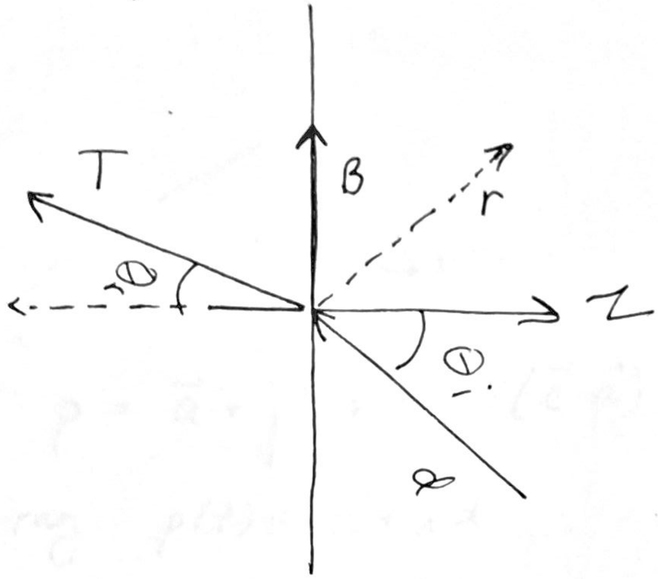
ransparency and Refraction



η = index of refraction

$\eta_{air} \equiv 1$

$\eta_{water} \equiv 1.33$

θ_i = angle of incidence
θ_r = angle of refraction

What is the transmitted ray T?

First, find θ_r using Snell's law:

$$\eta_r \sin\theta_r = \eta_i \sin\theta_i$$

using cosine's, this is

$$\eta_r^2(1-\cos^2\theta_r) = \eta_i^2(1-\cos^2\theta_i)$$

$$\Rightarrow \cos^2\theta_r = -\frac{\eta_i^2}{\eta_r^2}(1-\cos^2\theta_i)+1$$

$$\Rightarrow \cos\theta_r = \Big[\underbrace{1-\frac{\eta_i^2}{\eta_r^2}(1-\cos^2\theta_i)}_{\circledast}\Big]^{1/2}$$

if $\circledast < 0$, no refracted ray "total internal reflection"

$$-\vec{d} = \cos\theta_i\vec{N} - \sin\theta_i\vec{B} \quad\Rightarrow\quad \vec{B} = \frac{+d+\cos\theta_i\vec{N}}{+\sin\theta_i}$$

$$\vec{T} = -\cos\theta_r\vec{N} + \sin\theta_r\vec{B}$$

$$\vec{T} = -\cos\theta_r\vec{N} + \frac{\sin\theta_r}{\sin\theta_i}\left[\vec{d}+\cos\theta_i\vec{N}\right]$$

$$= -\cos\theta_r\vec{N} + \frac{\eta_i}{\eta_r}\vec{d} + \frac{\eta_i}{\eta_r}\cos\theta_i\vec{N} = -\cos\theta_r\vec{N} + \frac{\eta_i}{\eta_r}\left[\vec{d}-(N\cdot d)N\right]$$

$$\vec{T} = \frac{\eta_i}{\eta_r}\left[d-(N\cdot d)N\right] - \cos\theta_r N$$