CS130 : Computer Graphics
Curves

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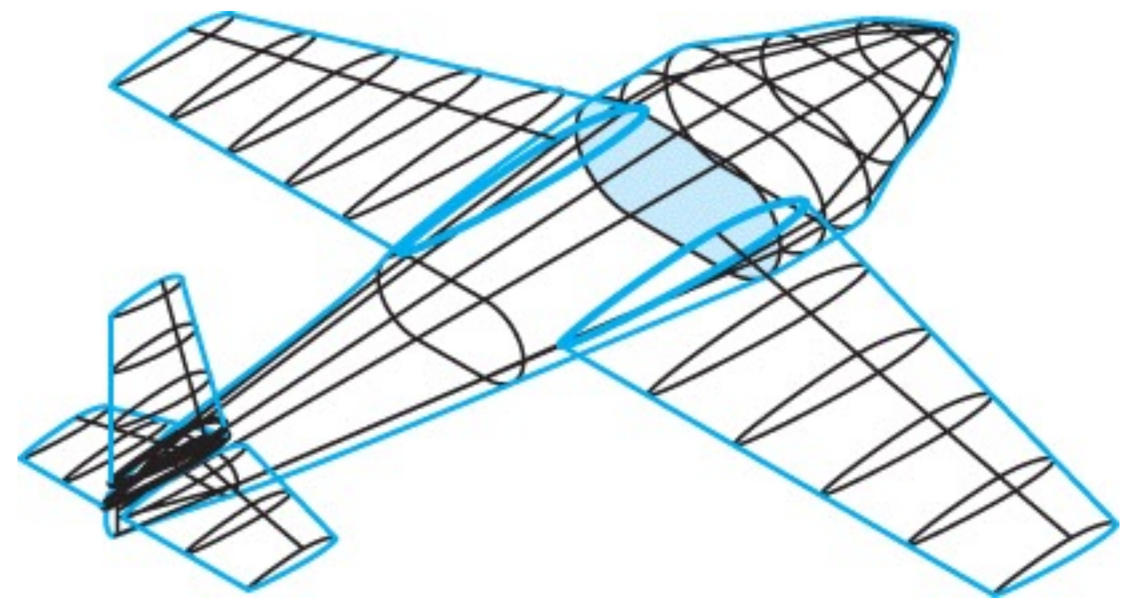
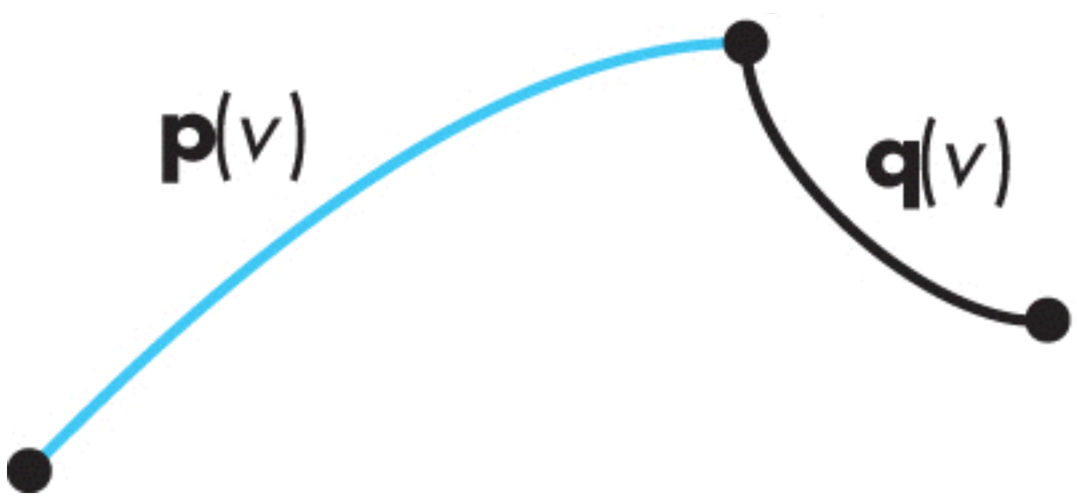
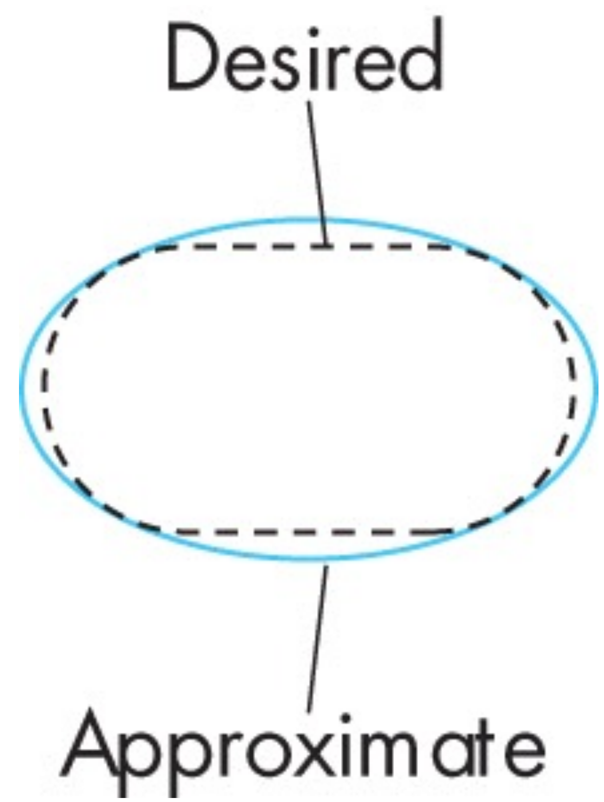
Design considerations

- local control of shape
  - design each segment independently
- smoothness and continuity
- ability to evaluate derivatives
- stability
  - small change in input leads to small change in output
- ease of rendering







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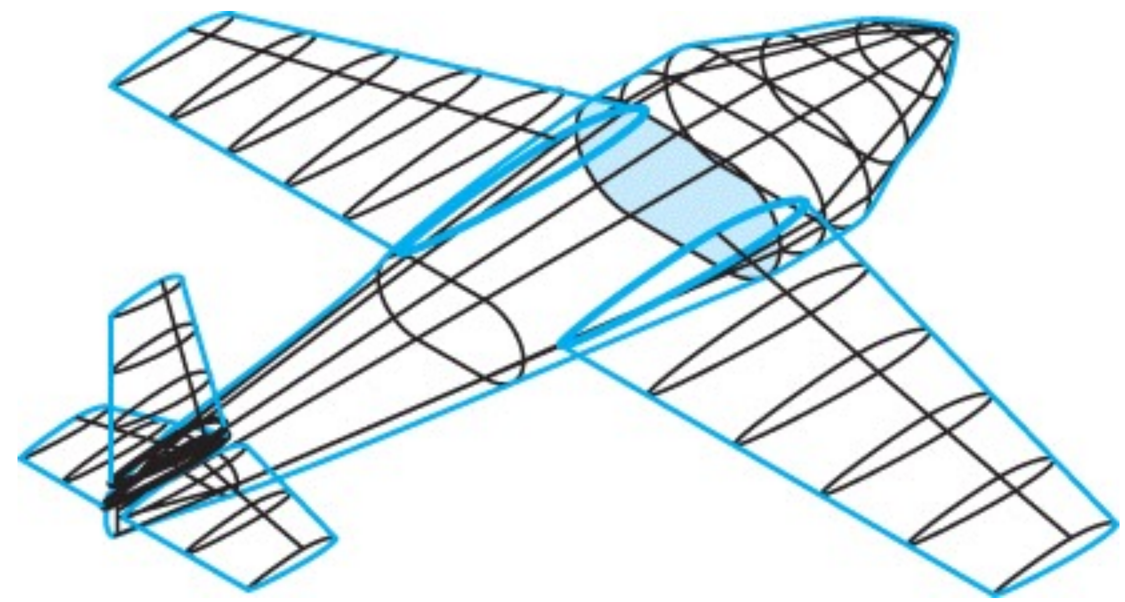

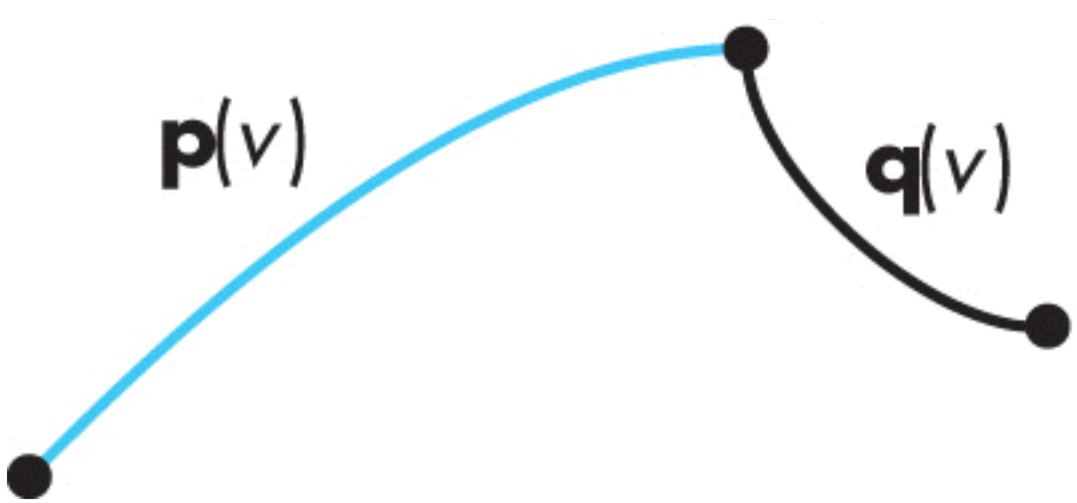

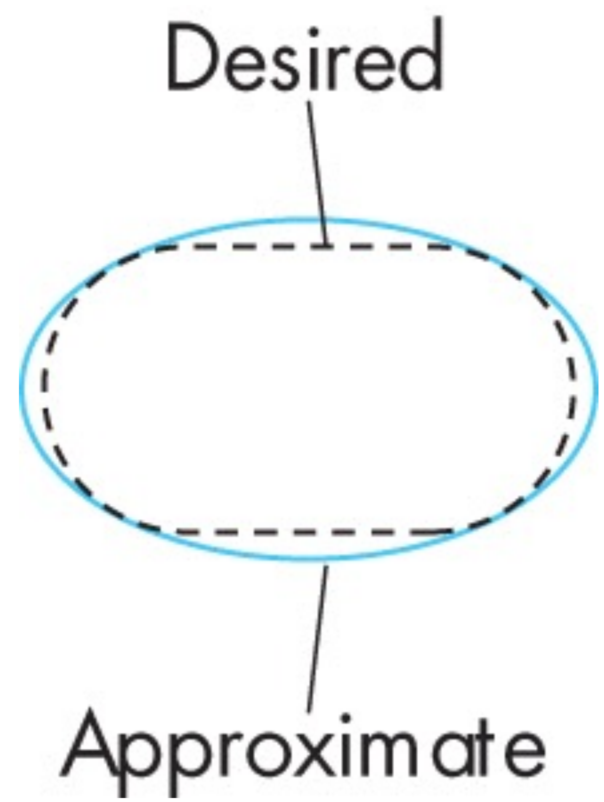

approximate out of a number of wood strips

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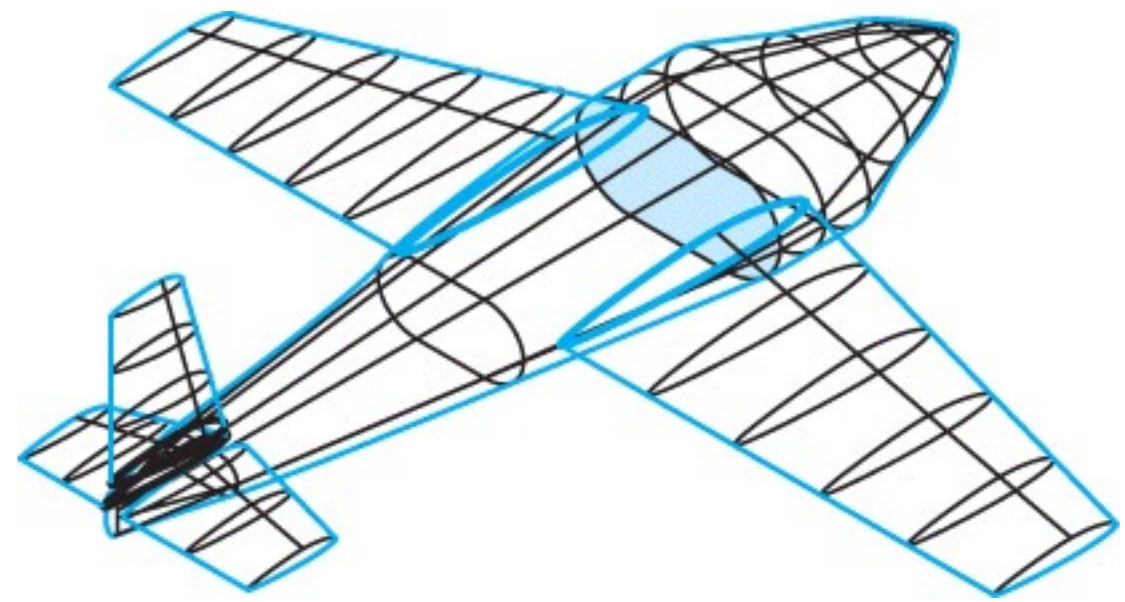
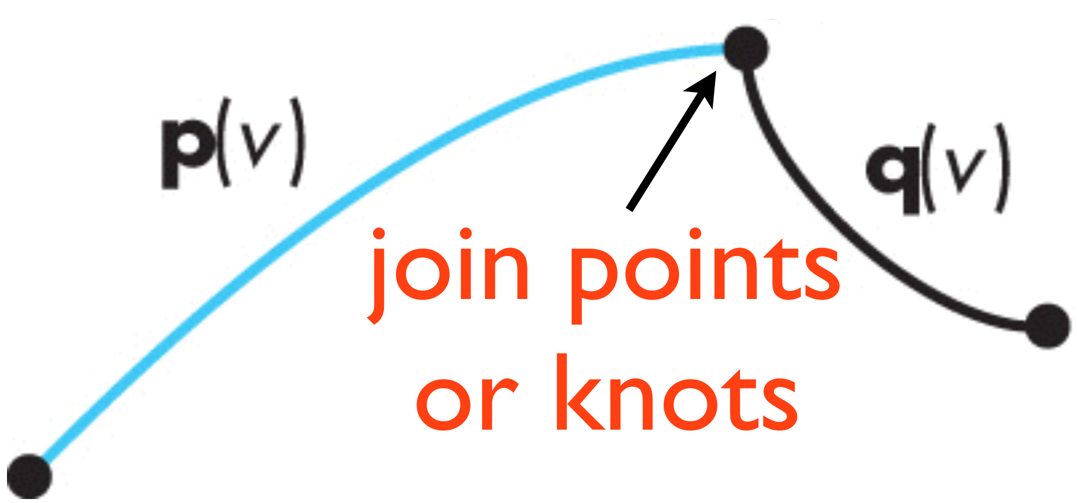
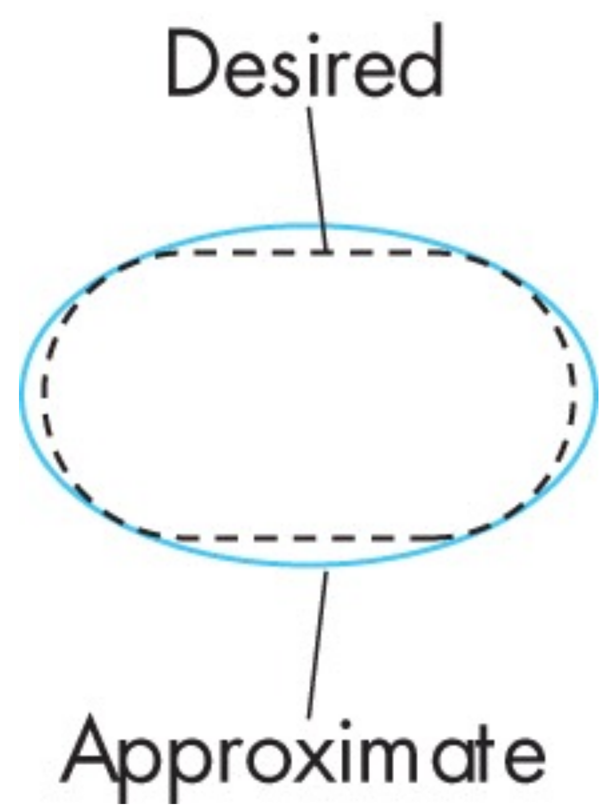




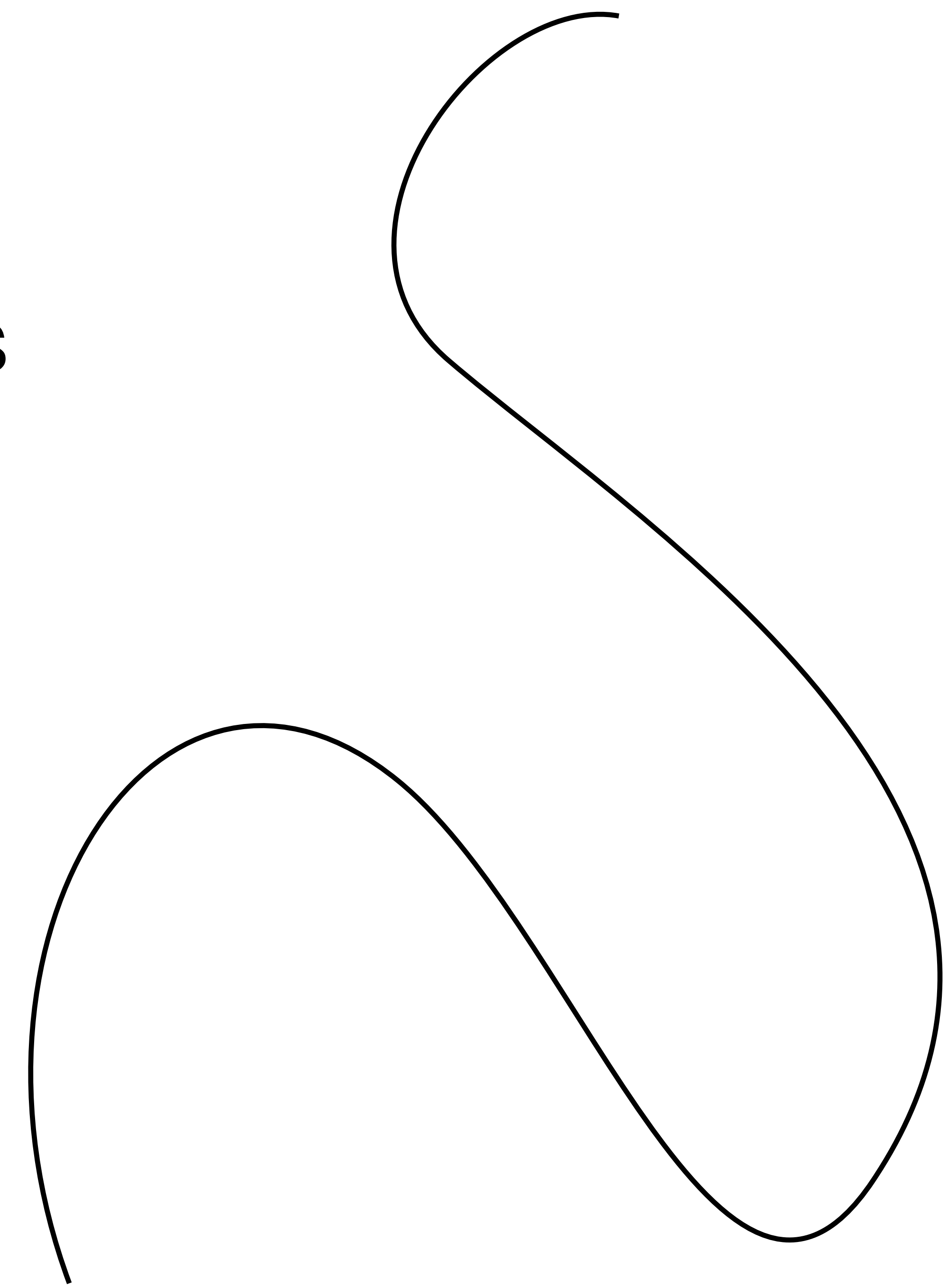


approximate out of a number of wood strips

What is a curve?

intuitive idea:
  draw with a pen
  set of points the pen traces

may be 2D, like on paper
or 3D, *space curve*



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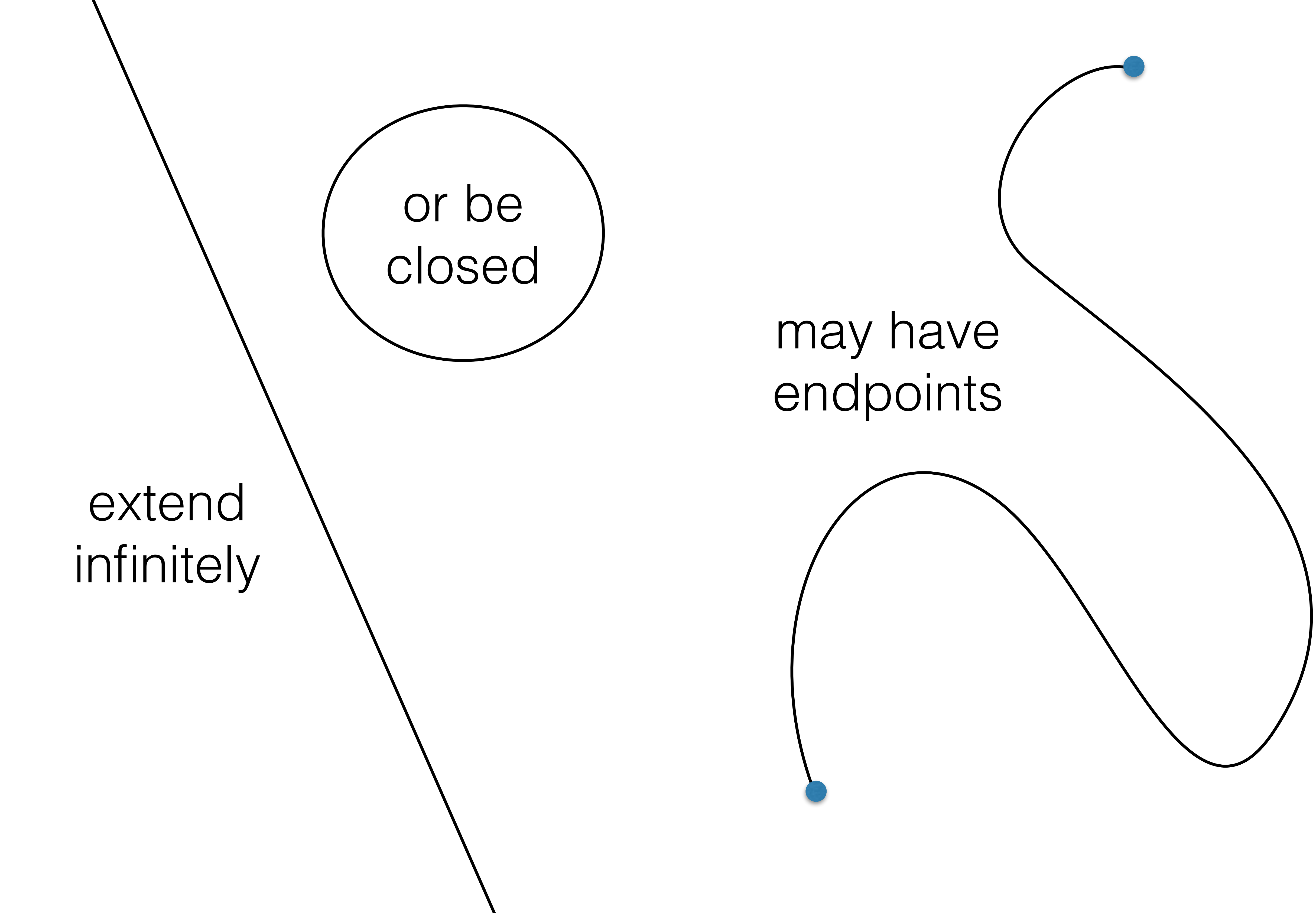
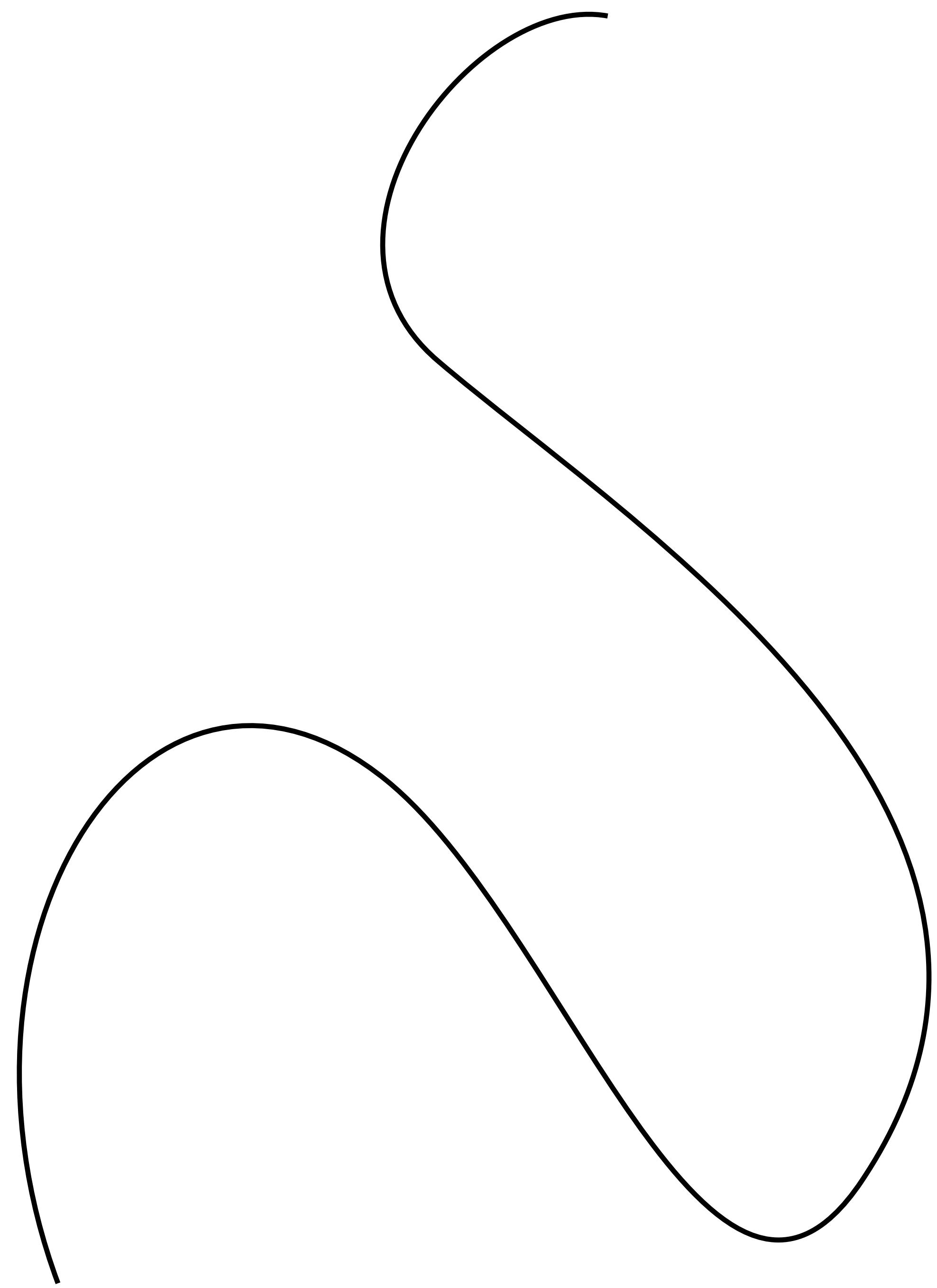


How do we specify a curve?

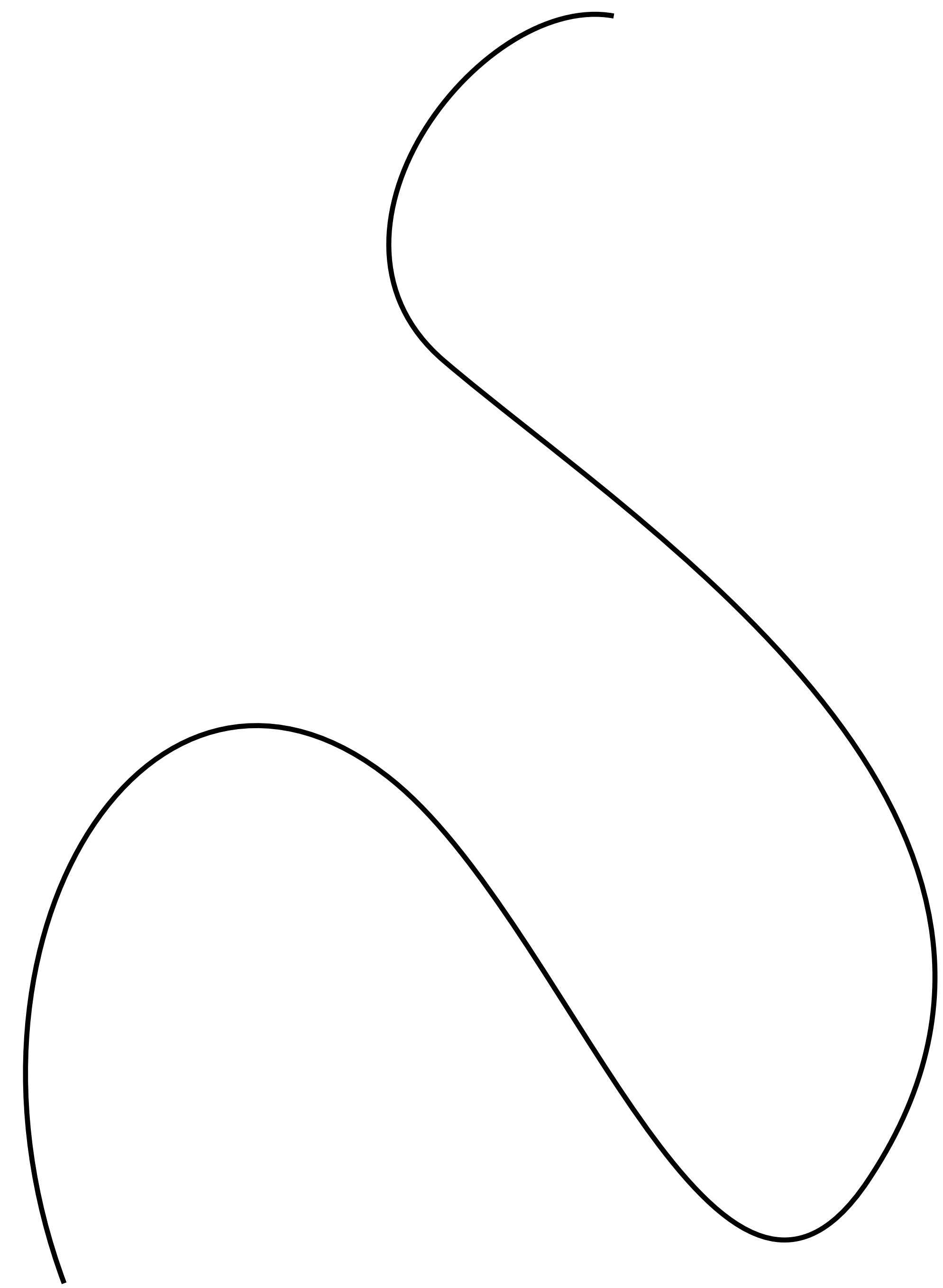


How do we specify a curve?

Implicit

(2D) $f(x,y) = 0$

test if (x,y) is on the curve



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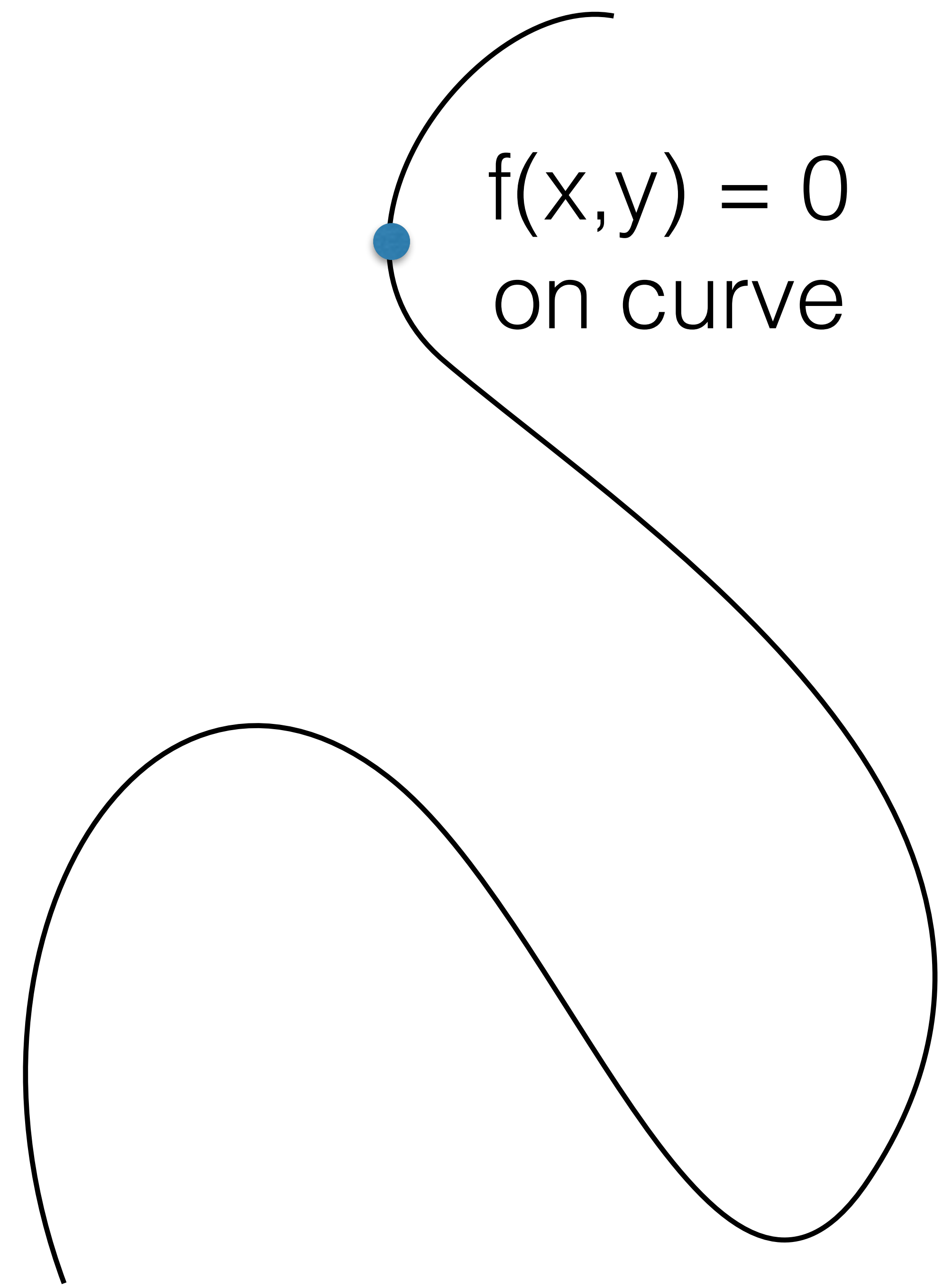

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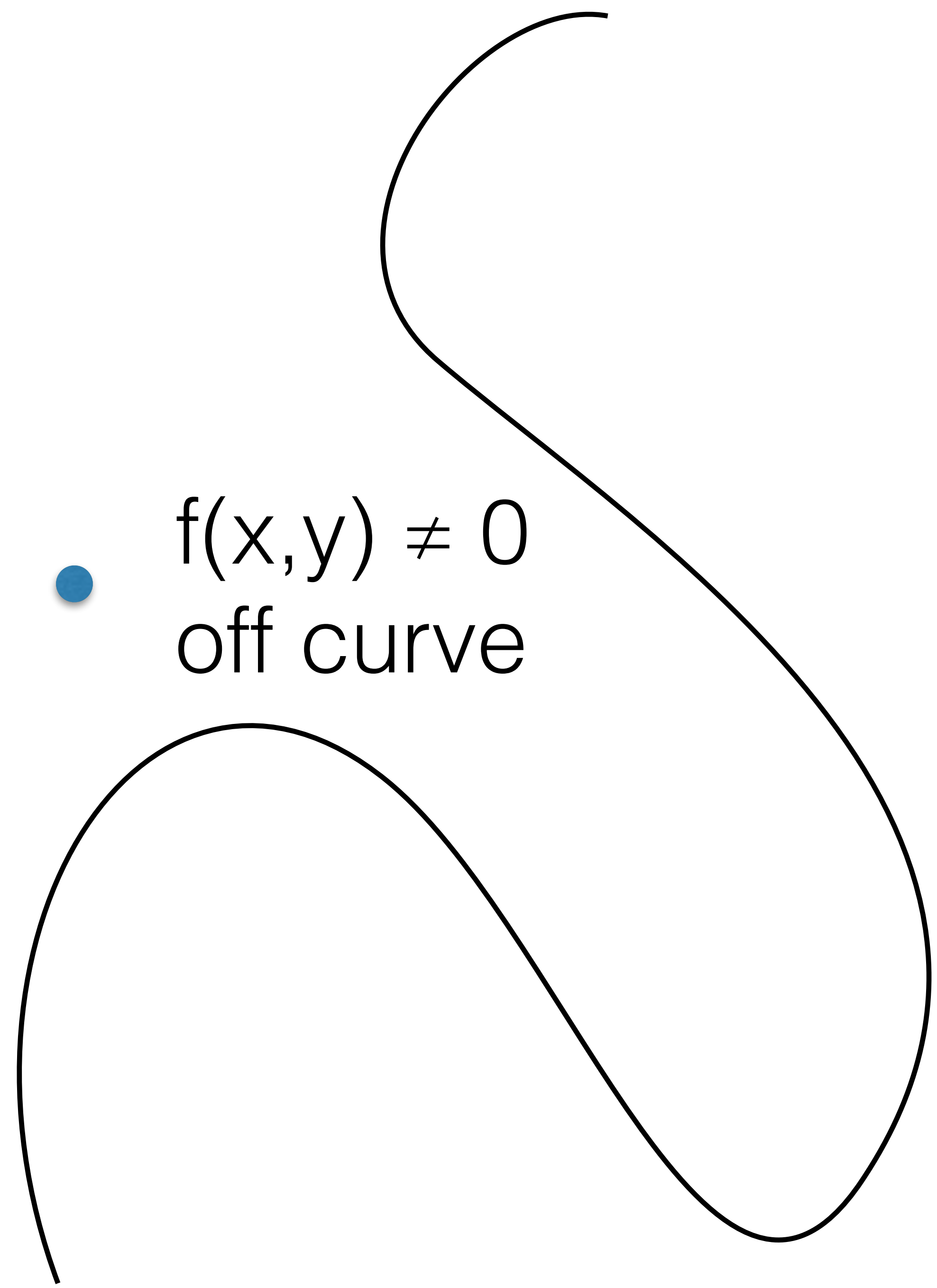

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(2D) $f(x,y) = 0$

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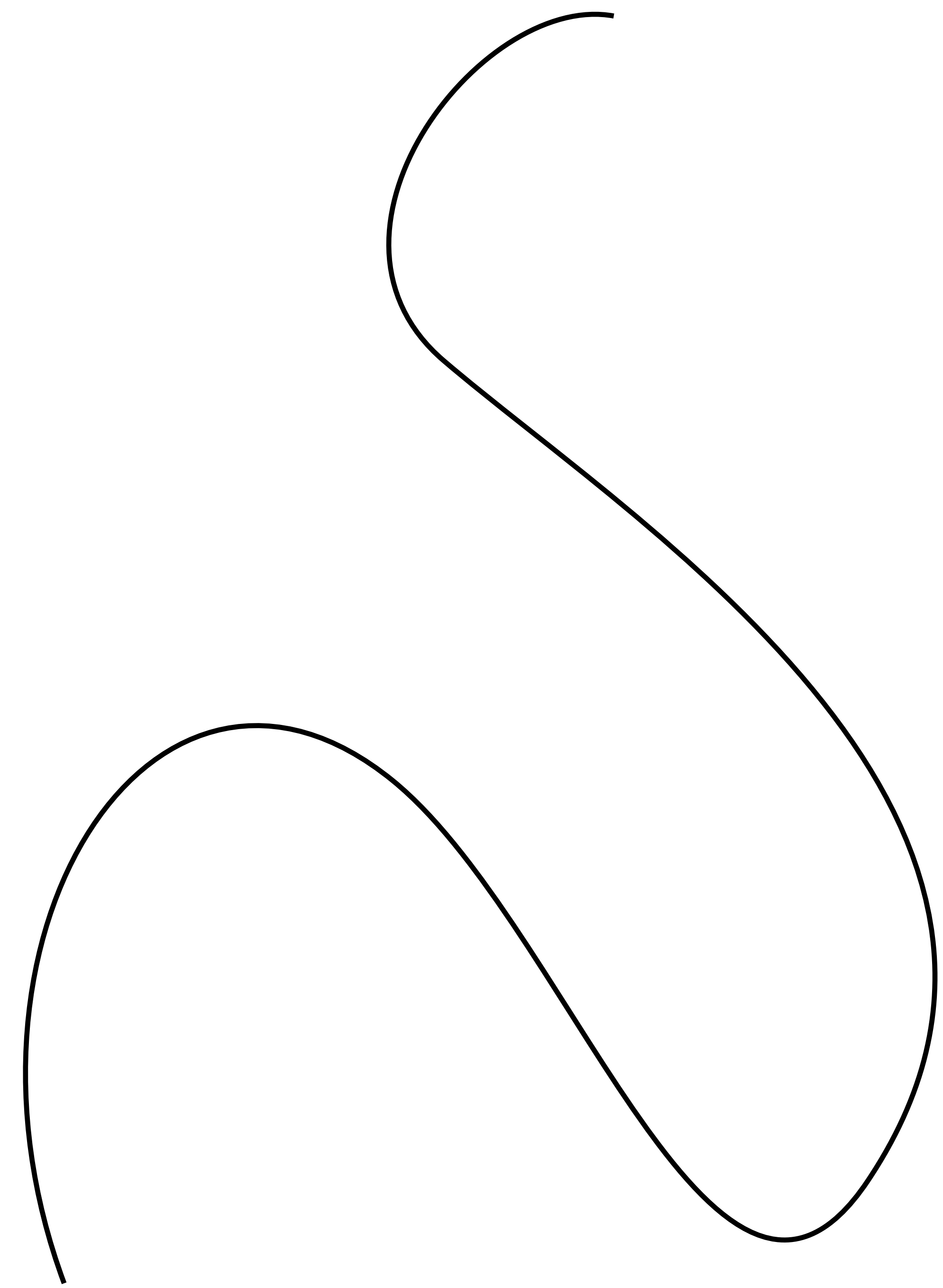
Parametric

(2D) $(x,y) = \mathbf{f}(t)$

(3D) $(x,y,z) = \mathbf{f}(t)$

map free *parameter* t

to points on the curve



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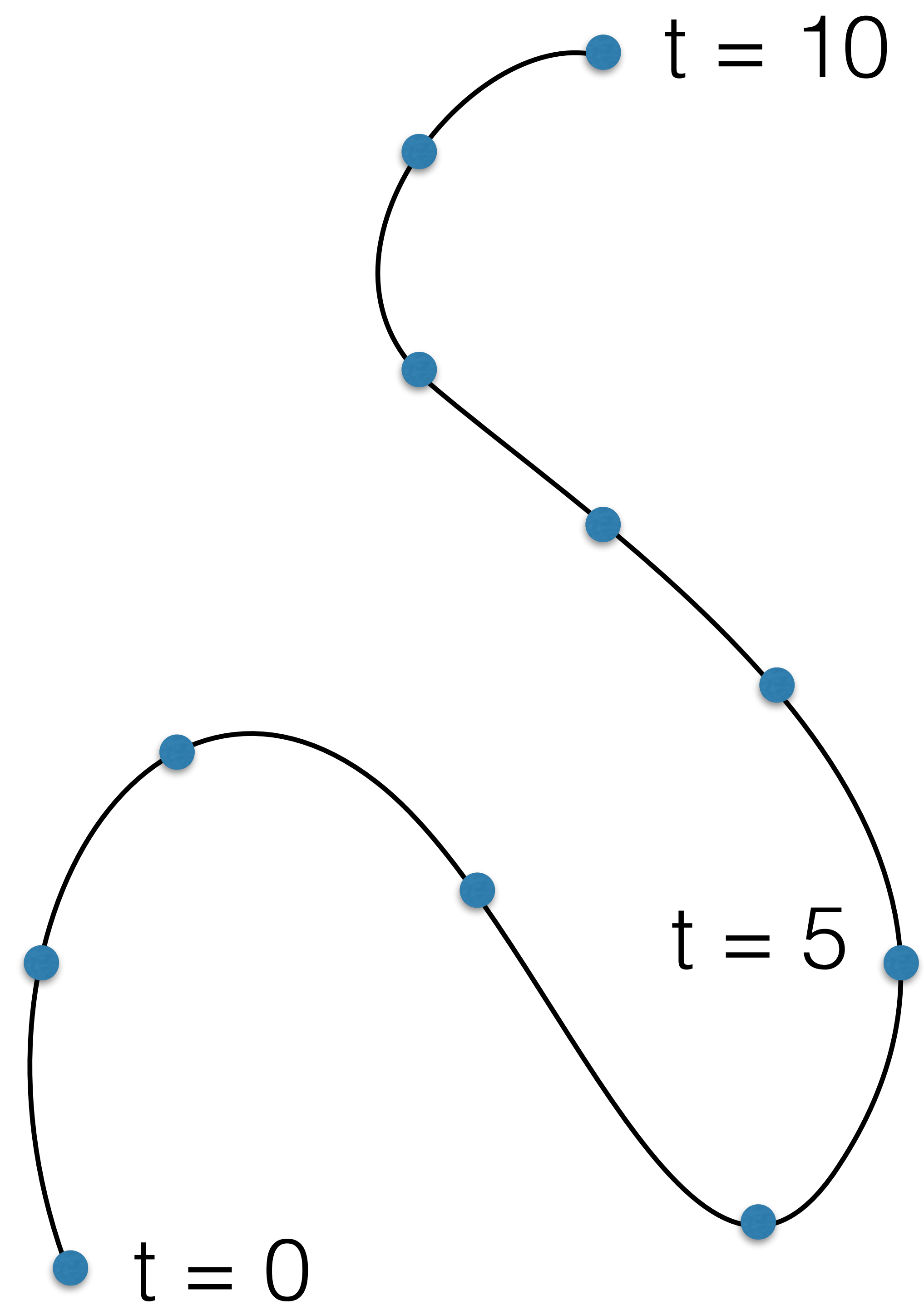

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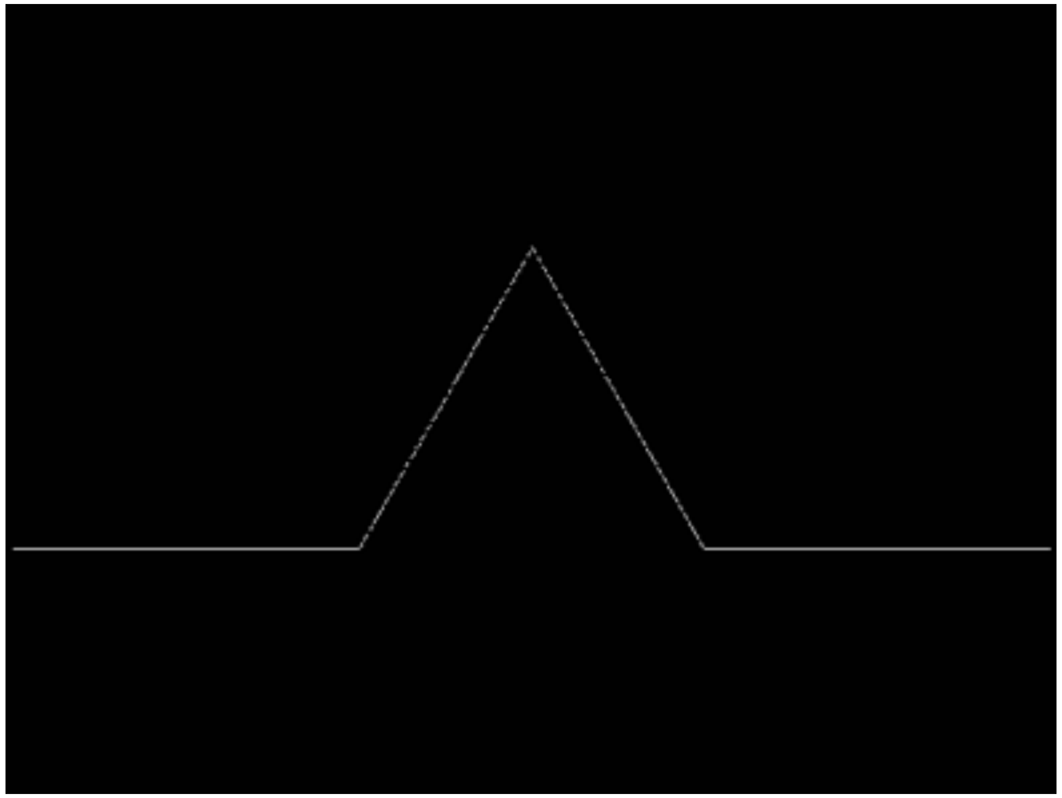
Procedural

e.g., fractals, subdivision schemes



[George Reese]

Fractal: Koch Curve

How do we specify a curve?

Implicit

(2D) f(x,y) = 0

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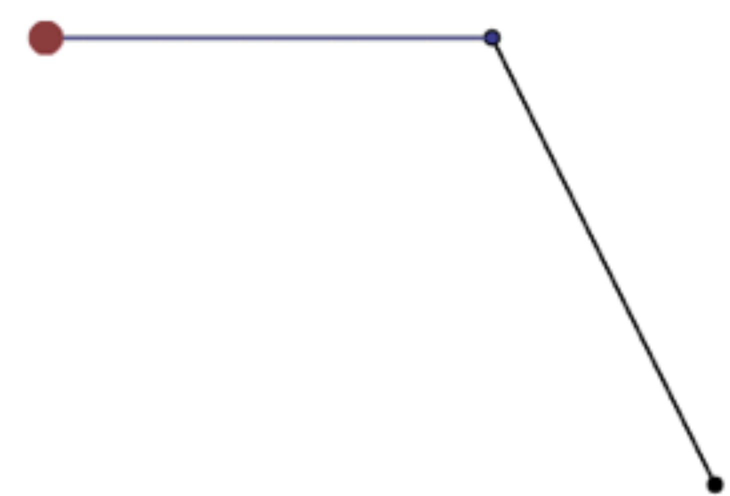
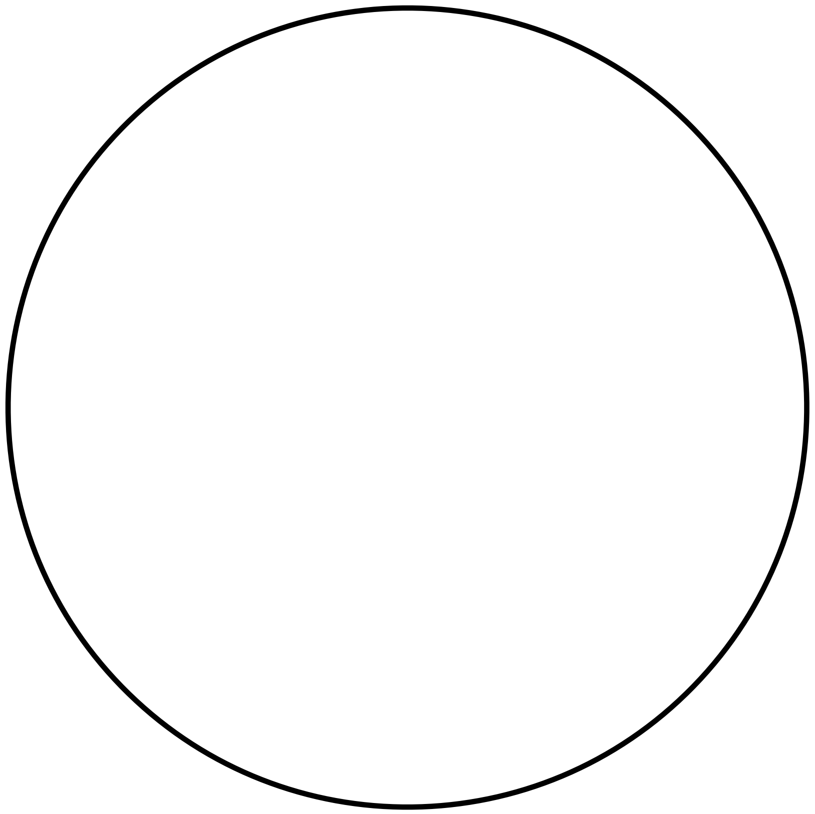


Bezier Curve

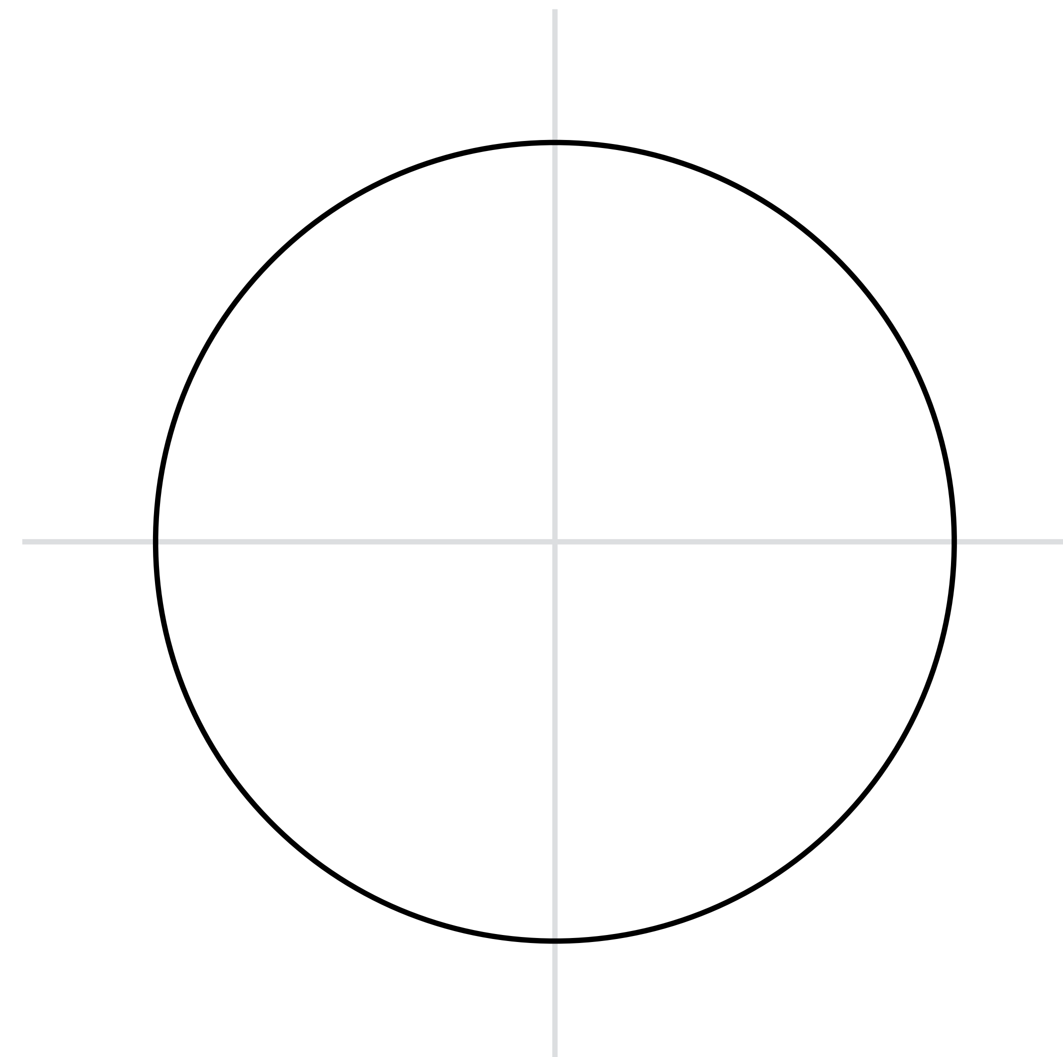
A curve may have multiple representations



A curve may have multiple representations

Implicit

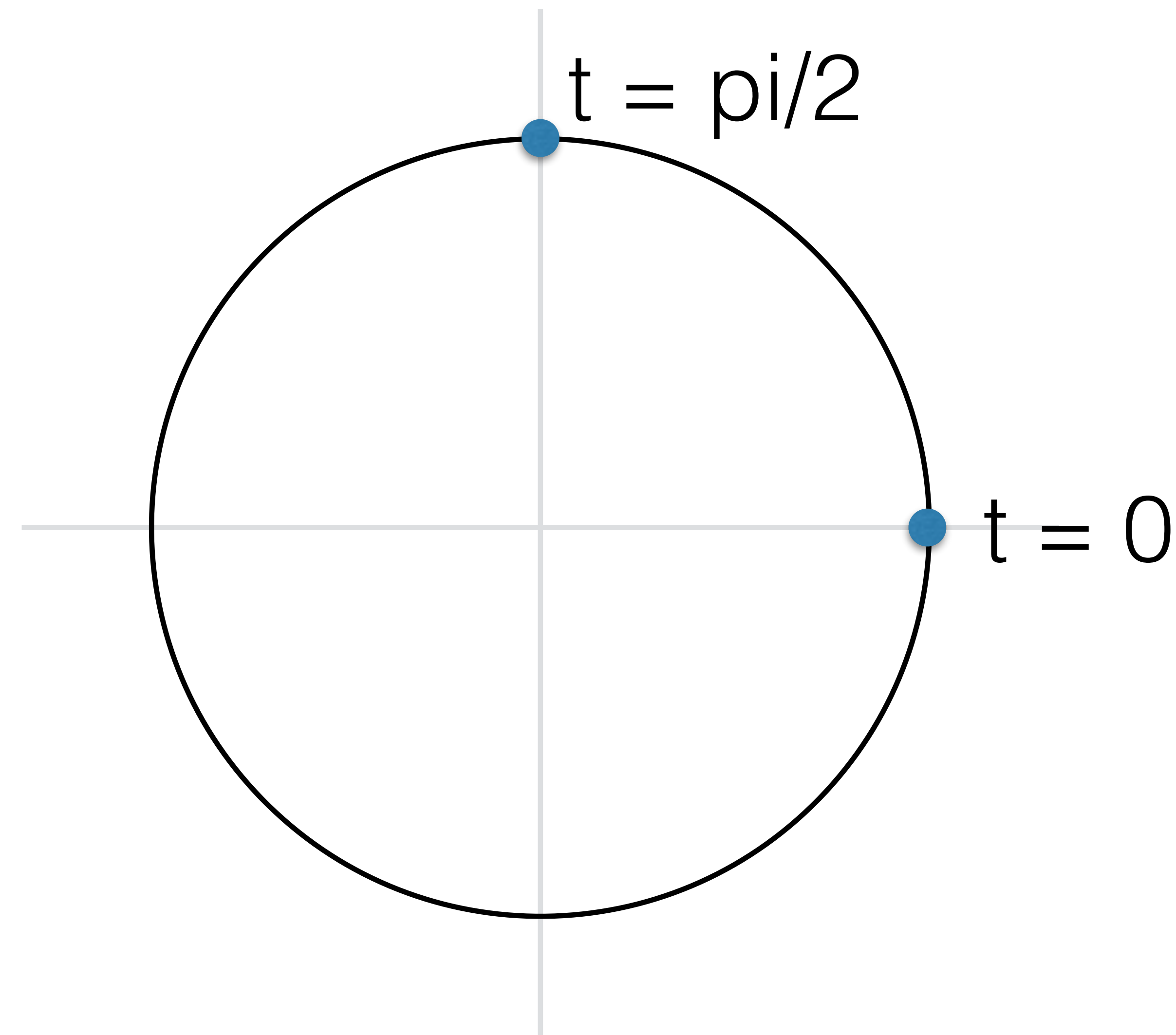
$f(x,y) = x^2 + y^2 - 1 = 0$



A curve may have multiple representations

Parametric

$(x,y) = \mathbf{f}(t) = (\cos t, \sin t)$



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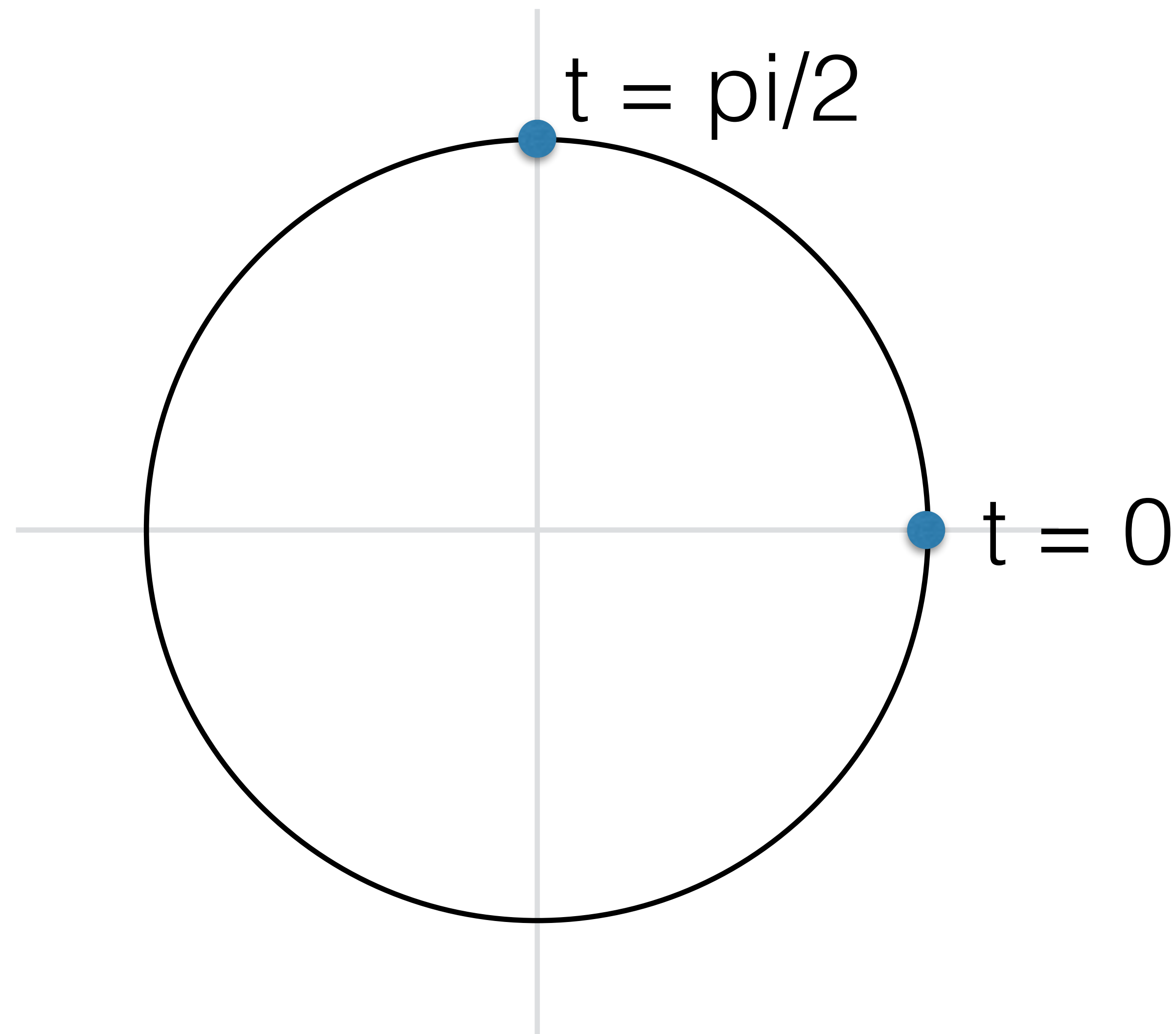


Parametric

(x,y) = **f**(t) = (cos t, sin t),
t in [0,2pi)

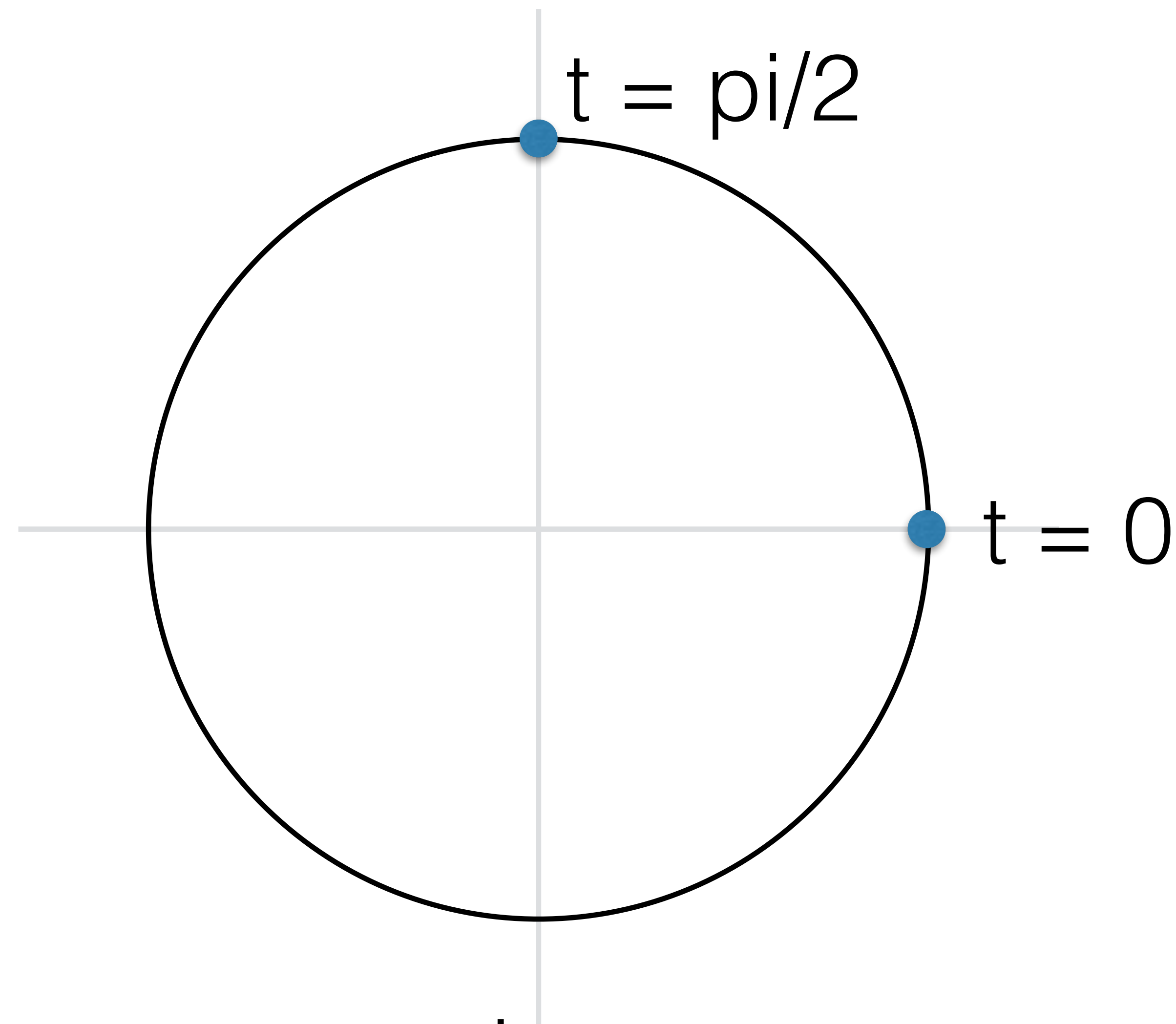
Same curve (set of points),
but different mathematical representation!

A curve may have multiple representations

Parametric

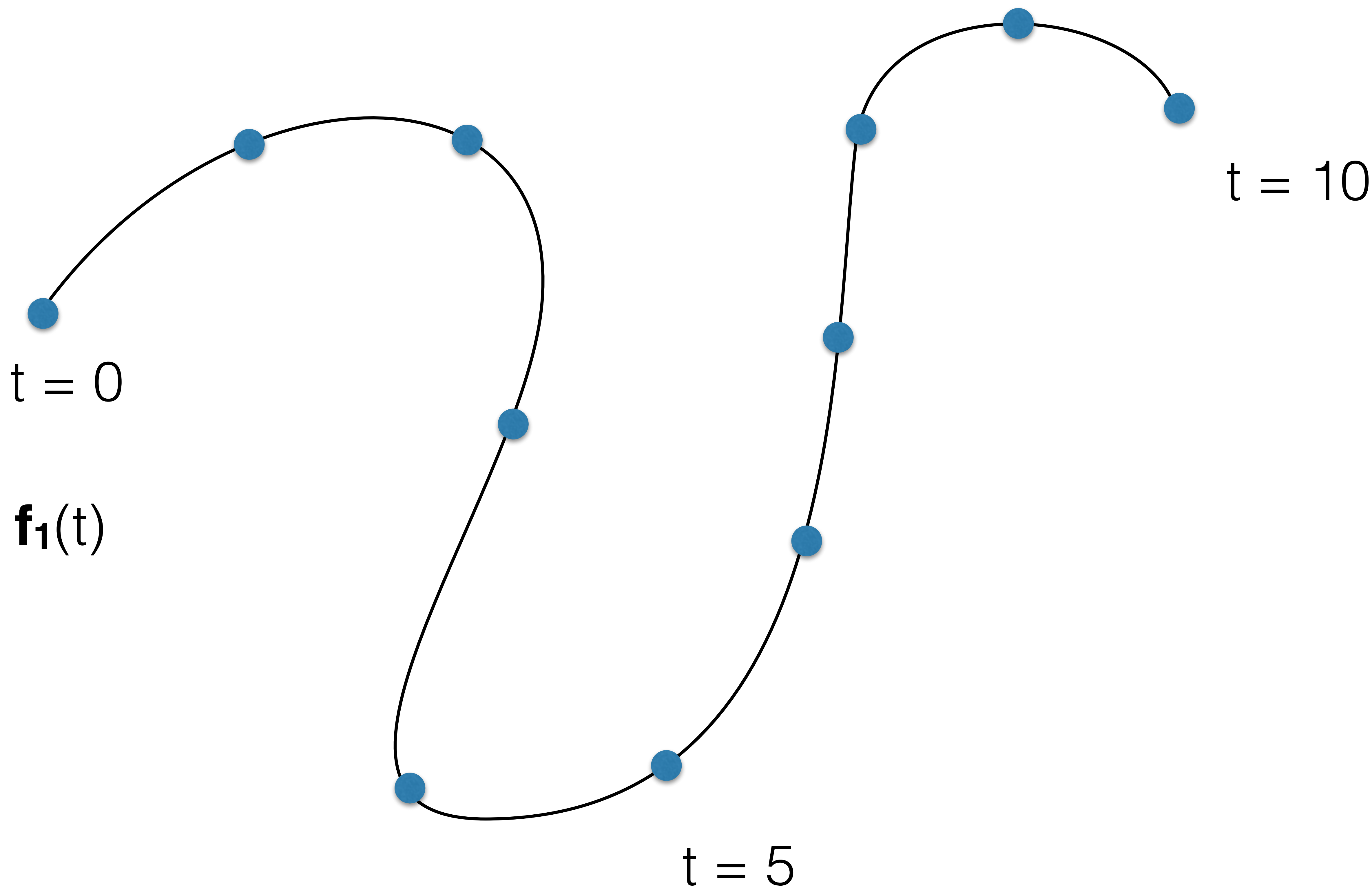
$(x,y) = \mathbf{f}(t) = (\cos t, \sin t),$

$t \text{ in } [0,2\pi)$



We will focus on parametric representations

Parameterization, re-parameterization



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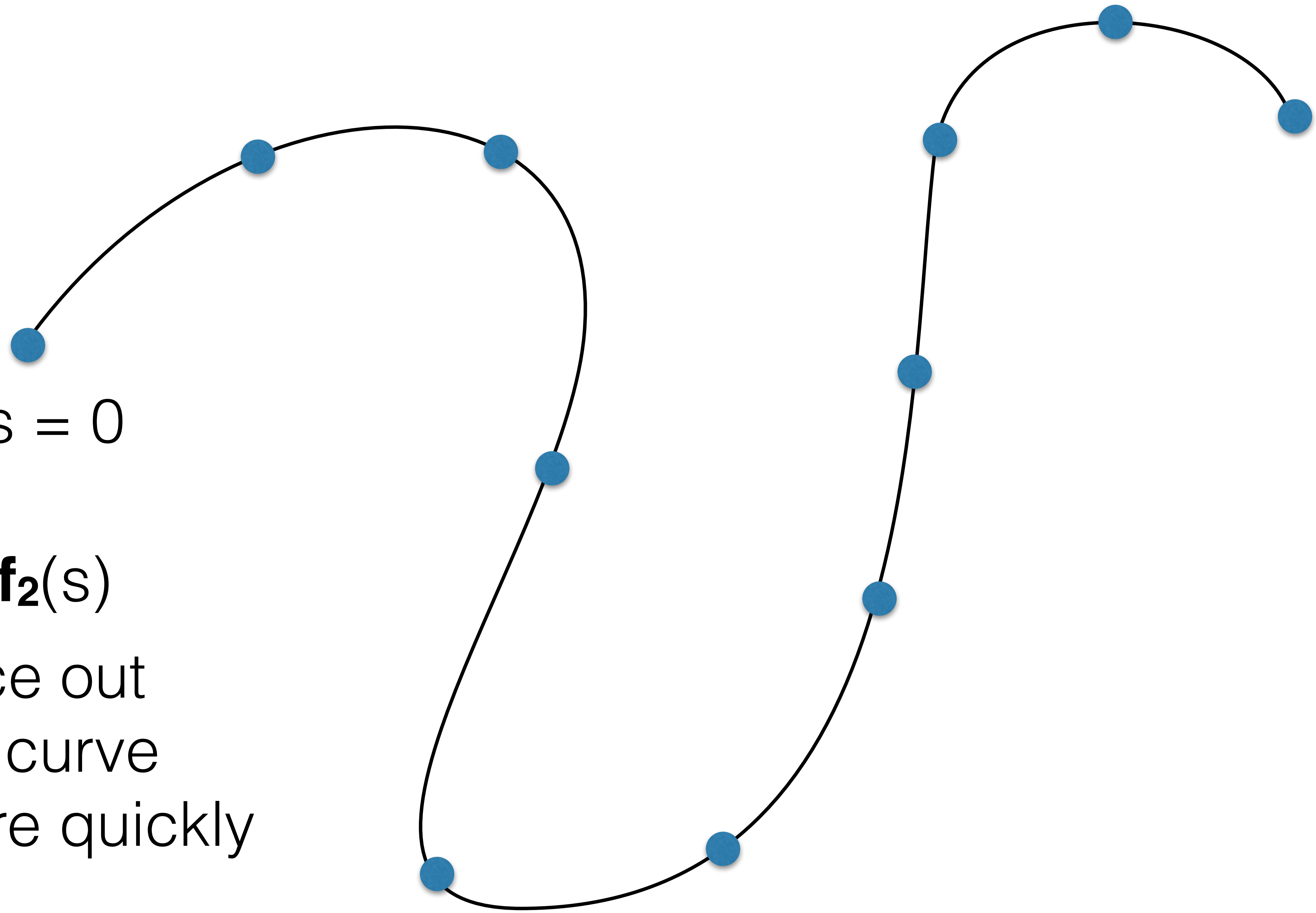
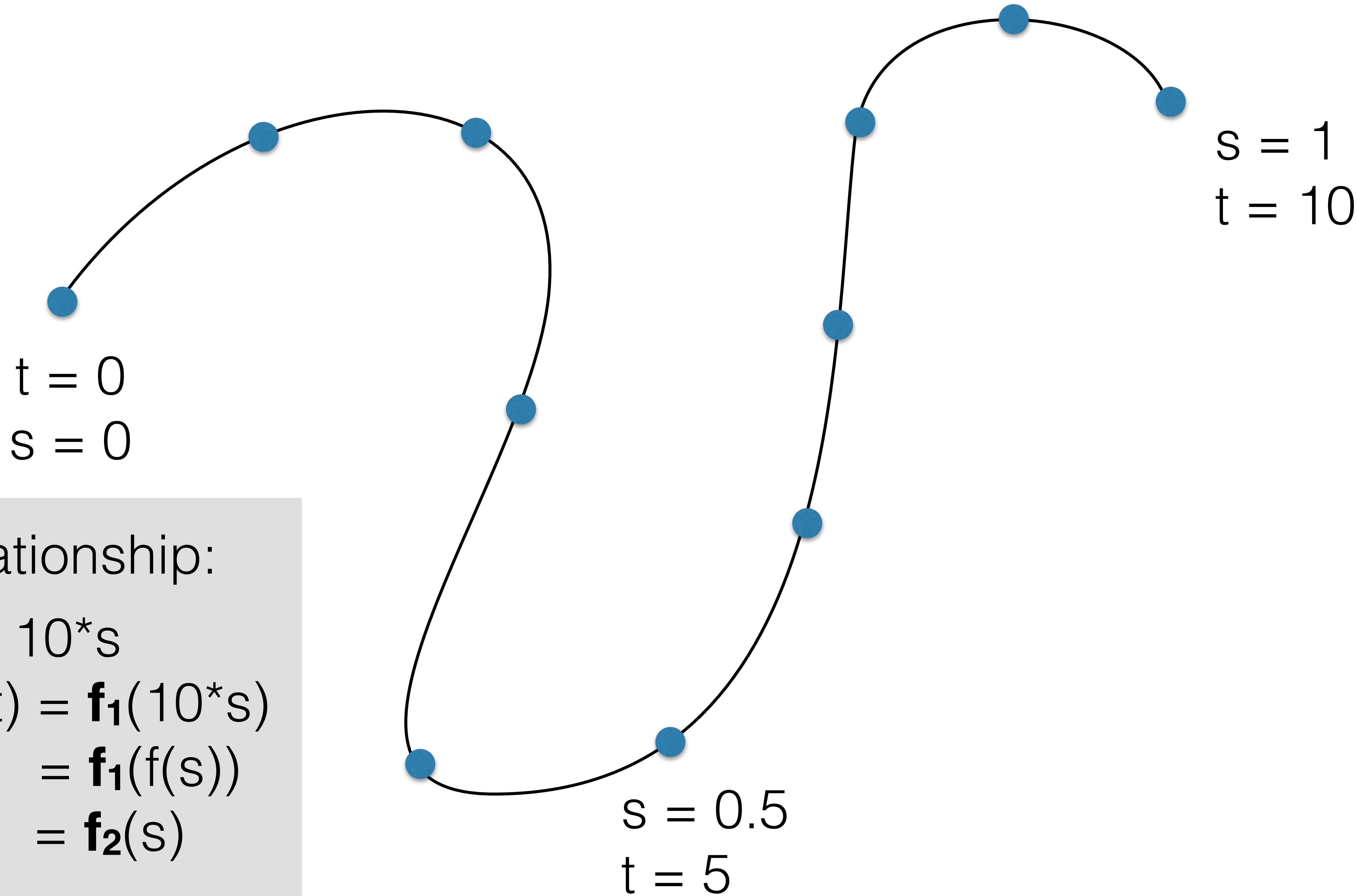


$\mathbf{f_2}(s)$

trace out
the curve
more quickly

Parameterization, re-parameterization



relationship:

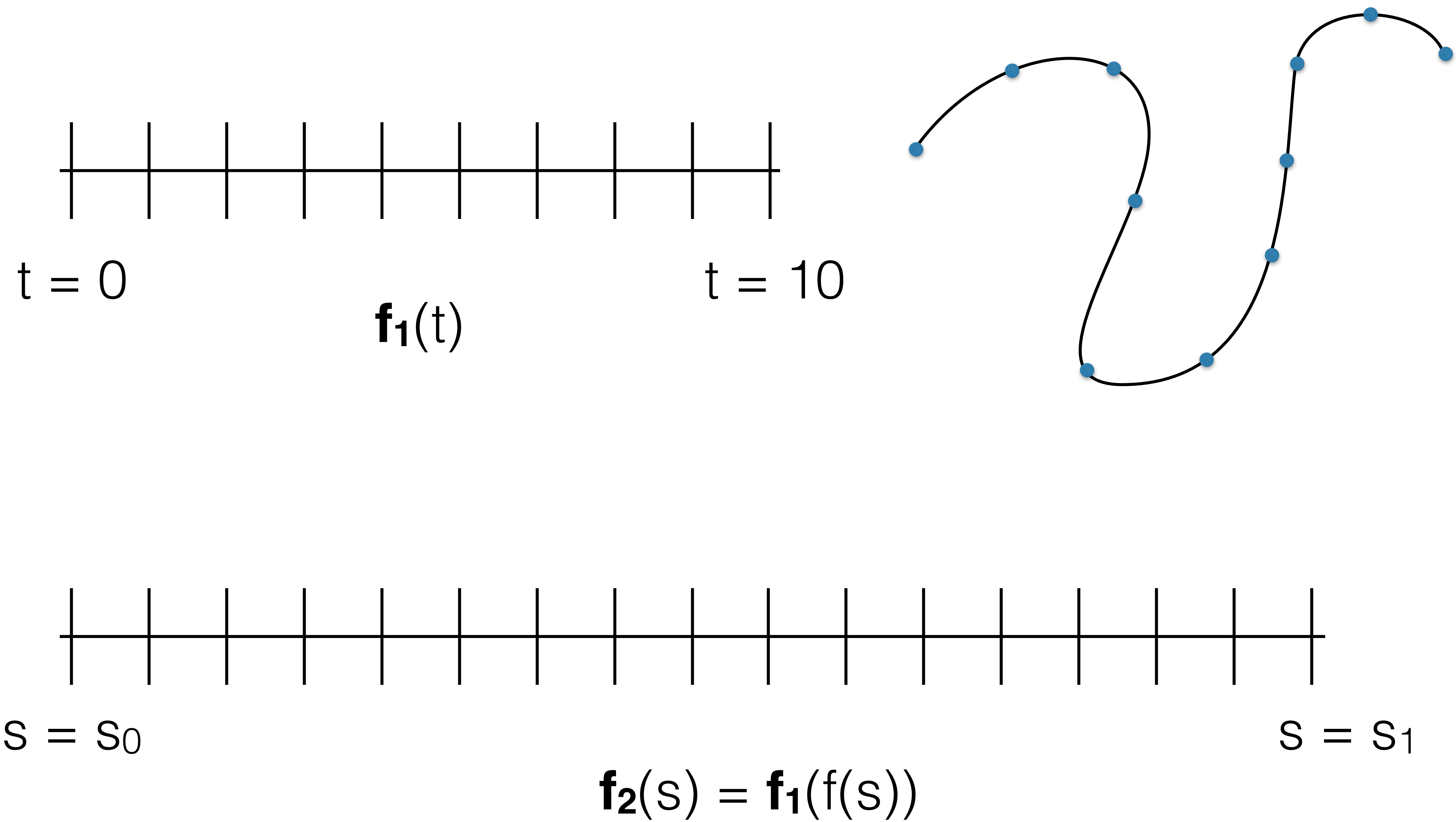
$t = 10^*s$

$\mathbf{f_1}(t) = \mathbf{f_1}(10^*s)$

$= \mathbf{f_1}(f(s))$

$= \mathbf{f_2}(s)$

Parameterization, re-parameterization



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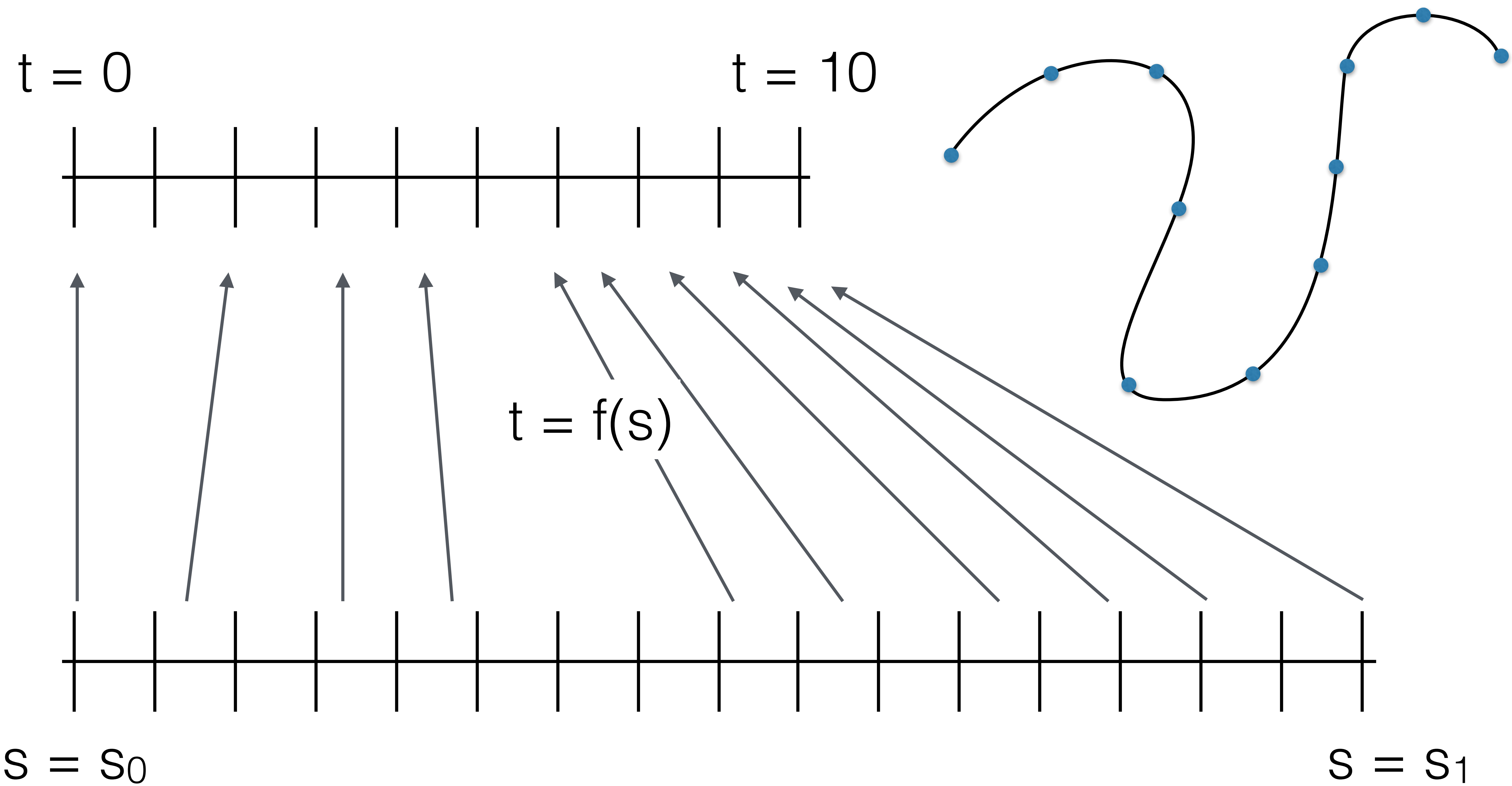
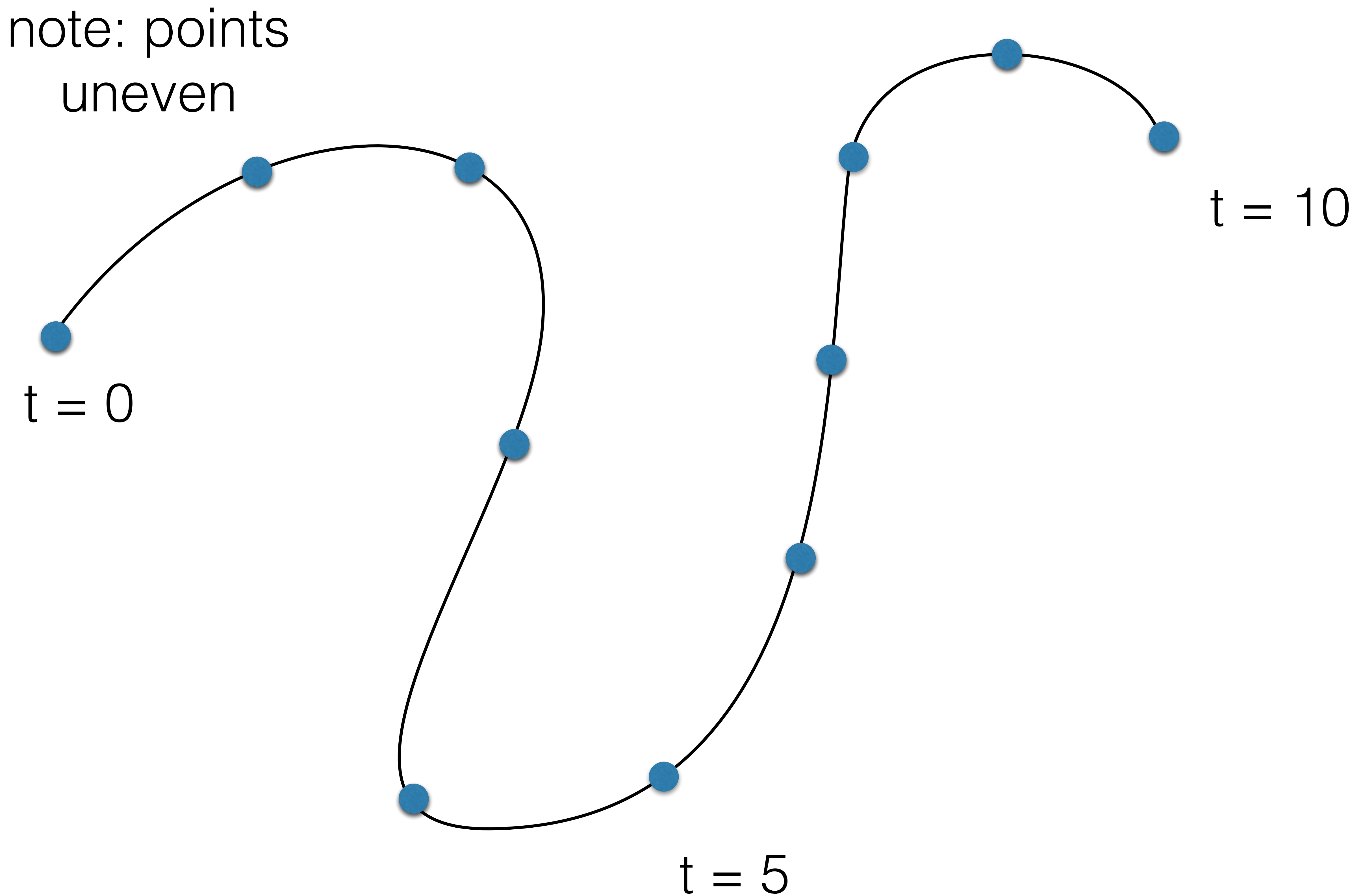


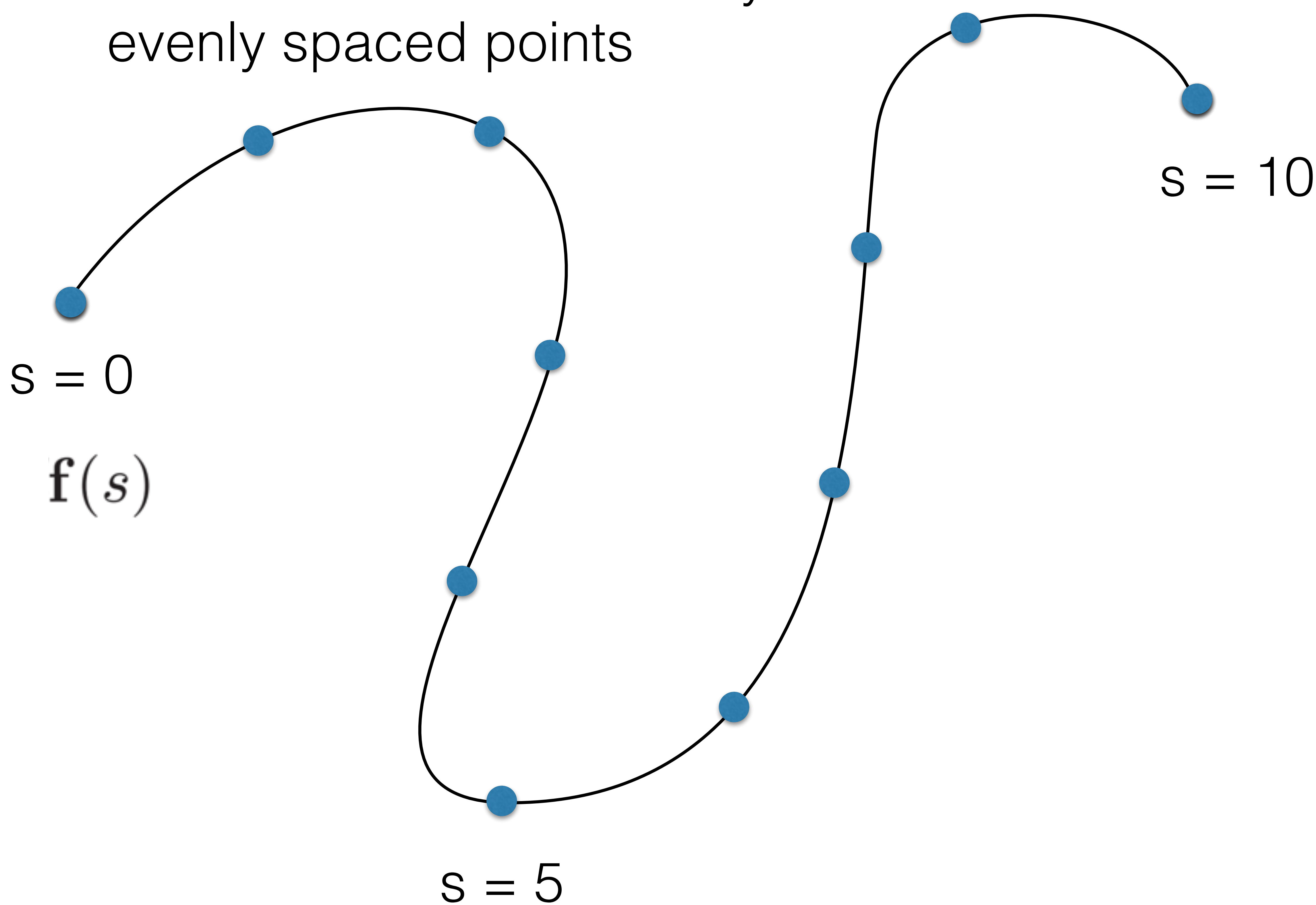
$$\mathbf{f_2}(s) = \mathbf{f_1}(f(s))$$

Natural parameterization



Natural parameterization

pen moves at a constant velocity:
evenly spaced points



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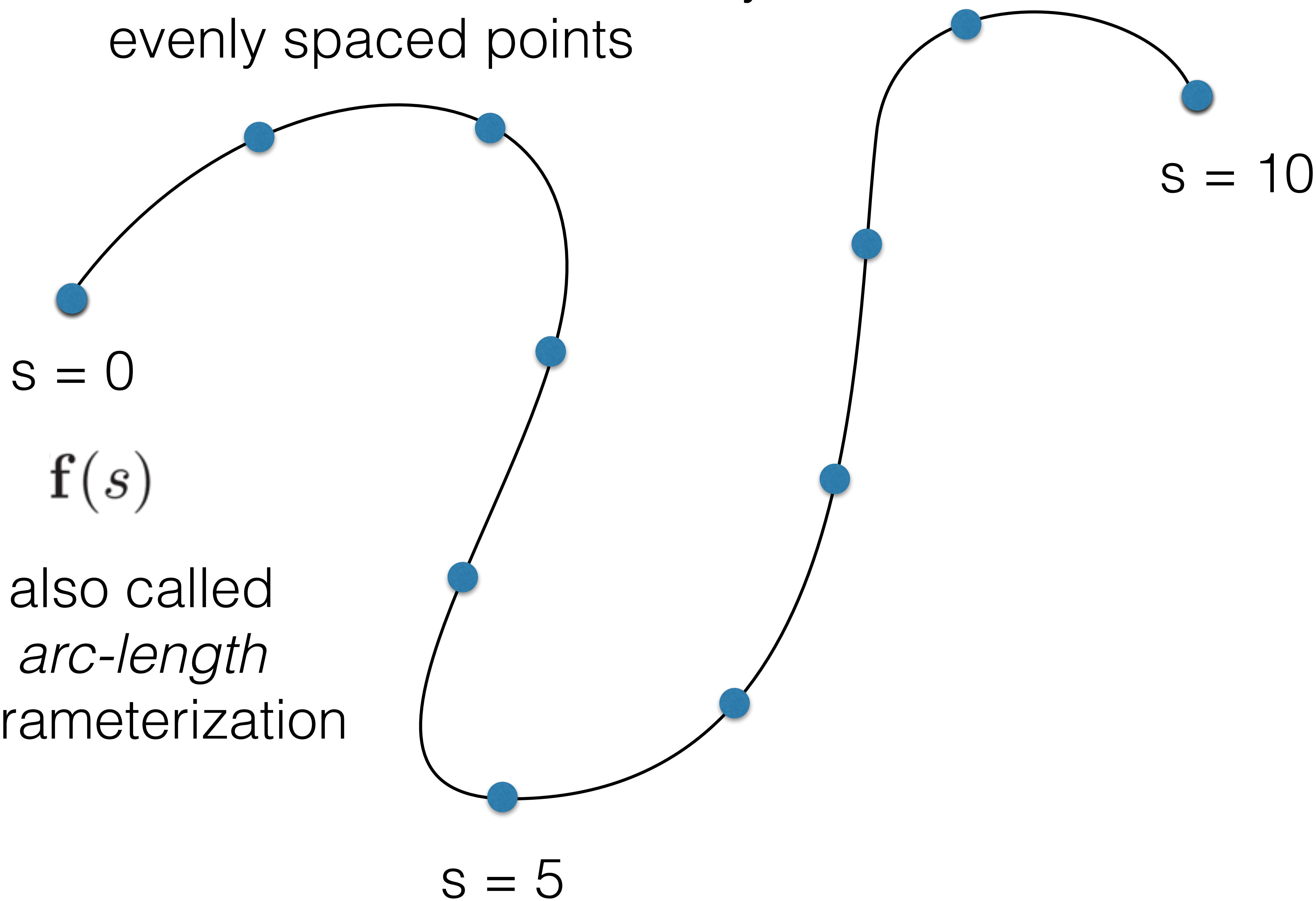

$\mathbf{f}(s)$

also called
arc-length
parameterization

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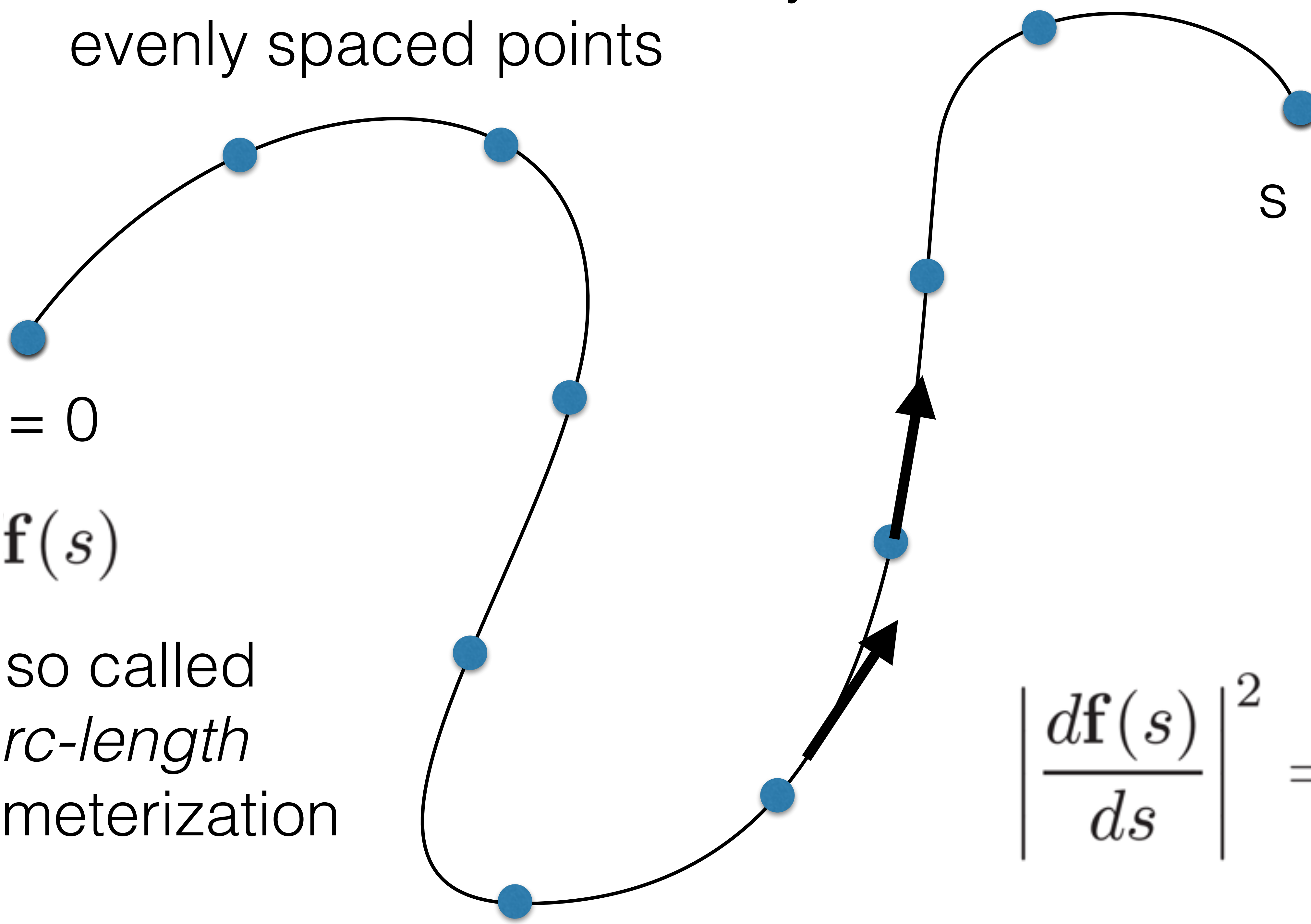

$\mathbf{f}(s)$

also called
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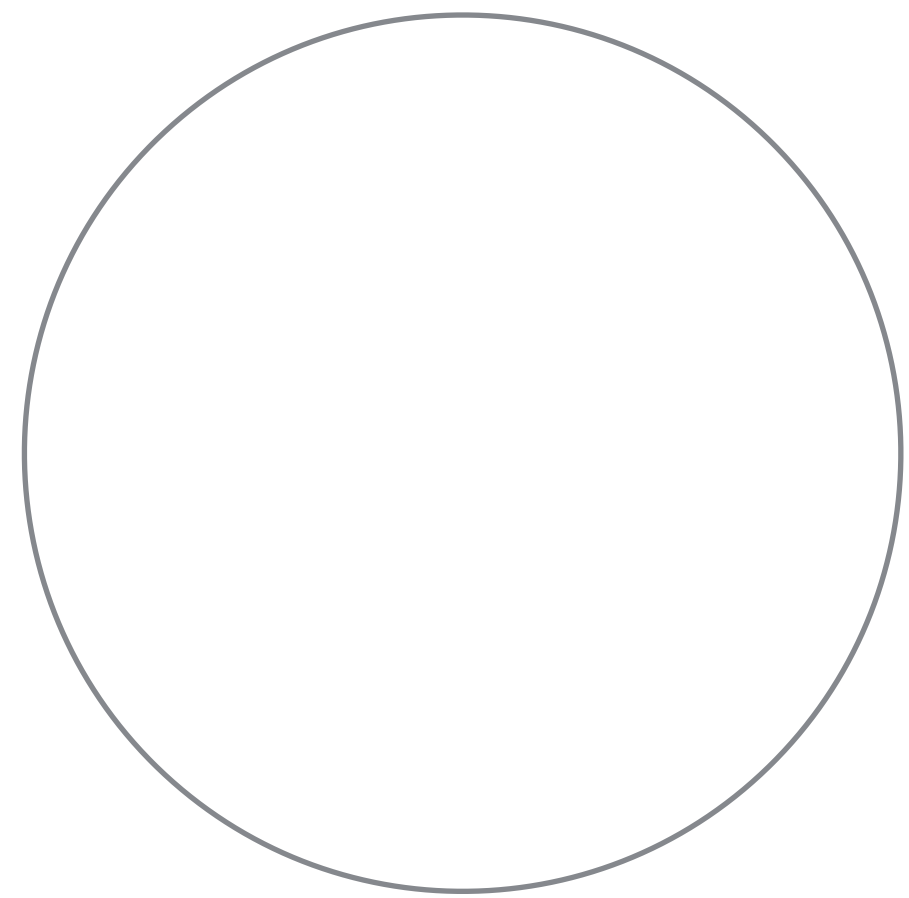
parameterization

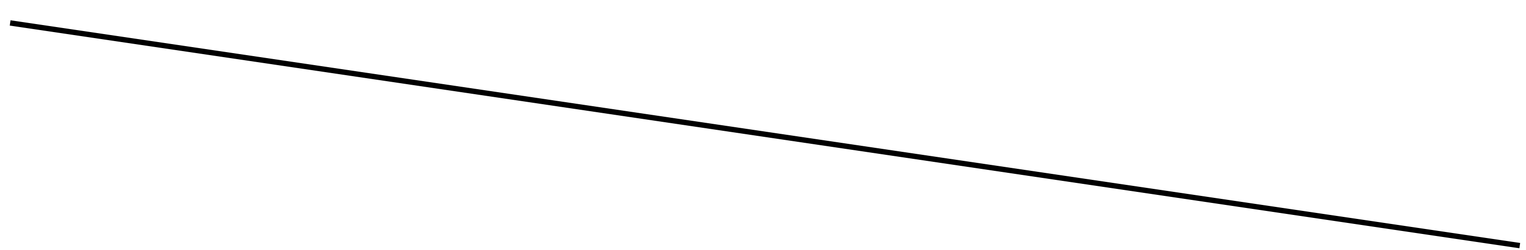
$$\left|\frac{d\mathbf{f}(s)}{ds}\right|^2 = c$$

piecewise parametric representation

sometimes easy
to find a parametric
representation

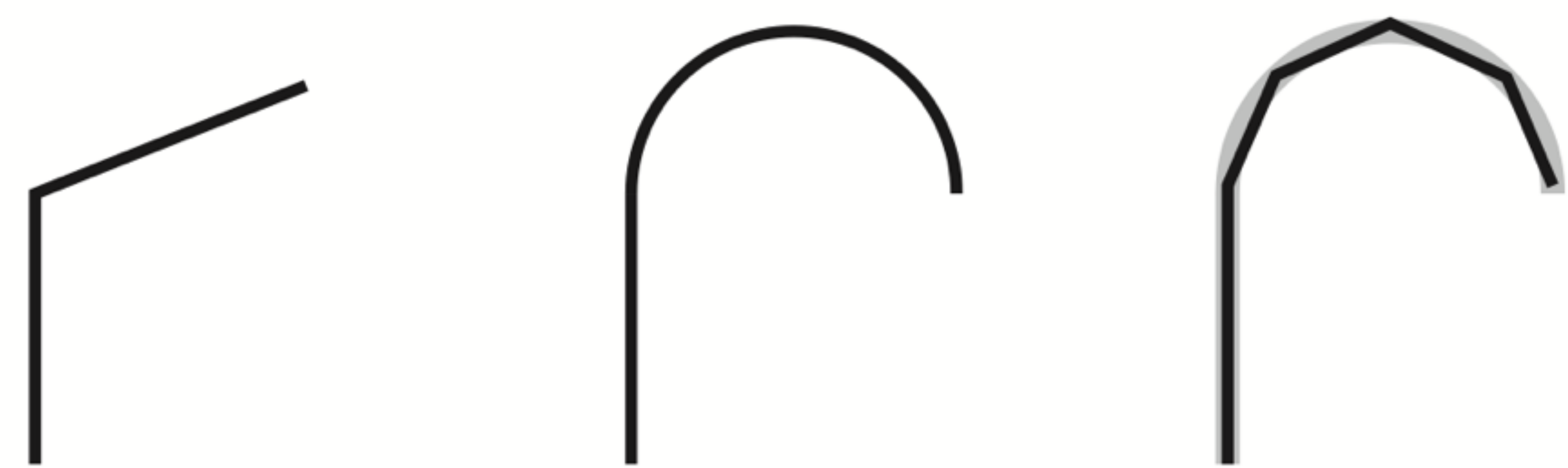
e.g., circle, line segment





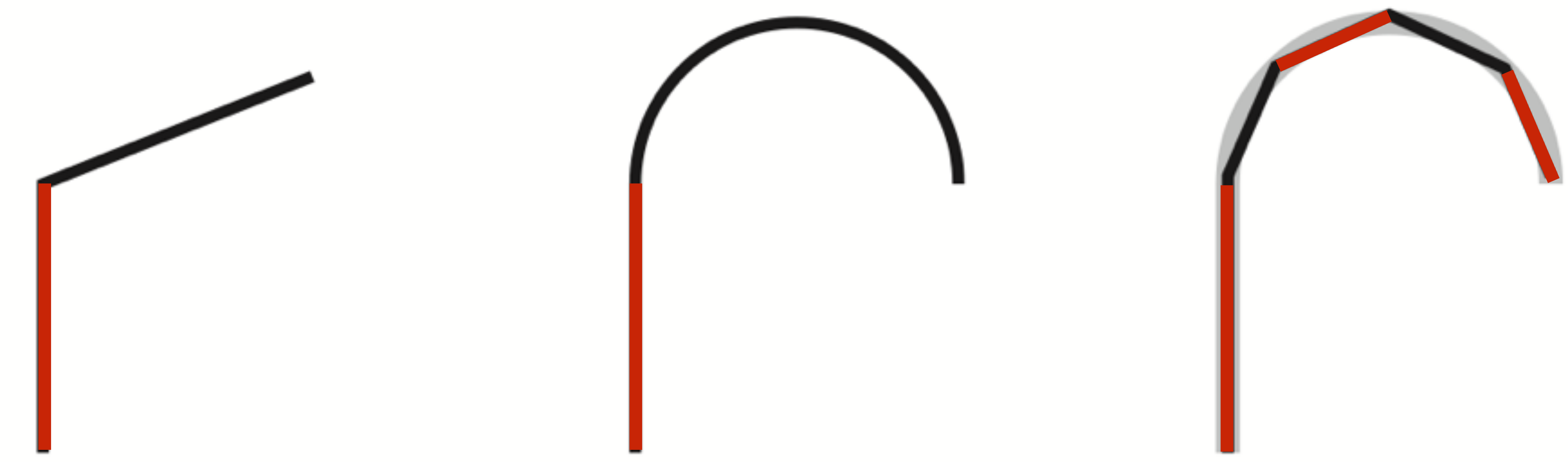
piecewise parametric representation

in other cases, not obvious



piecewise parametric representation

strategy: break into simpler pieces



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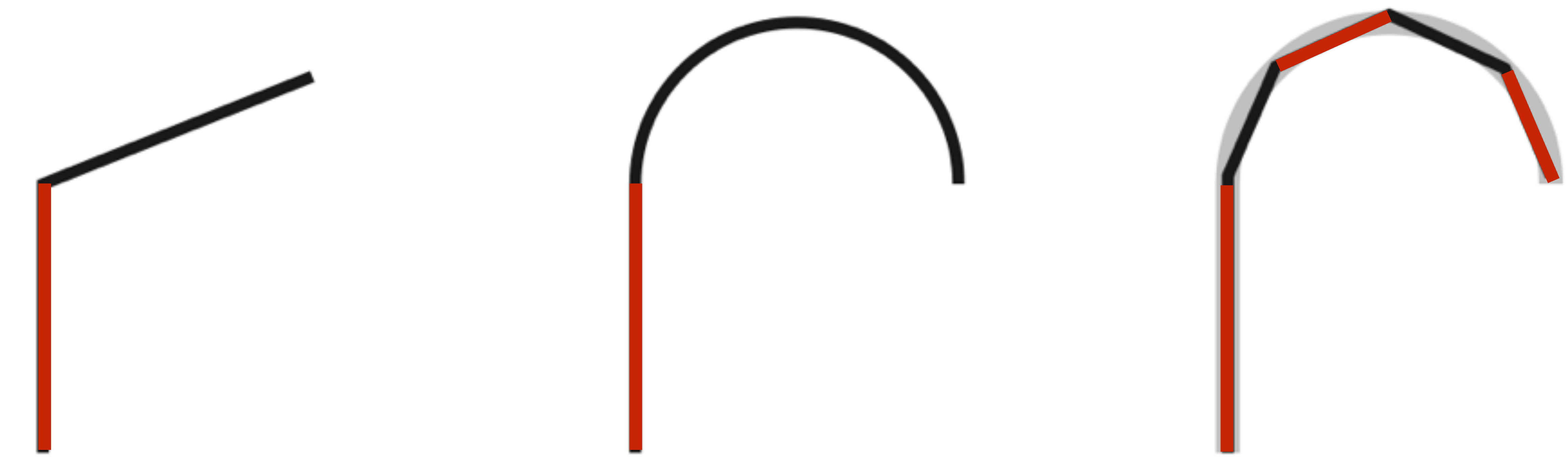

switch between functions that represent pieces:

$$\mathbf{f}(u) = \begin{cases} \mathbf{f}_1(2u) & u \leq 0.5 \\ \mathbf{f}_2(2u-1) & u > 0.5 \end{cases}$$

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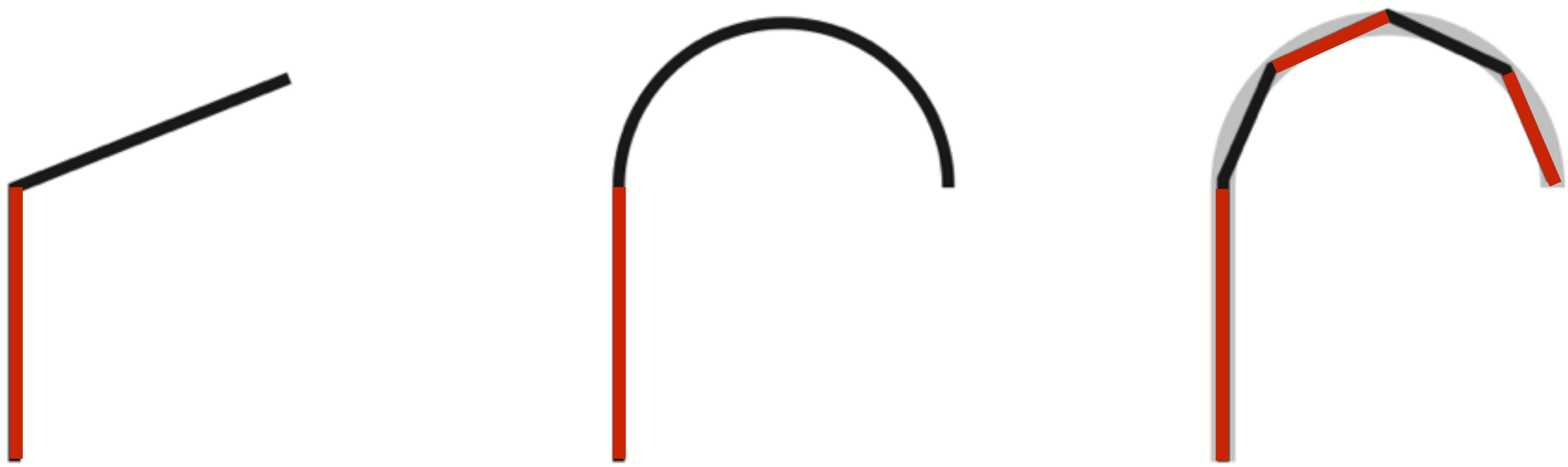

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map the inputs to
$\mathbf{f}_1$ and $\mathbf{f}_2$
to be from 0 to 1

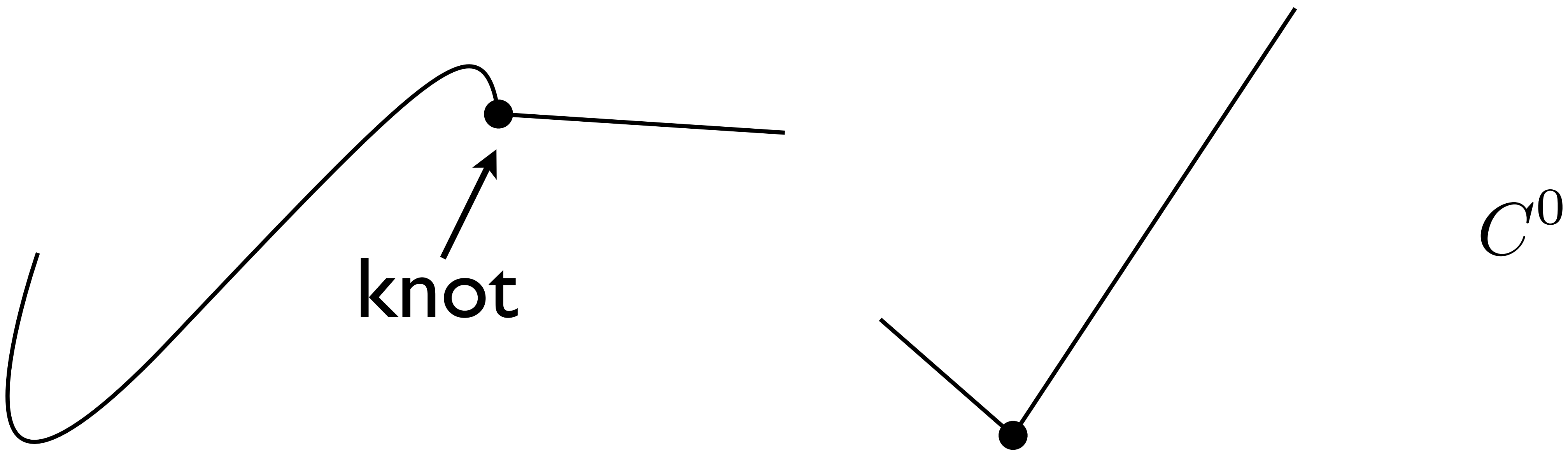
Curve Properties

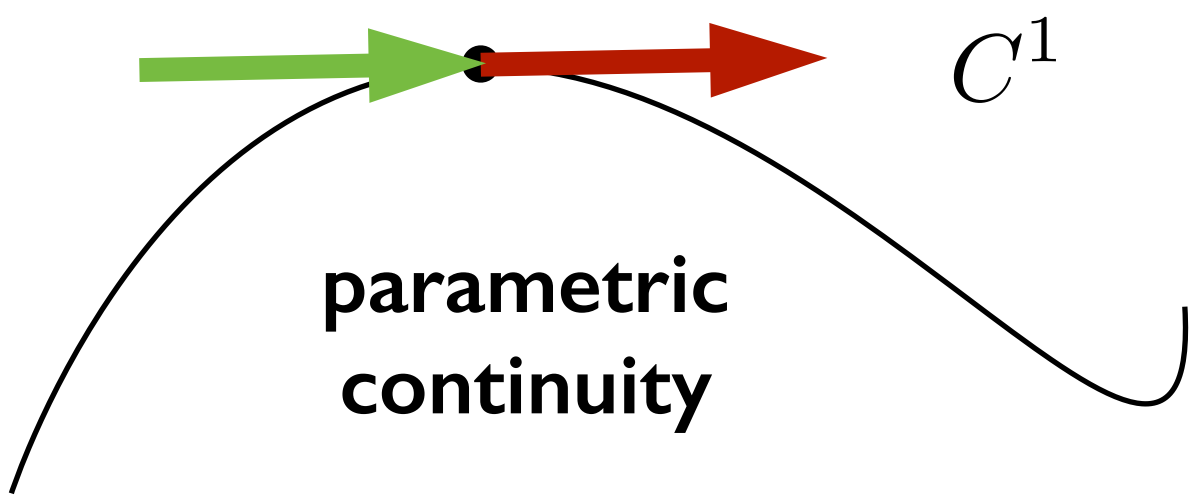
Local properties:
- continuity
- position
- direction
- curvature

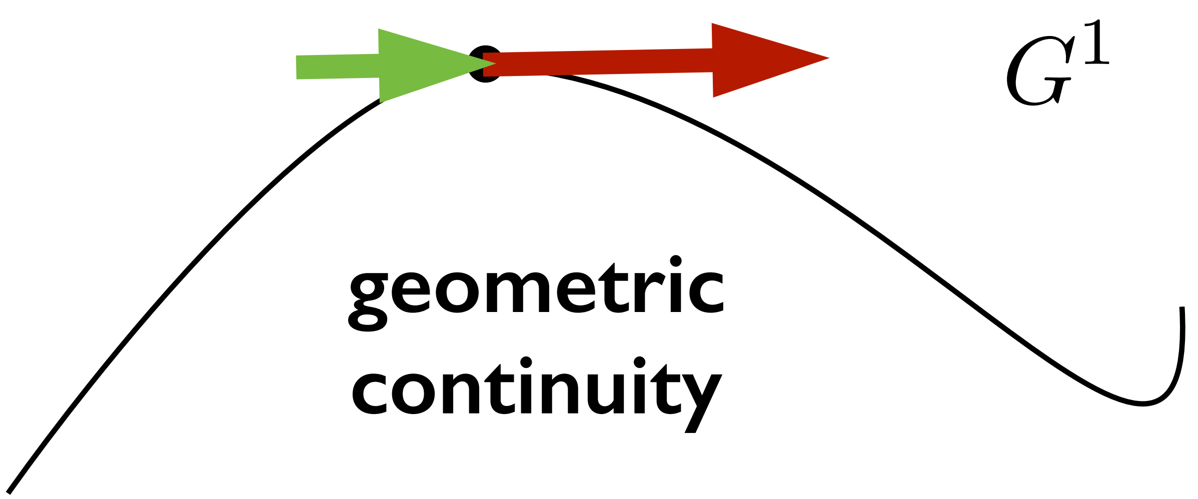
Global properties (examples):
- closed curve
- curve crosses itself

Interpolating vs. non-interpolating

Continuity: stitching curve segments together







Finding a Parametric Representation

Polynomial Pieces

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