You may not ask any questions during the test. If you believe that there is something wrong with a question, write down what you think the question is trying to ask and answer that.
<table>
<thead>
<tr>
<th>Question</th>
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1 True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively. You get −0.25 points for answering the question incorrectly and 0.5 points for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60% confident that your answer is correct).

1. (T/F) Texture data is passed to the fragment shader by OpenGL, in order to apply the texture map in fragment shader.

2. (T/F) Multiple texture maps can be used in the fragment shader to determine the final color of a fragment.

3. (T/F) Texture coordinates are typically assigned at vertices and interpolated to the interior of a triangle using the barycentric coordinates.

4. (T/F) A Bezier curve always passes through all of its control points.

5. (T/F) A Bezier curve can have 4 control points at most.

6. (T/F) Given N distinct points, there is a unique polynomial of degree N that goes through the points.

7. (T/F) If all control points of a Bezier curve lie on a line, then the Bezier curve lies on that line.

8. (T/F) In ray tracing, view rays are cast from the world position of a pixel towards camera position.

9. (T/F) One can decide if a line is intersecting with a sphere by combining the sphere and line equations and evaluating the discriminant of the quadratic formula.

10. (T/F) The intersection of a ray with a cube can be calculated analytically given the plane equations of each side of the cube.

11. (T/F) Semi-transparent objects can be rendered with ray tracing.

12. (T/F) Interpolation can be used to calculate intermediate points between two keyframes.

2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

13. Which of the following statements about mipmapping is true?

   (a) Using mipmapping, minification artifacts far from the camera can be alleviated.

   (b) A higher resolution texture is used further away from the camera, and a lower resolution texture is used closer to the camera.

   (c) The use of n resolution levels requires n times the amount of memory.

   (d) Magnification artifacts due to low texture resolution can be mitigated.

   (e) None of the above.
14. Which of the following statements about extended uses of texture maps is true?

(a) Bump mapping can be used to give the object a bumpy appearance in both the interior polygons and its silhouette.
(b) Normal mapping results in an increased polygon count.
(c) Shadow mapping can be used to add shadows in a z-buffer based rendering approach.
(d) Textures cannot be used to implement environment maps.
(e) None of the above.

15. Which statement about rotations is true?

(a) Interpolating rotations by linear interpolation of rotation matrices produces valid rotations.
(b) Interpolating rotations by SLERP provides visually superior results than linear interpolation of rotation matrices.
(c) A 45-degree rotation matrix can be calculated by taking $0.5 \times A + 0.5 \times B$, where $A$ and $B$ are rotation matrices of 0 and 90 degrees.
(d) It is not necessary to interpolate rotations in keyframe character animation.
(e) All of the above.

16. Which statement about ray intersections is true?

(a) If the direction of a ray is orthogonal to the normal of a plane, they can never intersect.
(b) A ray and a plane can intersect at most at one point.
(c) If the end point of a ray is inside a sphere, it intersects with the sphere exactly one time.
(d) All of the above.
(e) None of the above.

17. Given a ray tracing algorithm, if we add small random perturbations to each reflection ray, how will that change the resulting image?

(a) It will blur reflections in the image.
(b) The image will be distorted beyond recognition.
(c) It will appear grainy.
(d) It will increase aliasing artifacts.
(e) None of the above.
3 Written Response

18. Consider a ray with endpoint \( e \) and direction \( d \), given by the ray equation

\[
p(t) = e + td.
\]

and a triangle with vertices \( a, b, c \).

(a) Find an implicit equation for the plane containing the triangle, of the form

\[
f(p) = \mathbf{N} \cdot (p - \mathbf{q}) = 0
\]

where \( \mathbf{N} \) is a normal to the plane and \( \mathbf{q} \) is a point in the plane. Specify \( \mathbf{N} \) and \( \mathbf{q} \) in terms of the triangle vertices.

(b) Find the intersection point of the ray with the plane, if any, or specify how to determine that there is no intersection point.

(c) How would you determine whether the ray intersects the original triangle or not? You do not need to give all the mathematical details, but simply outline in words a procedure.
19. Quadratic Bezier curve. This problem will guide you through deriving the quadratic Bezier blending functions.

Given three control points $p_0$, $p_1$, and $p_2$, a quadratic Bezier curve

$$f(u) = a_0 + a_1 u + a_2 u^2$$

(1)

can be determined from the following conditions:

**condition 1** $f(0) = p_0$

**condition 2** $f(1) = p_1$

**condition 3** $f'(0) = 2(p_1 - p_0)$

(a) Fill in the right hand side of the equation below by differentiating equation (1).

$$f'(u) =$$

(2)

(b) Use conditions 1-3 and equations (1) and (2) to fill in the following linear system:

$$
\begin{pmatrix}
a_0 \\
a_1 \\
a_2
\end{pmatrix}
= 
\begin{pmatrix}
p_0 \\
p_1 \\
p_2
\end{pmatrix}
$$

(c) Given that

$$
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
a & b & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-a & -b & 1
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-a & -b & 1
\end{pmatrix}
$$

fill in the following linear system:

$$
\begin{pmatrix}
a_0 \\
a_1 \\
a_2
\end{pmatrix}
= 
\begin{pmatrix}
p_0 \\
p_1 \\
p_2
\end{pmatrix}
$$

(d) Use the above work to write down the quadratic Bezier blending functions $b_o(u)$, $b_1(u)$, $b_2(u)$, such that

$$f(u) = b_o(u)p_0 + b_1(u)p_1 + b_2(u)p_2.$$  

(hint: recall that $f(u) = \mathbf{u}^T \mathbf{a}$, where $\mathbf{u} = (1, u, u^2)^T$ and $\mathbf{a} = (a_0, a_1, a_2)^T$.)

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