

CS133 - Winter 2002 - Quiz 3 - Feb. 27, 2002

Name:

SSN (4 last digits):

You have to answer all 3 problems. Each is worth 33%. Good luck!

Only students that are registered for CS133 can take this exam.

1. Assume P is a convex polygon of n vertices, and Q is a convex polygon of m vertices.
Prove that the intersection of the two convex polygons $P \cap Q$ is a convex polygon.
What is the maximum number of vertices that the polygon $P \cap Q$ can have?
2. Let S_1 be a set of n points, S_2 be a set of m different points, $CH(S_1)$ be the convex hull of the set S_1 , and $CH(S_2)$ be the convex hull of the set S_2 . Show that, if $CH(S_1)$ and $CH(S_2)$ are known, then $CH(S_1 \cup S_2)$ can be computed in $O(n + m)$ time. (Hint: Show how you can apply the Graham's Scan convex hull algorithm to compute $CH(S_1 \cup S_2)$.)
3. Let $S = \{v_1, \dots, v_n\}$ be a set of n points, with $v_1.x < v_2.x < \dots < v_n.x$ (that is, the points are sorted in their x coordinate.) Show that the following algorithm:
 - (a) Set $S_1 = \{v_1\}$, $CH(S_1) = \{v_1\}$
 - (b) For ($i = 2$; $i \leq n$; $i++$)
 - i. Use $CH(S_1)$ to compute $CH(S_1 \cup \{v_i\})$
 - ii. Set $S_1 = S_1 \cup \{v_i\}$
 - (c) Return $CH(S_1)$

can be used to compute the convex hull of S in $O(n)$ time (you have to describe step b.i).

What is the worst case of this algorithm if the points are not sorted? Give an example.