrix}$$

Or use geometric property: bary. coords. are signed, scale distance from corresponding edge.





$$\beta = \frac{f_{ac}(p)}{f_{ac}(b)}$$

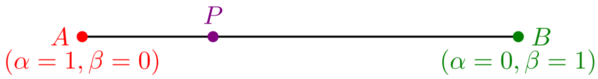


$$\alpha = \frac{A_a}{A}, \quad \beta = \frac{A_b}{A}, \quad \gamma = \frac{A_c}{A}$$

Barycentric Coordinates

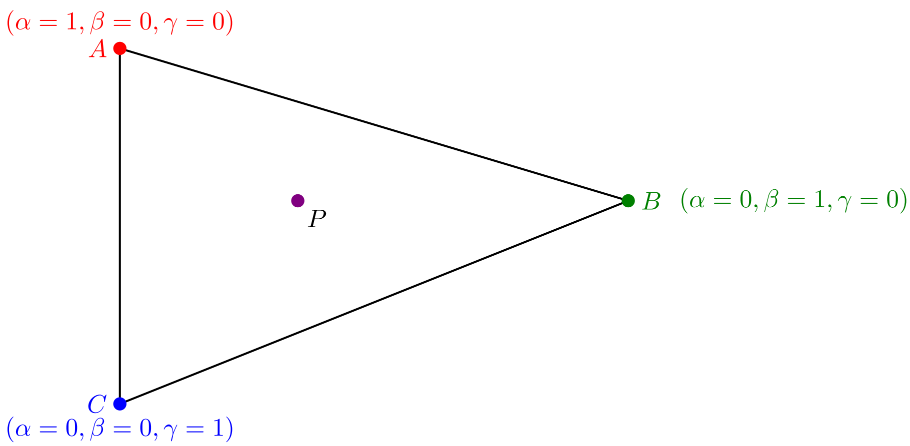
CS 130

1. Want to interpolate vertex data along a segment



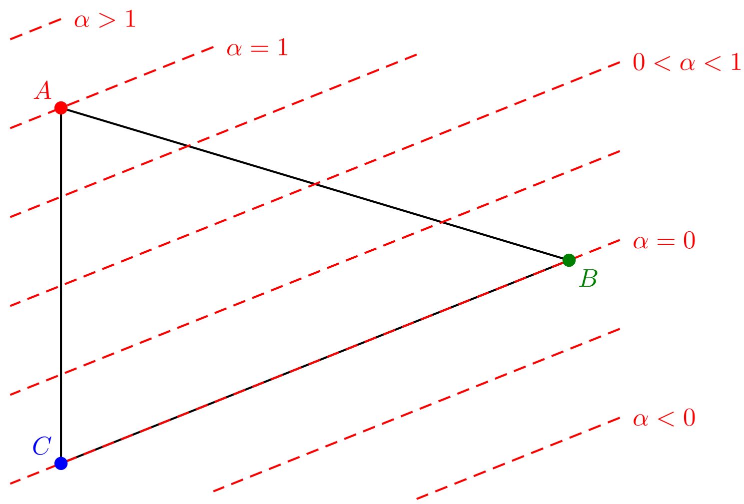
   - Define $f(\mathbf{x})$ for all points $\mathbf{x}$ on the line
   - Value at endpoints: f_A, f_B.
   - Interpolation should get the endpoints right: $f(A) = f_A$, $f(B) = f_B$
   - $f(P) = \alpha f(A) + (1 - \alpha) f(B)$.
   - $0 \leq \alpha \leq 1$.
   - Symmetry: define $\beta = 1 - \alpha$.
   - $f(P) = \alpha f(A) + \beta f(B)$, with $\alpha + \beta = 1$.
   - $\alpha = \frac{\text{len}(PB)}{\text{len}(AB)}$, $\beta = \frac{\text{len}(AP)}{\text{len}(AB)}$

2. Extend this to a triangle

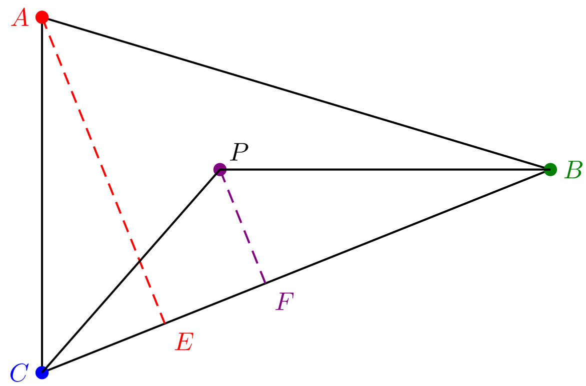


   - Define $f(\mathbf{x})$ for all points $\mathbf{x}$ on the triangle
   - Value at vertices: f_A, f_B, f_C.
   - Interpolation should get the vertices right: $f(A) = f_A$, $f(B) = f_B$, $f(C) = f_C$
   - $f(P) = \alpha f(A) + \beta f(B) + \gamma f(C)$, with $\alpha + \beta + \gamma = 1$.

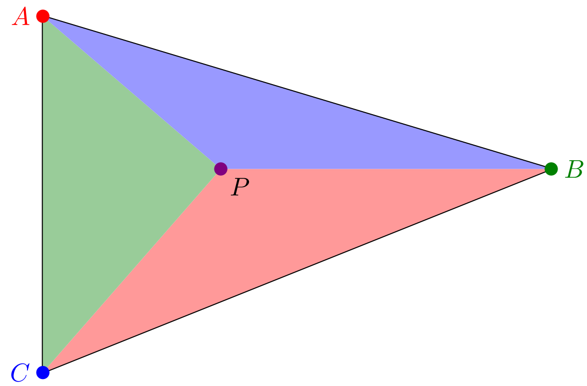
- Weights form isocontours:



- Note that $\alpha < 0$ or $\alpha > 1$ lies outside the triangle
- Compute using distance to edge:



- $\alpha = \frac{\text{len}(PF)}{\text{len}(AE)} = \frac{\frac{1}{2}\text{len}(PF)\text{len}(BC)}{\frac{1}{2}\text{len}(AE)\text{len}(BC)} = \frac{\text{area}(PBC)}{\text{area}(ABC)}$
- Similarly: $\beta = \frac{\text{area}(APC)}{\text{area}(ABC)}$, $\gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)}$



- Pattern of areas
- Since area(PBC) + area(APC) + area(ABP) = area(ABC), we have $\alpha + \beta + \gamma = 1$
- Barycentric interpolation is okay for z-values
- Barycentric interpolation is okay for colors in orthographic case
- Barycentric interpolation does not work for colors in the projective case

3. Inside/outside tests

   - $\alpha < 0$ or $\alpha > 1$ lies outside the triangle (Same for $\beta < 0$ or $\beta > 1$, $\gamma < 0$ or $\gamma > 1$)
   - Inside the triangle if $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ and $0 \leq \gamma \leq 1$.
   - Sufficient to check $\alpha, \beta, \gamma \geq 0$
   - For example if $\alpha \geq 0$ and $\beta \geq 0$ then $\gamma = 1 - \alpha - \beta \leq 1 - \beta \leq 1$.
   - Since we need the weights to compute the depth values when doing z-buffering, we might as well also use them to determine inside/outside.