CS130 : Computer Graphics
Lecture 3: Rasterizing Lines and Triangles

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Rendering approaches

1. **object-oriented**
   foreach object ...

2. **image-oriented**
   foreach pixel ...

vertices → **3D rendering pipeline** → image
Outline

clipping - clip objects to viewing volume
rasterization - make fragments from clipped objects
hidden surface removal - determine visible fragments
What is rasterization?

Rasterization is the process of determining which pixels are “covered” by the primitive.
What is rasterization?

**input**: primitives  **output**: fragments

- enumerate the pixels covered by a primitive
- interpolate attributes across the primitive
Rasterization

Compute integer coordinates for pixels covered by the 2D primitives

Algorithms are invoked many, many times and so must be efficient

Output should be visually pleasing, for example, lines should have constant density

Obviously, they should be able to draw all possible 2D primitives
Screen coordinates
Line Representation
Math Review

- 2D math for lines

How do we determine the equation of the line?
Math Review

• Explicit (functional) representation
  \[ y = f(x) \]

  \( y \) is the dependent, \( x \) independent variable

Find value of \( y \) from value of \( x \)

Example, for a line:
  \[ y = mx + b \]
Math Review

- 2D math for lines
  
  Slope-Intercept formula for a line
  
  \[
  \text{Slope} = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(Y - Y_1)}{(X - X_1)}
  \]
  
  Solving for Y
  
  \[
  Y = \left[\frac{(Y_2 - Y_1)}{(X_2 - X_1)}\right]X + \left[\frac{-(Y_2 - Y_1)}{(X_2 - X_1)}\right]X_1 + Y_1 \text{ or } Y = mX + b
  \]
Math Review

- Parametric Representation

\[ x = x(u), \ y = y(u) \]

where new parameter \( u \) (or often \( t \)) determines the value of \( x \) and \( y \) (and possibly \( z \)) for each point.

\( x, y \) treated the same, axis invariant.
Parametric formula for a line

\[ X = X_1 + t(X_2 - X_1) \]
\[ Y = Y_1 + t(Y_2 - Y_1) \]

for parameter \( t \) from 0 to 1

Therefore, when

- \( t = 0 \) we get \((X_1,Y_1)\)
- \( t = 1 \) we get \((X_2,Y_2)\)

Varying \( t \) gives the points along the line segment
Implicit Line Equation

\[ f(X) = N \cdot (X - X_0) = 0 \]

\(<\text{whiteboard}>\)
Implicit Line Equation

$$f(X) = N \cdot (X - X_0) = d$$

- $d > 0$
- $d < 0$
- $d = 0$

$x_0 = (x_0, y_0)$
Implicit Line Equation

\[ f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = d \]

- \( d > 0 \)
- \( d < 0 \)
- \( d = 0 \)

Decision variable, \( d \)

\( \mathbf{X}_0 = (x_0, y_0) \)
Implicit Line Equation

\[ f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = d \]

- \( d > 0 \)
- \( d < 0 \)
- \( d = 0 \)

Let \( \mathbf{X} = (x_0, y_0) \)
Implicit Line Equation

Decision variable, \( d \)

\[ f(X) = N \cdot (X - X_0) = d \]

- \( d > 0 \)
- \( d < 0 \)
- \( d = 0 \)

\( X_0 = (x_0, y_0) \)
Line Drawing
DDA algorithm for lines

Parametric Lines: the DDA algorithm (digital differential analyzer)

\[ Y_{i+1} = m \cdot x_{i+1} + B \]

\[ = m(x_i + \Delta x) + B \quad \Delta x = (x_{i+1} - x_i) \]

\[ = y_i + m(\Delta x) \quad \text{<- must round to find int} \]

If we increment by 1 pixel in \(X\), we turn on \([x_i, \text{Round}(y_i)]\) or same for \(Y\) if \(m > 1\)
Scan conversion for lines

DDA includes Round(); and this is fairly slow

For Fast Lines, we want to do only integer math +,-

We do this using the **Midpoint Algorithm**

To do this, let's look at lines with \( y \)-intercept \( B \) and with slope between 0 and 1:

\[
y = \frac{dy}{dx}x + B \quad \Longrightarrow \quad f(x,y) = (dy)x - (dx)y + B(dx) = 0
\]

Removes the division => slope treated as 2 integers
Which pixels should be used to approximate a line?

Draw the thinnest possible line that has no gaps
Line drawing algorithm
(case: $0 < m \leq 1$)

\[
y = y_0 \\
\text{for } x = x_0 \text{ to } x_1 \text{ do} \\
\quad \text{draw}(x,y) \\
\quad \text{if } (<\text{condition}> ) \text{ then} \\
\quad \quad y = y + 1
\]

• move from left to right
• choose between 
  \((x+1,y)\) and \((x+1,y+1)\)
Line drawing algorithm

(case: $0 < m \leq 1$)

$y = y_0$
for $x = x_0$ to $x_1$ do
  draw$(x, y)$
  if ($<\text{condition}>$) then
    $y = y + 1$

• move from left to right
• choose between $(x+1, y)$ and $(x+1, y+1)$
Use the midpoint between the two pixels to choose
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Use the midpoint between the two pixels to choose

implicit line equation:
\[ f(X) = N \cdot (X - X_0) = 0 \]

<whiteboard>
evaluate \( f \) at midpoint:
\[ f(x, y + \frac{1}{2}) \not= 0 \]
Use the midpoint between the two pixels to choose

implicit line equation:

\[ f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0 \]

evaluate \( f \) at midpoint:

\[ f(x, y + \frac{1}{2}) > 0 \]
Line drawing algorithm

(case: $0 < m \leq 1$)

\[ y = y_0 \]
\[ \text{for } x = x_0 \text{ to } x_1 \text{ do} \]
\[ \quad \text{draw}(x,y) \]
\[ \quad \text{if } \left( f(x + 1, y + \frac{1}{2}) < 0 \right) \text{ then} \]
\[ \quad y = y + 1 \]
We can make the Midpoint Algorithm more efficient

\[
y = y_0\\
\text{for } x = x_0 \text{ to } x_1 \text{ do}\\
\quad \text{draw}(x, y)\\
\quad \text{if } \left( f(x + 1, y + \frac{1}{2}) < 0 \right) \text{ then}\\
\quad \quad y = y + 1
\]
We can make the Midpoint Algorithm more efficient by making it incremental!

\[ f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0 \]

\[ f(x + 1, y) = f(x, y) + (y_0 - y_1) \]

\[ f(x + 1, y + 1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0) \]
We can make the Midpoint Algorithm more efficient

\[ f(x + 1, y + \frac{1}{2}) > 0 \]

\[ f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0 \]

\[ f(x + 1, y) = f(x, y) + (y_0 - y_1) \]

\[ f(x + 1, y + 1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0) \]
We can make the Midpoint Algorithm more efficient

\[ f(x + 1, y + \frac{1}{2}) < 0 \]

\[ f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0 \]

\[ f(x + 1, y) = f(x, y) + (y_0 - y_1) \]

\[ f(x + 1, y + 1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0) \]
We can make the Midpoint Algorithm more efficient

\[
y = y_0 \\
d = f(x_0 + 1, y_0 + 1/2) \\
\text{for } x = x_0 \text{ to } x_1 \text{ do} \\
\quad \text{draw}(x, y) \\
\quad \text{if } (d < 0) \text{ then} \\
\quad \quad y = y + 1 \\
\quad \quad d = d + (y_0 - y_1) + (x_1 - x_0) \\
\quad \text{else} \\
\quad d = d + (y_0 - y_1) \\
\]

\[f(x + 1, y) = f(x, y) + (y_0 - y_1)\]

\[f(x + 1, y + 1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)\]
Adapt Midpoint Algorithm for other cases

case: $0 < m \leq 1$
Adapt Midpoint Algorithm for other cases

case: \(-1 \leq m < 0\)
Adapt Midpoint Algorithm for other cases

case: $l \leq m$ or $m \leq -l$
Line drawing references

• the algorithm we just described is the *Midpoint Algorithm* (Pitteway, 1967), (van Aken and Novak, 1985)

• draws the same lines as the *Bresenham Line Algorithm* (Bresenham, 1965)
Triangles
barycentric coordinates
barycentric coordinates
barycentric coordinates
barycentric coordinates
barycentric coordinates
barycentric coordinates

\[ p = \alpha a + \beta b + \gamma c \]

What are \((\alpha, \beta, \gamma)\)?