You may not ask any questions during the test. If you believe that there is something wrong with a question, write down what you think the question is trying to ask and answer that.
<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
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<tbody>
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<td>Written Response</td>
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<td><strong>Total</strong></td>
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</table>
1 True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively. You get −0.25 points for answering the question incorrectly and 0.5 points for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60% confident that your answer is correct).

1. (T/F) Shaders are programs that run on the GPU.
2. (T/F) The fragment shader determines the position of the vertexes.
3. (T/F) For nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v} = 0$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are parallel.
4. (T/F) For nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, if and only if $\mathbf{u}$ and $\mathbf{v}$ are parallel.
5. (T/F) The product of a matrix with a vector is a vector.
6. (T/F) Barycentric coordinates cannot be negative.
7. (T/F) For any point $\mathbf{p}$ inside a triangle $T$, the sum of barycentric coordinates of $\mathbf{p}$ with respect to $T$ is always equal to 1.
8. (T/F) Orthographic projections preserve parallel lines.
9. (T/F) If the rotation matrix $R$ rotates a point by the angle $\theta$, then $R^T$ rotates the point by $-\theta$.
10. (T/F) Scaling preserves the area of polygons.
11. (T/F) DDA provides antialiasing effects when rasterizing lines.
12. (T/F) For any vector $\mathbf{v}$, $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||$.

2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

13. Which statement about OpenGL is true?

(a) The `glPushMatrix` function restores to the top of the stack the matrix that was returned by the most recent `glPopMatrix` operation.
(b) `glColor` sets the color of the most recently drawn primitive or object.
(c) After modifying only the vertex shader, the C++ application code must be recompiled in order to see the effects of changes.
(d) A program can utilize the OpenGL programmable shader pipeline, by providing only a vertex shader.
(e) None of the above
14. Match the type of transformation in the left column with the example transformation matrix in the right by drawing lines between the matching boxes.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Matrix</th>
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</table>
| Translation   | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] |
| Rotation      | \[
\begin{pmatrix}
\cos \theta & 0 & \sin \theta & 0 \\
0 & 1 & 0 & 0 \\
-\sin \theta & 0 & \cos \theta & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] |
| Perspective   | \[
\begin{pmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] |
| Identity      | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -3 & -2
\end{pmatrix}
\] |

15. OpenGL perspective projection
   (a) preserves parallel lines.
   (b) is a linear transformations in $z$.
   (c) is an affine transformations.
   (d) preserves the $z$ ordering between the near and far planes, but not necessarily everywhere else.
   (e) preserves the $z$ ordering everywhere.

16. Which statement about transformation matrices is true?
   (a) A uniform scaling transformation, $S$, can be represented as $S = \alpha I$, where $\alpha$ is a scalar and $I$ is the identity matrix.
   (b) Two translation matrices, $T_1$ and $T_2$ can be combined into another translation matrix $T$ by adding them, i.e. $T = T_1 + T_2$.
   (c) The inverse of a scaling matrix is always equal to its transpose.
   (d) For a rotation matrix $R$ and a translation matrix $T$, $RT = TR$, even though matrix multiplication is not generally commutative.
   (e) None of the above

17. Which sentence completion is false? The Phong reflectance model
   (a) calculates the red, green, and blue color channels independently.
   (b) captures ambient, diffuse, and specular components.
   (c) does not consider the shadow of an object on itself in its calculations.
   (d) does not consider the geometry of the object being shaded.
   (e) requires the normal of the object at the point being shaded.
3 Written Response

18. For each of the four triangles and points \( p \) shown below, give the barycentric coordinates of the point \( p \) with respect to the triangle.
19. Come up with a series of transformation matrices as well as an order of multiplication (you don’t need to actually perform the multiplication) to transform the rectangle with vertices 

\[ a = (1, 0), \quad b = (3, 0), \quad c = (3, 1), \quad d = (1, 1) \]

into the rectangle with vertices 

\[ a' = (-1, 0), \quad b' = (-1, 1), \quad c' = (-2, 1), \quad d' = (-2, 0) \]

(specifically, \(a, b, c, d\) map to \(a', b', c', d'\), respectively). Sketch the rectangle at every step of the transformation.
20. Consider a ray with endpoint \( a \) and a normalized direction \( u \), and a plane with normal \( n \) and point \( q \). The parametric equation for the ray is

\[ p(t) = a + tu, \quad t \geq 0. \]

The implicit equation for the plane is

\[ f(p) = n \cdot (p - q) = 0. \]

Determine if the ray intersects the plane, and find any intersection points. Show your work.